Problem Solving with Multiple Representations

One of the biggest challenges physics students face is understanding a given problem enough to solve it. Often problems are presented in "word problem" form and one has to be fluent in translating the information given into both a diagram that helps visualize the situation and pulling out of the words the mathematics required to find a solution. This unit will focus on this issue and will give you practice on becoming fluent enough in physics to make sense of what you are trying to solve.
Lets begin with an example:

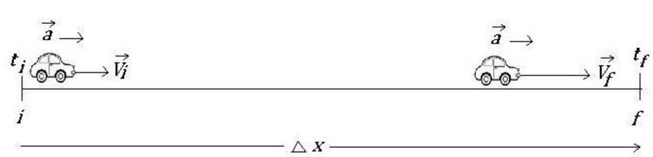
A car is traveling at 10 m/s. It accelerates for 10 seconds at a rate of 1 m/s^2. What is its final velocity?

Here is the equation that will help solve this problem:

$$\vec{v}_f = \vec{v}_i + \vec{a}\triangle t$$

This equation says: The final velocity vector of an object ($\vec{v}_f$) equals (=) its initial velocity vector ($\vec{v}_i$) plus (+) its acceleration vector ($\vec{a}$) times the change ($\triangle$) in time (t). Understanding what an equation means is crucial to being able to apply it. Math is a language. Just as you have learned the symbols of your native language (the letters and words) and the way to structure them into meaningful sentences, math also has symbols (+, -, $\div$, x_i, etc.) and is organized into sentences that have meaning (equations). In math class, you learned what many of these symbols mean and how to organize them meaningfully. Here in physics class, we will expand upon that knowledge and learn how to use the language of math to describe accurately the processes we observe in the physical world.

Every time you are presented with a physics problem, draw a diagram of the situation and label it. This will help you to "see" the situation clearly. For the above problem, we could draw a picture of the situation like this:



The point of drawing diagrams is to maximize your chances of success by expanding visually your knowledge of the problem. It is another tool in your toolbox and a well drawn diagram can be instrumental in your ability to fully understand a problem. Notice that this diagram has all of the elements of the problem, including a final velocity vector that is longer than the initial velocity vector. We know that this car accelerated, so we know that it will have a final velocity that is bigger than the initial velocity. Since it is a vector, the magnitude of the final velocity vector is drawn to be longer to accurately reflect this. The point is to draw the diagram so it makes the visualization of the problem more understandable.

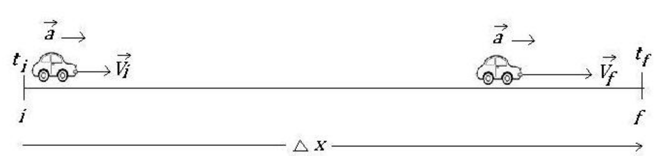
The problem is asking us to find the final velocity vector of the car ($\vec{v}_f$). What we are given is (a) its initial velocity ($\vec{v}_i = 10m/s$), (b) how long the car accelerates ($\triangle t = 10$ seconds), and (c) the rate of acceleration ($\vec{a} = 1m/s^2$). Let's take these values and plug them into our equation. Then we have:

$$\begin{aligned}\vec{v}_f &= 10m/s + (1m/s^2)(10s)\\ &= 10m/s + 10m/s\\ &= 20m/s\end{aligned}$$

So the car has a final velocity of 20 m/s.

As you can see, we have used three different ways, or representations, to help us to solve this problem. Each tool provides a different way of "seeing" the situation, and aids in both the understanding of the problem and in solving it. This is what we used:

Words	Math
A car is traveling at 10 m/s. It accelerates for 10 seconds at a rate of 1 m/s^2. What is its final velocity?	Equation: $\vec{v}_f = \vec{v}_i + \vec{a}\triangle t$ Knowns: $\vec{v}_i = 10m/s$ $\vec{a} = 1m/s^2$ $\triangle t = 10$ seconds Unknowns: $\vec{v}_f$ This was the diagram we used:



Each of these elements contribute to our understanding of the problem and aid in solving it.

Let's look at another equation:

$$\sum_{i=1}^{n} \vec{F} = m\vec{a}$$

First, translate it:

"The sum ($\sum$) over all (i=1 (the first vector) to n (the last vector)) of the force vectors ($\vec{F_i}$) is equal (=) to the mass (m) times the acceleration vector ($\vec{a}$)."

Then we can draw a picture of a problem that includes arrows indicating all of the forces acting upon an object. This should include arrows of different sizes to reflect different magnitudes, if necessary, and their directions (remember they are vectors). It is a good idea to represent the object as a point, then draw the arrows going away from this point. Once you have done this, plug what you know into the equation and solve for what you don't.

Questions:

1) Is $\sum_{i=1}^{n} \vec{F} = m\vec{a}$ just one equation? (*Hint:* Vector arrows indicate that there component vectors involved here.)

2) If the above represents more than one equation, write the equations.

3) Three ropes are tied together in a knot and you and your friends are having a three way tug-o-war. One of your friends pulls on one rope with a force of 3 Newtons due north, and another pulls on a second rope with a force of 5 Newtons due west. How hard and in what direction must you pull on the third rope to keep the knot from moving? Draw a diagram of this situation.

a) First, draw a dot representing the knot.
b) Then, draw arrows pointing in the directions given and one representing your rope.

5) A student, who is late for physics class is hurrying toward class on her scooter at 6 m/s. If she is constantly accelerating at a rate of 0.5 m/s^2 for the 120 seconds it takes to get to class, what is her final velocity?

a) Write the equation needed to solve this problem. (*Hint:* It appears somewhere in this worksheet.)

b) Draw a diagram of this situation.

c) Plug what you know into the equation in (a). First make a list of knowns and unknowns.

d) Translate part (c) into English.

e) Solve the problem.

Here is a final problem. In the third column, you are given a diagram. Write words that describe this situation in the first column. Then, write the math (equation, knowns and unknowns) in the second column. Don't worry about solving any problems, just set it up.

Words	Math	Diagram

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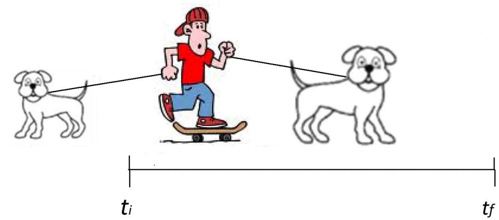
Kinematics



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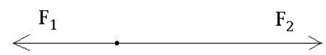
Forces



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