Kinetic Theory of Gases

Basics You Should Know

1. Assumptions for kinetic theory of ideal gas:
(i) volume of molecules negligible.
(ii) intermolecular attraction negligible.
(iii) elastic collision.
(jv) time of collision negligible compared with time between collisions

2. Pressure $p = {}^{1}/_{3}\rho c^{2}$ where c^{2} is mean square speed.
Note that root-mean square (r.m.s.) speed, $\sqrt{c^{2}}$ is different from mean speed, c.
In proving pressure formula, note that (i) force = momentum change per second at walls, (ii) $c^{2} = u^{2} + v^{2} + w^{2}$ and $u^{2} = v^{2} = w^{2} = {}^{1}/_{3}\, c^{2}$ for random motion and large number of molecules.

3. Other formulae (i) $pV = {}^{1}/_{3}Nmc^{2}$ or $p = {}^{1}/_{3}nmc^{2}$ where n is N/V, number per unit volume, (ii) $pV = {}^{1}/_{3}\, Mc^{2} = RT$ (M = molar mass) (iii) r.m.s. speed $\sqrt{(3RT/M)}$ so r.m.s. speed $\alpha \sqrt{T}$ or $1/\sqrt{M}$

4. For 1 mole of monatomic gas, kinetic energy of translation $= {}^{1}/_{2}Mc^{2} = {}^{3}/_{2}RT$=internal energy of monatomic gas.
Kinetic energy of translation of 1 molecule $= {}^{3}/_{2}kT$, where k = Boltzmann constant = R/NA = $1.38X10^{-23}$ J K^{-1} $molecule^{-1}$

5. In kinetic theory, we assume kinetic energy of translation of gas α T.
Note. Mean square speed c^{2} also printed as $< c^{2} >$

Worked Example

A vessel contains 5 g of helium, molar mass 4 g, at 27°C and 1.5×10^{5} Pa pressure. Calculate (i) the kinetic energy of the gas.
(ii) the root-mean-square speed of the molecules.
(iii) the number of molecules per unit volume present.

(Assume $R = 8.3JK^{-1}\, mol^{-1}$, $N_{A} = 6.0 X 10^{23}\, mol^{-1}$)

(i) From kinetic theory, $pV = {}^{1}/_{3}\, Nmc^{2} = RT$ for 1 mole
So kinetic energy for 1 mole = ${}^{1}/_{2}Mc^{2} = {}^{3}/_{2}RT$.
Thus energy for ${}^{5}/_{4}$ moles $= {}^{5}/_{4} \times {}^{3}/_{2} \times 8.3 \times 300 =$ **4670 J**
Since M = 4g = 0.004kg.

(ii). R.m.s speed = $\sqrt{(3RT/M)} = \sqrt{(3 \times 8.3 \times 300/0.004)} =$ **1367m/s**

(iii) From pV=nRT, n/V= p/RT

Since 1 mole has N_A particles = 6.0×10^{23}

Then the no. of molecules per unit volume = $\frac{6.0 \times 10^{23} \times 1.5 \times 10^5}{8.3 \times 300}$

$= 3.6 \times 10^{25}$

Questions:

1. List the fundamental assumptions of the kinetic theory of gases. A cube contains N molecules of a gas. On simple theory, how many molecules on the average may be assumed to move (i) along each of three perpendicular axes, (ii) in a direction towards, and perpendicular to, one particular face of the cube?

Ans: i.N/3 ii. N/6

2. What is the difference between the root-mean-square speed and the mean speed of a group of gas molecules. The r.m.s. velocity is given by the expression $\sqrt{(3p/\rho)}$, where p is the gas pressure and ρ is the density. Show that this formula is dimensionally correct.

3. Calculate the r.m.s. speed by hydrogen molecules at (i) 0°C, (ii) 27°C if the density of hydrogen is 0.09 kgm^{-3} at 0°C and 1.0×10^5 Pa.

Ans: i.1840ms^{-1} ii. 1930 ms^{-1}

4. Show how Boyle's law is explained by the kinetic theory of gases. State the assumptions made.

5. Derive an expression for the root-mean-square speed of the molecules of a gas. Calculate the root-mean-square speed of oxygen molecules at 0^0C and at 100°C, if the density of oxygen at 0°C and 1.0×10^5 Pa is 1.42 kg m^{-3}.

Ans: 460ms^{-1} 540ms^{-1}

6. (i) The r.m.s. speed of oxygen gas molecules is 500 ms^{-1} at a certain temperature and pressure. What would be the r.m.s speed of hydrogen gas molecules at the same temperature and pressure. if the molar mass of oxygen is 16 times that of hydrogen?

(ii) At what temperature will the r.m.s. speed of hydrogen gas molecules be twice that at 300 K?

(iii) At what temperature will the r.m.s. speed of nitrogen molecules equal that of oxygen molecules at 300 K, if the relative molecular masses of nitrogen and oxygen are 28 and 32 respectively?

Ans: 2000ms^{-1} 1200K 262.5K

7 Using the motion of the molecules, explain

(i) the pressure of a gas,

(ii) the increase in pressure when the volume of a gas is reduced at constant temperature,
(iii) the increase in pressure of a gas at constant volume when its temperature rises,
(iv) the rise in temperature when the air in the barrel of an air pump is compressed.

8. The mass of one mole of helium gas is 4 x 10^{-3} kg. Calculate the r.m.s. speed of its molecules at 0°C and 1x 10^{5} Pa. What is the r.m.s. speed of oxygen gas at the same temperature and pressure if the mass of one mole of oxygen is 32 x 10^{-3} kg?

Ans:1304ms^{-1}, 461ms^{-1}

9. The pressure and heat capacity of a given mass of gas depends on the mean square-speed of all its molecules, whereas its rate of diffusion through a porous partition depends on the mean speed of all its molecules.
From the kinetic theory, account for this difference. Name another property of a gas which depends on its mean speed.

10 The number of molecules per m^3 in a given gas is 1.2 x $10^{24}m^{-3}$ at 27^0C and 1.0 x 10^5 Pa pressure. Calculate the number of molecules per unit volume of this gas at a temperature of -23°C and a pressure of 2x 10^3 Pa.

Ans: 2.9 x 10^{22}

11 Derive the ideal gas equation pV = RT from kinetic theory. State the assumption made about the temperature of the gas.

12 Hydrogen gas escapes much more quickly from a sealed container than chlorine gas due to leakage through a small hole. Explain why this occurs, given that the densities of hydrogen, air and chlorine are respectively 0.09,1,2 and 3.2kgm^{-3}.

13. The speed of sound in air is given by $c = \sqrt{(1.4p/\rho)}$, where p is the pressure and ρ is the density of the air. Why is this relationship for v similar to that for the r.m.s. speed of a gas, $c_r = \sqrt{(3p/\rho)}$?

14. 2 g of helium gas, molar mass 4 g, in a container has a temperature of 47^0C and a pressure of 1.2X10^5Pa.
i. Calculate the internal energy of the gas and the number of molecules.
ii. 5g of neon gas molar mass 20g and temperature 47^0C is added to the helium container at constant volume. Find the new pressure inside the container and the new value of the internal energy.

Ans: i.1992J, 3 x10^{23}, ii.1.8 x 10^5 Pa, 2988J