Problem Set 10: Rotation
AP Physics C Supplementary Problems

1. Show that 1 rev/min = 0.1047 rad/s.

2. The angle turned through by the flywheel of a generator during a time interval t is given by
$$\theta = at + bt^3 - ct^4,$$
where a, b, and c are constants. Determine the expression for its (a) angular velocity and (b) angular acceleration.

3. The angular position of a point on the rim of a rotating wheel is described by $\theta = 4.0t - 3.0t^2 + t^3$, where θ is in radians if t is given in seconds. (a) Calculate the angular velocity at t = 2.0 s and at t = 4.0 s. (b) Calculate the average angular acceleration for the time interval that begins at t = 2.0 s and ends at t = 4.0 s. (c) Calculate the instantaneous angular acceleration at the beginning and end of this time interval.

4. A wheel rotates with an angular acceleration given by
$$\alpha = 4at^3 - 3bt^2,$$
where t is the time and a and b are constants. If the wheel has an initial angular speed ω_o, write the equations for (a) the angular speed and (b) the angle turned through as functions of time.

5. A wheel has eight spokes and a radius of 30 cm. It is mounted on a fixed axle and is spinning at 2.5 rev/s. You want to shoot a 20 cm arrow parallel to this axle and through the wheel without hitting any of the spokes. Assume that the arrow and the spokes are very thin. (a) What minimum speed must the arrow have? (b) Does it matter where between the axle and rim of the wheel you aim? If so, where is the best location?

6. The flywheel of an engine is rotating at 25 rad/s. When the engine is turned off, the flywheel decelerates at a constant rate and comes to rest after 20 s. Calculate (a) the angular acceleration (in rad/s^2) of the flywheel, (b) the angle (in rad) through which the flywheel rotates in coming to rest, and (c) the number of revolutions made by the flywheel in coming to rest.

7. A wheel has a constant angular acceleration of 3.0 rad/s^2. In a 4.0 s interval, it turns through an angle of 120 rad. Assuming the wheel started from rest, how long had it been in motion at the start of this 4.0 s interval?

8. A uniform disk rotates about a fixed axis starting from rest and accelerates with constant angular acceleration. At one time it is rotating at 10 rev/s. After undergoing 60 more complete revolutions its angular speed is 15 rev/s. Calculate a) the angular acceleration, (b) the time required to complete the 60 revolutions mentioned, (c) the time required to attain the 10 rev/s angular speed, and (d) the number of revolution from rest until the time the disk attained the 10 rev/s angular speed.

Problems selected from Halliday, D., & Resnick, R. (1993). Fundamentals of Physics (4th ed.). New York: John Wiley & Sons, Inc.

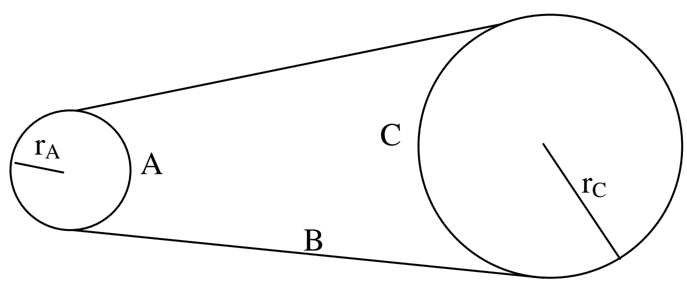
9. Calculate the angular speed of a car rounding a circular turn of radius 110 m at 50 km/h.

10. The earth's orbit about the sun is almost a circle. (a) Calculate the angular velocity of the earth (regarded as a particle) about the sun. (b) Calculate its linear speed in its orbit. (c) Calculate the acceleration of the earth with respect to the sun.

11. An astronaut is being tested in a centrifuge. The centrifuge has a radius of 10 m and, in starting, rotates according to $\theta = 0.3t^2$, where t in seconds gives θ in radians. When t = 5.0 s, calculate the astronaut's (a) angular velocity, (b) tangential velocity, (c) tangential acceleration, and (d) radial acceleration.

12. A coin of mass M is placed a distance R from the center of a phonograph turntable. The coefficient of static friction is μ_s. The angular speed of the turntable is slowly increased to a value ω_o at which time the coin slides off. Find ω_o in terms of the quantities M, R, g, and μ_s.

13. Wheel A of radius r_A = 10 cm is coupled by a belt B to wheel C of radius r_C = 25 cm. Wheel A increases its angular speed from rest at a uniform rate of 1.6 rad/s^2. Determine the time for wheel C to reach a rotational speed of 100 rev/min, assuming the belt does not slip. (Hint: If the belt does not slip, the linear speeds at the rims of the two wheels must be equal.)



14. Calculate the rotational inertia of a wheel that has a kinetic energy of 24,400 J when it is rotating at 600 rev/min.

15. The masses and coordinates of four particles are as follows: 50 g, x = 2.0 cm, y = 2.0 cm; 25 g, x = 0, y = 4.0 cm; 25 g, x = -3.0 cm, y = -3.0 cm; 30 g, x = -2.0 cm, y = 4.0 cm. Calculate the rotational inertia of this collection with respect to the (a) x, (b) y, and (c) z axes.

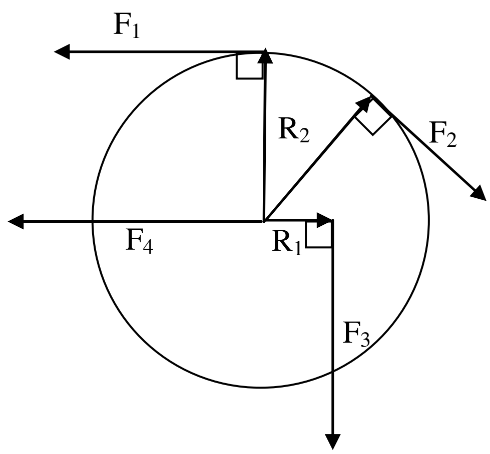
16. A communications satellite is a uniform cylinder with mass 1,200 kg, diameter 1.2 m, and length 1.7 m. Prior to launching from the shuttle cargo bay, it is set spinning at 1.5 rev/s about the cylinder axis. Calculate the satellite's (a) rotational inertia about the rotation axis and (b) rotational kinetic energy.

17. Calculate the rotational inertia of a meter stick, with mass 0.56 kg, about an axis perpendicular to the stick and located at the 20 cm mark.

18. A bicyclist of mass 70 kg puts all his weight on each downward-moving pedal as he climbs up a steep road. Take the diameter of the circle in which the pedals rotate to be 0.40 m and determine the maximum torque he exerts in the process.

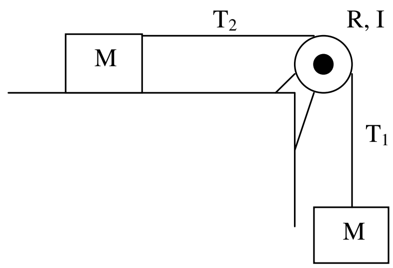
19. When a torque of 32 N·m is applied to a certain wheel, it acquires an angular acceleration of 25 rad/s^2. Calculate the rotational inertia of the wheel.

20. A cylinder having a mass of 2.0 kg rotates about an axis through its center. Forces are applied as shown below. $F_1 = 6.0$ N, $F_2 = 4.0$ N, $F_3 = 2.0$ N, $F_4 = 5.0$ N. Also, $R_1 = 5.0$ cm and $R_2 = 12$ cm. Find the magnitude and direction of the angular acceleration of the cylinder.

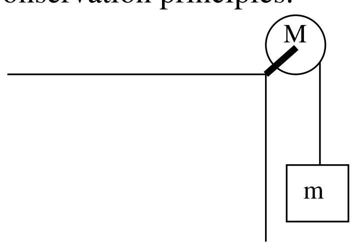


21. A pulley having a rotational inertia of 1.0×10^{-3} kg·m^2 and a radius of 10 cm is acted on by a force, applied tangentially at its rim, that varies in time as $F = 0.50t + 0.30t^2$, where F is in newtons if t is given in seconds. If the pulley was initially at rest, calculate (a) its angular acceleration and (b) it angular velocity after 3.0 s.

22. Two identical blocks, each of mass M, are connected by a light string over a frictionless pulley of radius R and rotational inertia I. The string does not slip on the pulley, and it is not known whether or not there is friction between the plane and the sliding block. When this system is released, it is found that the pulley turns through an angle θ in time t and the acceleration of the blocks is constant. (a) Calculate the angular acceleration of the pulley. (b) Calculate the acceleration of the two blocks. (c) Calculate the tensions in the upper and lower sections of the string. All answers are to be expressed in terms of M, I, R, θ, g, and t.

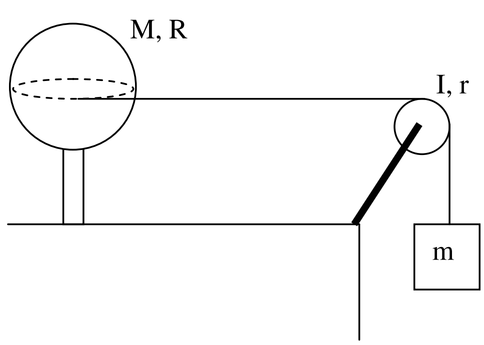


23. A falling block causes a disk to rotate. If R = 12 cm, M = 400 g, and m = 50 g, find the speed of m after it has descended 50 cm starting from rest. Solve the problem using energy-conservation principles.



24. A meter stick is held vertically with one end on the floor and is then allowed to fall. Find the speed of the other end when it hits the floor, assuming that the end on the floor does not slip. (Hint: Use conservation of energy.)

25. A uniform spherical shell rotates about a vertical axis on frictionless bearings. A light cord passes around the equator of the shell, over a light, frictionless pulley, and is attached to a small object that is otherwise free to fall under the influence of gravity. Calculate the speed of the object after is has fallen a distance h from rest. Use the work-energy theorem.



<u>Answers:</u>

1. proof
2. a) $a + 3bt^2 - 4ct^3$ b) $6bt - 12ct^2$
3. a) at 2.0 s: 4.0 rad/s; at 4.0 s: 28 rad/s b) 12 rad/s^2 c) at 2.0 s: 6 rad/s^2; at 4.0 s: 18 rad/s^2
4. a) $at^4 - bt^3 + \omega_0$ b) $\frac{1}{5}at^5 - \frac{1}{4}bt^4 + \omega_o t$
5. a) 4.0 m/s b) No, angular speed is the same regardless of r
6. a) -1.25 rad/s^2 b) 250 rad c) 39.8 rev
7. 8 s
8. a) 6.5 rad/s^2 b) 4.8 s c) 9.7 s d) 48 rev
9. 0.13 rad/s
10. a) 1.99 x 10^{-7} rad/s b) 29,886 m/s c) 0.006 m/s^2
11. a) 3.0 rad/s b) 30 m/s c) 6 m/s^2 d) 90 m/s^2
12. $\omega_o = \sqrt{\frac{\mu_s g}{R}}$
13. 16.4 s
14. 12.4 kg·m^2
15. a) 1305 g·cm^2 b) 545 g·cm^2 c) 1841 g·cm^2
16. a) 216 kg·m^2 b) 9583 J
17. 0.097 kg·m^2
18. 137 N·m
19. 1.28 kg·m^2
20. -9.72 rad/s^2, counterclockwise
21. a) 420 rad/s^2 b) 495 rad/s
22. a) $2\theta/t^2$ b) $2\theta R/t^2$ c) T_1: $M(g - 2\theta R/t^2)$ T_2: $M(g - 2\theta R/t^2) - 2I\theta/Rt^2$
23. 1.4 m/s
24. 5.4 m/s
25. $\sqrt{\frac{6r^2 mgh}{2Mr^2 + 3I + 3mr^2}}$