Lesson Plan 5 – Rotational Kinematics

Background

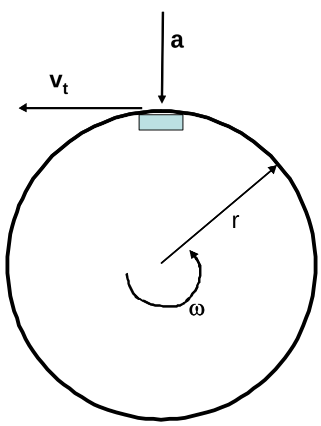
Circular Kinematics are very similar to linear kinematics, the only difference is that in circular kinematics the bodies are always rotating, like a weight at the end of a string you are swinging around. For the purposes of working with a roller coaster we will be combining the Kinematic equations with Newton's second law so that students can calculate velocity and acceleration required to keep roller coasters on their tracks. The equations that are needed for this lesson are as follows: The equation for rotational position is

$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2 \qquad [1]$$

where θ is the rotational position, ω is the rotational velocity, and α is the rotational acceleration. The equation for α is

$$\alpha = \omega^2 r \qquad [2]$$

where *r* is the radius of the circle.



It is often easier to work in linear distances and velocities, even though the object is traveling in a circular pathway. By definition, the tangential velocity of the body is

$$v_t = r\omega \qquad [3]$$

And the acceleration is

$$\alpha = \frac{v_t^2}{r} \qquad [4]$$

Centripetal force is described as

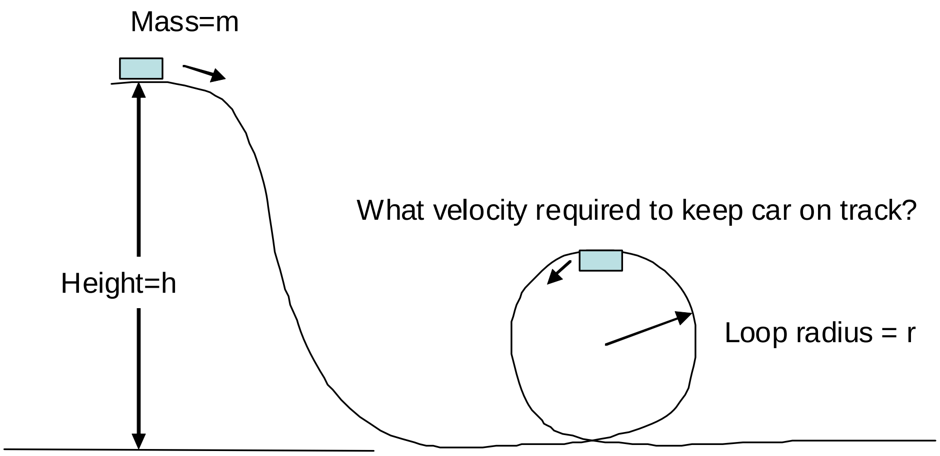
$$F_c = m\alpha = m\frac{v_t^2}{r} \qquad [5]$$

[4]

where *m* is the mass of the body.

In order to keep the car on the track, the acceleration (α) at the top of a loop needs to be greater than the acceleration of gravity. Thus, the minimum tangential velocity (v_t) of the car on the track around a loop of radius *r* can be calculated from equation [4] when $\alpha = g$:

$$v_t = \sqrt{rg} \qquad [6]$$



Theoretically, the roller coaster design that uses gravitational forces to achieve this velocity (no extra forces from a motor on the roller coaster) can be established from the energy equations and the assumption that the potential energy at the peak before the loop equals the kinetic energy plus potential energy at the top of the loop.

At start, energy = potential energy = mgh (no velocity) [7]

A top of loop, energy = PE + KE = mg(2r) + ½ mv_t^2 [8]

For a loop of radius *r* and car of mass *m*, the initial height of the starting point (*h*) can be determined by equating starting energy [7] = energy at top of loop [8], and by substituting eqn [6] for the velocity in the kinetic energy term:

$$mgh = 2mgr + \frac{1}{2}mv_t^2 = 2mgr + \frac{1}{2}mrg \qquad [9]$$

Solving for h:

$$h = 2r + \frac{r}{2} = 2.5r \qquad [10]$$

Note that this is independent of m!

Would you want to build a roller coaster with this design? This analysis assumed that the potential energy at the start was equal to the energy at the top of the loop. Thus, it is ignoring any “losses” due to friction. In reality, h would have to be higher both to account for frictional losses and to provide a margin of safety. Given what you know about friction, do you think that it really is important?

A roller coaster starting for a point H > h (to account for safety and friction) would have a velocity defined by eqn. [9] and centripetal force defined by eqn. [5]:

$$v_t = \sqrt{2gH - 4gr} \qquad [11]$$

$$F_c = m\alpha = m\frac{v_t^2}{r} = \frac{m}{r}(2gH - 4gr) \qquad [12]$$

Concepts

1. Rotational Kinematics is the study of bodies moving in a circular path.
2. Acceleration, velocity and position are all inter-related by the equations of motion
3. The rotational kinematics equations are a direct parallel to the linear kinematic equations
4. Rotational kinematics can be used to help design a roller coaster.
5. A roller coaster has to have a rotational acceleration greater than the acceleration of gravity in order to stay on the track.

Student Learning Objectives	NYS Standards
Students should be able to understand and use the kinematic equations to describe a body moving in a circular path.	
Students should be able to apply rotational kinematics and law of conservation of energy to provide an initial design of a roller coaster loop	

Key Terms

Position	Velocity	Centripetal Acceleration
Tangential Velocity		

Activities

- (5 mins) bring class to order and introduce the lesson to the students. Do demo to illustrate the importance of velocity and centripetal force and acceleration to keep a roller coaster on tracks. A vertically rotated bucket of water is best. If bucket twirled fast enough, the teacher does not end up with a wet head. Describe that our overall goal is to estimate the velocity necessary to allow a roller coaster to do vertical loops. If you are not tired of it yet, show the video again of the roller coaster, paying especial attention to the vertical loops . Key concept – centripetal force must be greater than gravitational force at the top of the loop

- (10 mins) Start lesson by putting linear and rotational kinematic equations on the board and explaining each part of the equations and the similarities between linear and rotational kinematics. Aspects of roller coaster specific equation development can be included in initial lecture.

- Do a practice problem with them.

- (20 Mins) Have students work on Kinematic equations worksheet #2 and work through the problems, teachers will circulate and help students, students may work in pairs or alone on this part of the project.
- (5mins) Wrap up lesson with what was covered focus on findings for roller coaster (#3). Draw free body diagram on board to review forces acting on the roller coaster. Note that the mass of the roller coaster not important, only the acceleration. Hand out kinematic equations HW #2 (this is just an extension of the kinematic equations worksheet)

Supply list

Kinematic Equations worksheet #2
Kinematic Equations HW #2
Example Problems to put on the board, these can be made up by the instructor as they see fit. (Have fun with the examples you use)

Resources

Kinematic Equations Worksheet #2.doc
Kinematic Equations HW#2.doc

NAME: ______________________________________

Kinematic Equations Worksheet #2

1. John Doe gets to go to the race track for a day. The track he is driving on is a perfect circle 750m in radius. If he is driving at 30m/s around the track, what is the centripetal acceleration experienced by the car, and what is the rotational velocity of the car?

2. Josh Beckett starts throwing his pitch; we assume that his arm moves in a perfect circular arc, and his arm is 0.7 m long. If he needs to release the ball with a tangential velocity of 35m/s what centripetal acceleration is required to achieve this?

3. Six Flags New England is installing a new roller coaster and they want to put in a 20m radius loop in the new coaster. How much speed must be carried into the loop in order for the centripetal acceleration to counteract gravity at the top of the loop? And what is the tangential velocity at this point?

NAME:______________________________________

Kinematic Equations HW #2

3. John Doe is cruising down the highway in his 1967 Shelbey 427 Cobra. While there he realizes that there is a stone in his tire. If the tire is 0.33 m in radius and John is traveling at 35 m/s what is the tangential velocity of the stone, and what centripetal acceleration does it experience.

4. Particle Man is riding on Tim Wakefield's curve ball which is spinning at 8 rps. Ignoring the linear velocity that Tim has imparted on the ball, what is Particle Man's tangential velocity and what is the centripetal acceleration that he experiences? The radius of a baseball is 3.64 cm.

3. A woodworker is building a set of shelves for the living room, if he makes his cuts with a DeWalt Miter Saw that spins at 5000 rpm. How fast is one tooth of the saw moving as he makes his cuts? The saw blade is .38 m in diameter.