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AP Physics
April 23, 2012

Chapter 11: Angular Momentum

Summary: This unit builds upon the Chapter 10 analysis of rotational motion by focusing upon torque and the conservation of angular momentum. Like linear momentum, angular momentum is conserved when no external torques act on the system.

Key Equations:

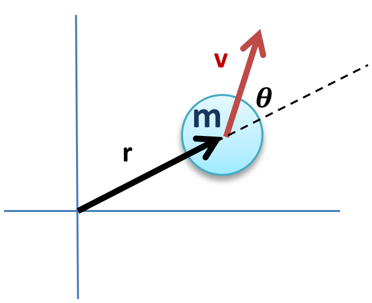
$$\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F} = \boldsymbol{I\alpha}$$

$$\boldsymbol{\tau} = \frac{\boldsymbol{dL}}{\boldsymbol{dt}}$$

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = \mathbf{I}\boldsymbol{\omega}$$

$$\mathbf{L_i} = \mathbf{L_f}$$

Diagram: setup to calculate angular momentum of a particle mass relative to the origin.



$$\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p} = \boldsymbol{rmv}\sin\boldsymbol{\theta}$$

Major Topics:

- **Cross products:** Torque and angular momentum are both evaluated using cross products. The resultant direction can be found using the right hand rule and the magnitude can be found using the sin of the angle between the two vectors. (ex: $\tau = r \times F = rF\sin\theta$)
- The cross product $\mathbf{L = r \times p}$ can be used to define the instantaneous angular momentum of any particle mass relative to another point in space.
- The equation $\mathbf{L = I}\boldsymbol{\omega}$ can be used to determine the angular momentum of a rotating object.
- **Law of Conservation of Angular Momentum:** if no external torques act on the system, the angular momentum remains constant.

Practice problems:

1. A 1.50 kg particle moves in the *xy* plane with a velocity of **v** = (4.20**i** – 3.60**j**) m/s. Determine the angular momentum of the particle when its position vector is r = (1.50**i** + 2.20**j**) m.

Solution 1:

This is a relatively simple problem that is good for practicing the calculation of a cross product.

L = r x p = r x mv
L = (1.50**i** + 2.20**j**) x 1.50(4.20**i** – 3.60**j**)
L = (1.50**i** + 2.20**j**) x (6.30**i** – 5.40**j**)

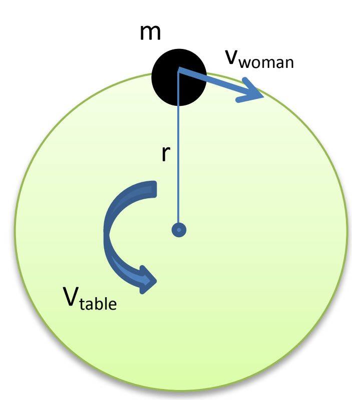
Remember that from the right hand rule and the definition of a cross product, **i x i = 0 ; j x j = 0 ; i x j = k ; j x i = –k.**

L = (–22.0 kg•m^2/s)**k**

2. A 65.0 kg woman stands at the rim of a horizontal turntable that has a moment of inertia of 400 kg • m^2 and a radius of 3.00 m. The turntable is initially at rest and is free to rotate about a frictionless, vertical axle through its center. The woman then starts walking around the rim clockwise (as viewed from above the system) at a constant speed of 2.0 m/s relative to the Earth. A) In what direction and with what angular speed does the turntable rotate? B) How much work does the woman do to set herself and the turntable into motion?

Solution 2:

First, just for clarity, draw a picture of the scenario.



Given information:

m=65.0 kg

I=400kg•m^2

r=3.00m

v=2.0 m/s

A) Recognize that this problem focuses on the law of conservation of angular momentum. So…

$$L_i = L_f$$

We know that initially, L=0 because we are told that the turntable is initially at rest. Letting the subscript *w* stand for the woman and *t* for the table, we can write:

$$0 = I_w\omega_w + I_t\omega_t$$

Using knowledge from previous chapters, we can substitute for I and ω and solve for the final ω_t. Realize that this accomplishes the same thing as just using *rmv*sin(90°) for $L_{woman.}$

$$0 = mr^2\left(\frac{v}{r}\right) + I_t\omega_t$$

$$0 = (65)(3^2)(\frac{-2}{3}) + 400\omega_t$$

$$\boxed{\omega_t = .98 \text{ rad/s counterclockwise}}$$

Note: The negative sign indicates the clockwise direction of the woman's velocity.

B) This part reviews the concept of work introduced in previous chapters

$$W = \Delta K = K_f - K_i$$
$$W = \frac{1}{2}m_w v_w^2 + \frac{1}{2}I\omega_t^2 - 0$$
$$W = \frac{1}{2}(65)2^2 + \frac{1}{2}(400).98^2$$

$$\boxed{W = 322 \text{ J}}$$

3. Two astronauts, each having a mass M, are connected by a rope of length *d* having negligible mass. They are isolated in space, orbiting their center of mass at speeds *v*. Treating the astronauts as particles, calculate A) the magnitude of the angular momentum of the system and B) the rotational energy of the system. By pulling on the rope, one of the astronauts shortens the distance between them to *d*/2. C) What is the new angular momentum of the system? D) What are the astronauts' new speeds? E) What is the new rotational energy of the system? F) How much work does the astronaut do in shortening the rope?

Solution 3:

A) Treating the astronauts as 2 point masses, we find:

L = r x p = r x mv = $(\frac{d}{2})$Mv + $(\frac{d}{2})$Mv

L = dMv

B) Here we know that the rotational energy will be the kinetic energy of the system:

K = $\frac{1}{2}$mv^2 = 2($\frac{1}{2}$Mv^2)

K=Mv^2

C) Using the law of conservation of momentum, the total angular momentum will be the same before and after the astronaut shortens the rope.

$L_i = L_f$ = Mvd

D) To find the new speeds, we set the expression for the constant L equal to the equation for angular momentum while plugging in the new r value:

L = r x mv

Mv_id = 2($\frac{d}{4}$Mv_f)

$v_f = 2v_i$

E) To find the new rotational energy, we plug in this new velocity to the kinetic energy equation:

K = $\frac{1}{2}$mv^2 = 2($\frac{1}{2}$M$(2v_i)^2$)

K = 4Mv_i^2

F) To find the work done in shortening the rope:

$W = \Delta K = K_f - K_i$

W = 4Mv^2 - Mv^2

W = 3Mv^2