Problems for Chapter 5 Statics

PROBLEMS FOR SECTION 5.2-5.3

5.1 Consider the sketch of the movable crane. Make a free-body diagram of the mobile crane. The wheels are 4 ft apart and the center of gravity is halfway between the wheels. Assume the ballast is an external force acting one foot behind the left axle and that the weight hanging off the crane at point B is also treated as an external force. Draw the free body diagram, label all the forces and dimensions. Be sure to include a coordinate axis.

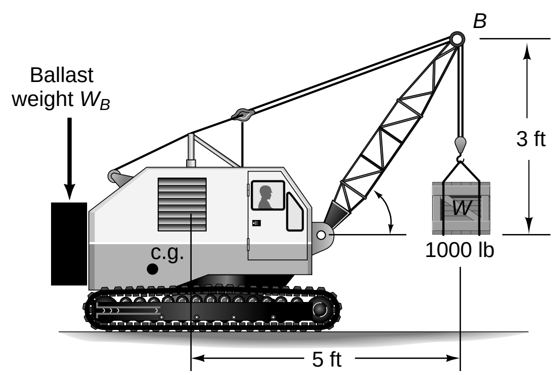

FIGURE P5.1

Solution:

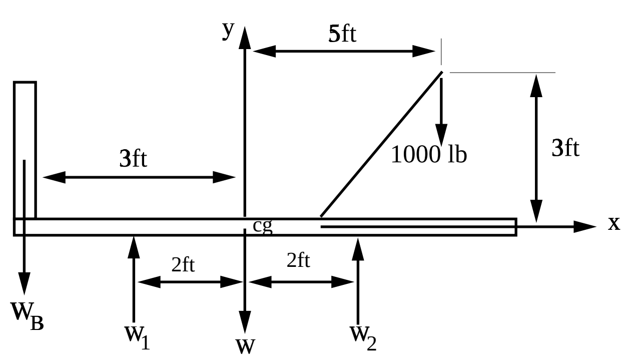

FIGURE P5.1

5.2 A small bridge on a walking path has a pin at one end, and a roller at the other. Separate the bridge from its supports and make a free body diagram. The center of gravity of the bridge is at its geometric center, midway between the supports.

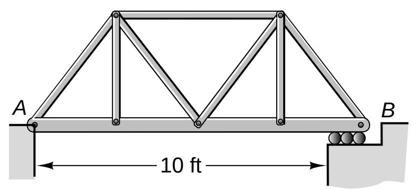

FIGURE P5.2

Solution:

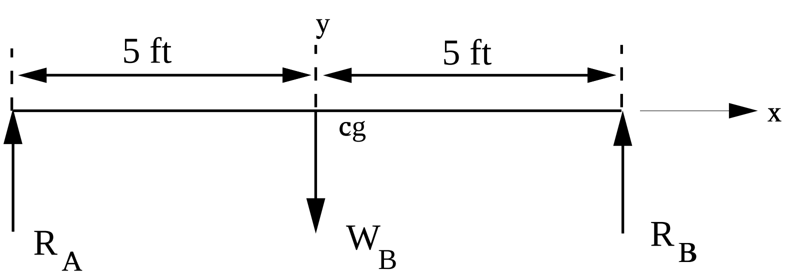

FIGURE S5.2

5.3 An adjustable shelving system consists of a rod mounted to a shaft on the left and a frictionless support on the right. Isolate the shelf brace and make a free body diagram of the system.

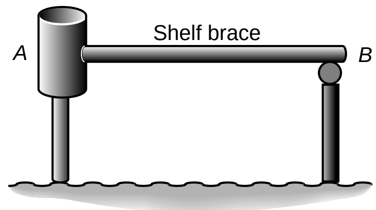

FIGURE P5.3

Solution:

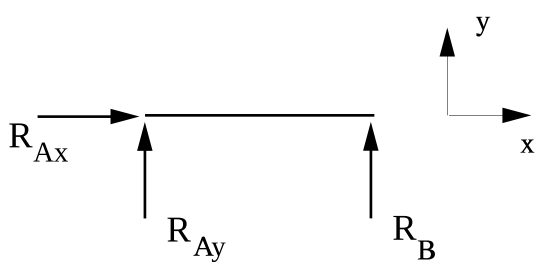

FIGURE S5.3

5.4 Draw a free body diagram of the automobile side mirror pictured, by isolating the mirror from the car body. Include the mirrors weight. The mirror forms a fixed connection with the car body.

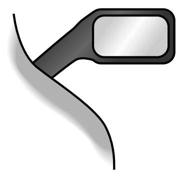

FIGURE P5.4

Solution: Treat the point of attachment to the body as fixed

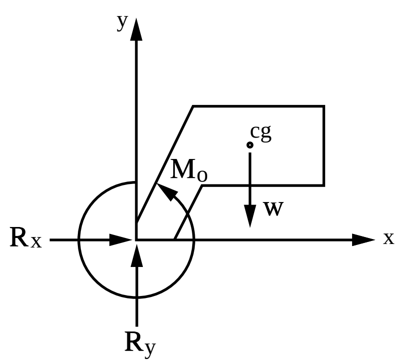

FIGURE S5.4

5.5 Make a free body diagram of the beam used to hold up the sign illustrated. The sign has a weight of 10 lb and the uniform beam weighs 5 lb.

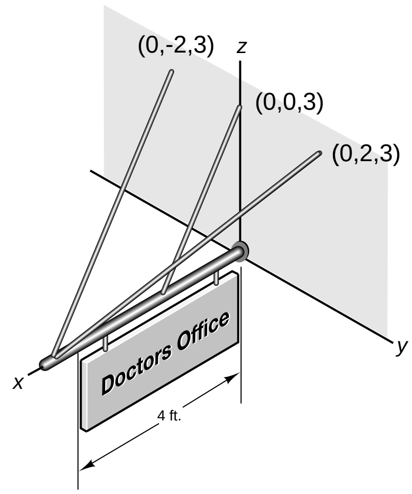

FIGURE P5.5

Solution:

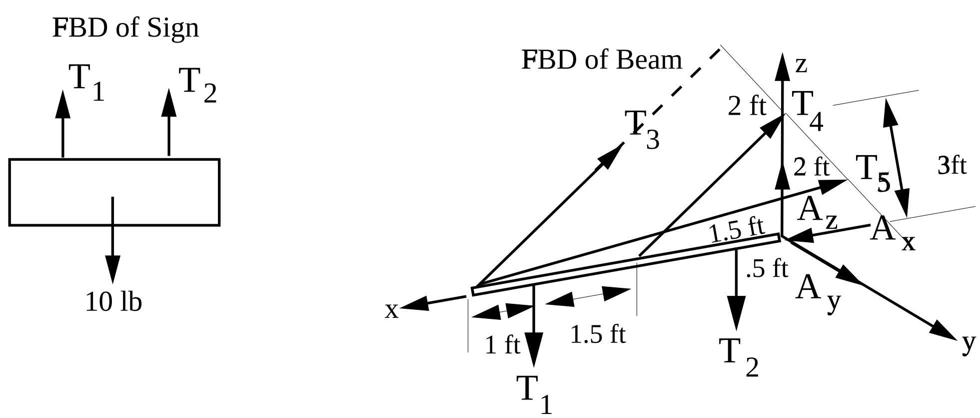

FIGURE S5.5

5.6 A uniform, 4-meter long slab is supported by a cable at A and two rigid links at points B and C. The slab has a mass of 5.1 kg. Draw a free-body diagram of the slab.

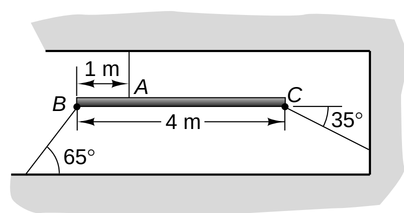

FIGURE P5.6

Solution: Because the slab is uniform its center of gravity is at the center. Encourage students to dimension the FBD's.

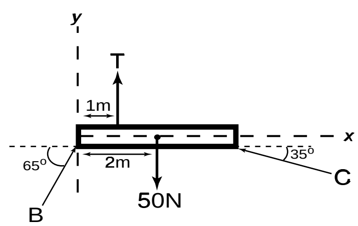

FIGURE S5.6

5.7 A utility pole is subject to a wind load F. Construct the free-body diagram.

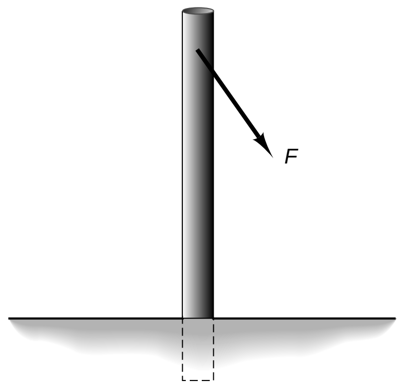

FIGURE P5.7

Solution:

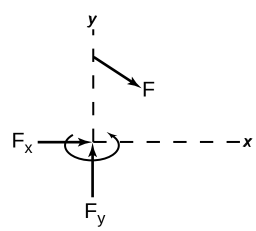

FIGURE S5.7

5.8 An old ice box is placed on a ramp having a slope of 15°. The box is .4 m × 1 m in cross section with its center of gravity at its geometric center. The box is held in place by a small rib which can be modeled as applying a force along the ramp. Draw a free body diagram of the box which has a mass of 50 kg.

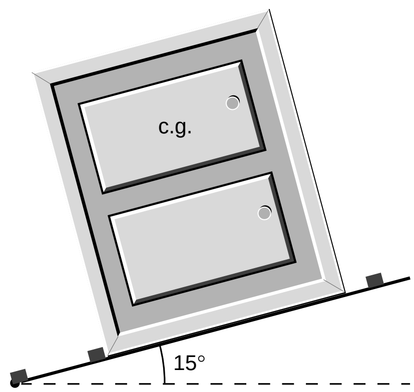

FIGURE P5.8

Solution: This is a good time to point out that the exact location of the normal force N is not known until the equation of equilibrium determines it.

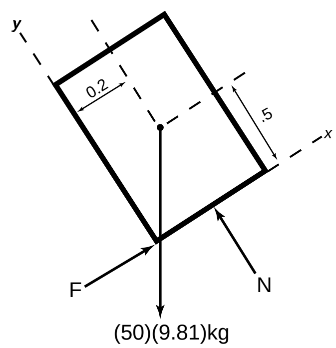

FIGURE S5.8

5.9 A 50-N bar is pinned at one end (A) and held up by a rope at B. A 100-N sign hangs by ropes as illustrated. a) Draw a free-body diagram of the sign, and b) draw a free-body diagram of the bar.

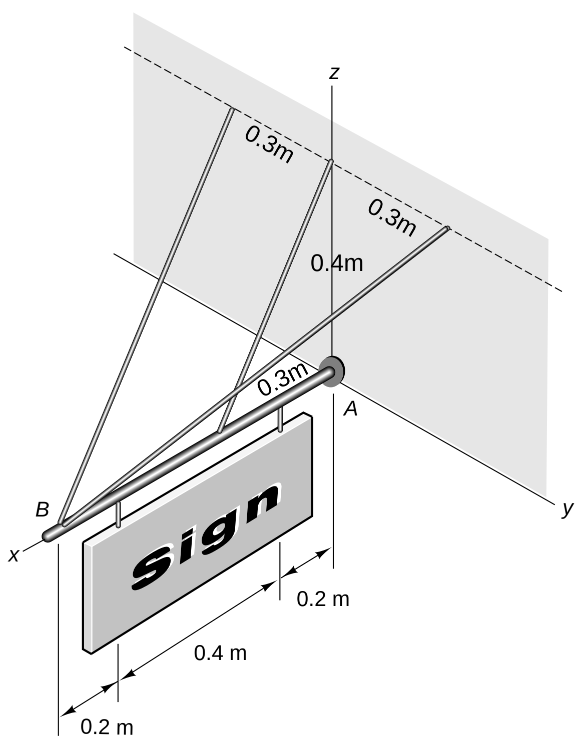

FIGURE P5.9

Solution: This is a good introduction to systems that require more than one free body to solve.

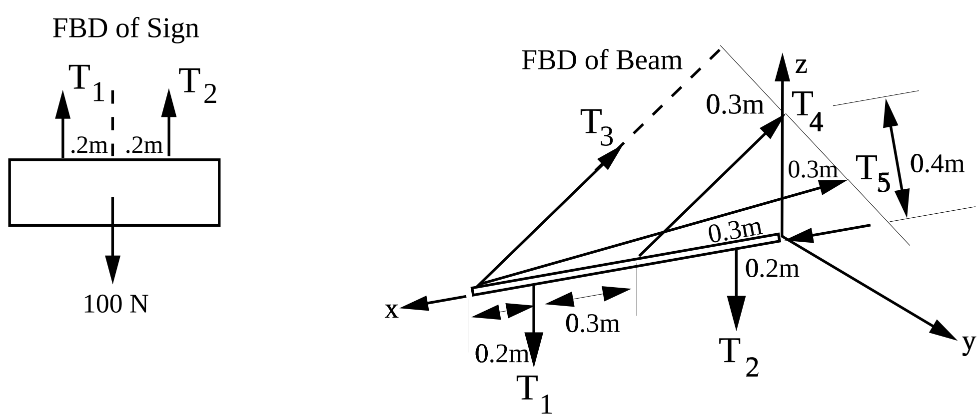
FBD of Sign
T1
T2
.2m
.2m
100 N
FBD of Beam
z
T3
0.3m
T4
0.3m
T5
0.4m
0.3m
0.2m
x
0.3m
0.2m
T1
T2
y

FIGURE S5.9

5.10 Draw a free body diagram of the lifting mechanism illustrated which has a mass of 10 kg.

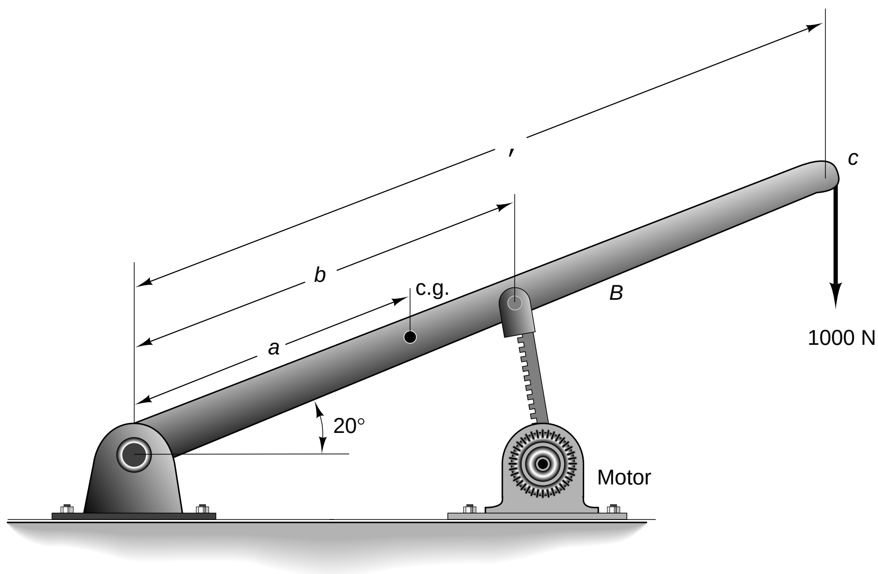

FIGURE P5.10

Solution:

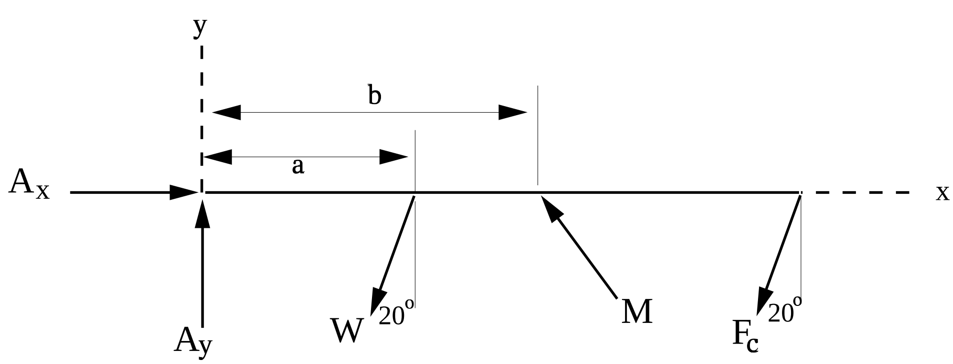

FIGURE S5.10

Problems for Sections 5.5

5.11 Two children are swinging from a 12 ft long jungle gym that weighs 50 lbs. Child C weighs 70 lb and the child at D weighs 60 lbs. Calculate the reactions for the pin at A and the roller at B.

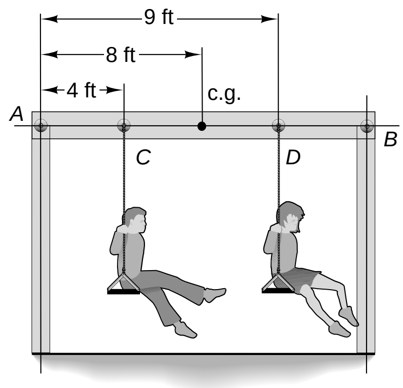

FIGURE P5.11

Solution: First construct a free body diagram of the top beam:

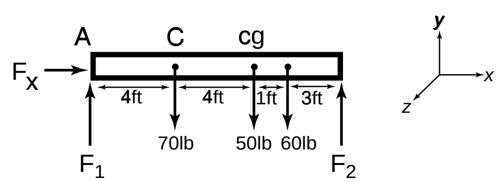

FIGURE S5.11

Note that these are all parallel forces so we expect to have only two independent equations available:

$$\Sigma M = 0, \ \Sigma F_y = 0.$$

Taking moments about A yields

$\Sigma \mathbf{M}_A = \mathbf{0} = -(4)(70)\hat{\mathbf{k}}$ lbft $-(8)(50)\hat{\mathbf{k}}$ lbft $-(9)(60)\hat{\mathbf{k}}$ lbft $+(12)F_2\hat{\mathbf{k}}$

(continued)

which is simply

$$F_2 = \frac{1}{12}(280 + 400 + 540) = \underline{101.67} = \underline{102 \text{ lb}}.$$

Next summing the forces in the y direction yields

$$\Sigma F_y = F_1 - 70 - 50 - 60 + 102 = 0$$

or

$$F_2 = 180 - 102 = \underline{78 \text{ lb}}.$$

$\blacksquare$

5.12 A small pedestrian bridge is supported by a rocker at an end (B) and a pin at the other (A). A 200-lb person is standing 4 ft out from point A. The bridge has a weight of 500 lb. Assuming the center of gravity is in the middle of the bridge, calculate the reaction at points A and B.

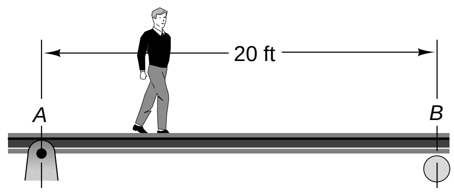

FIGURE S5.12

Solution: First draw the free body diagram. Taking moments about point A yields

$$\Sigma \mathbf{M}_A = -(10)(500)\hat{\mathbf{k}} - (4)(200)\hat{k} + 20B_y\hat{\mathbf{k}}$$

or

$$B_y = \frac{1}{20}(5000 + 800) = \underline{290 \text{ lb}}.$$

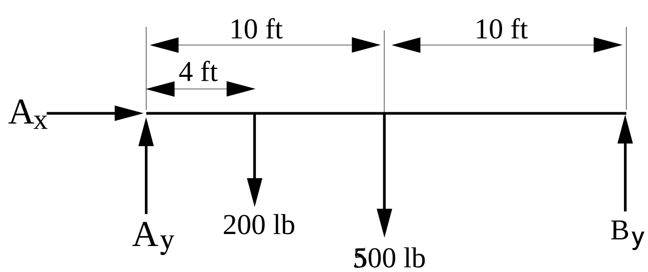

FIGURE S5.12

Next sum the forces in the y direction yields

$$\Sigma F_y = 0$$

or

$$0 = A_y - 200 - 500 + 290$$

or

$$\underline{A_y = 410 \text{ lb}}.$$

Note $Ax = 0$ since it is the only force in the x direction. ∎

5.13 Consider a deck beam of mass 45 kg with a 90 kg man standing near the edge. Calculate the reaction forces at the pin at point A and the roller at point B.

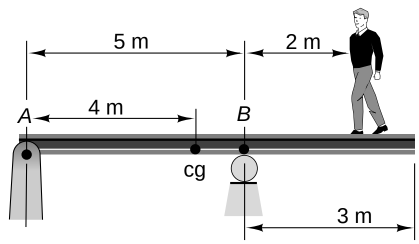

FIGURE P5.13

Solution: The free body diagram is

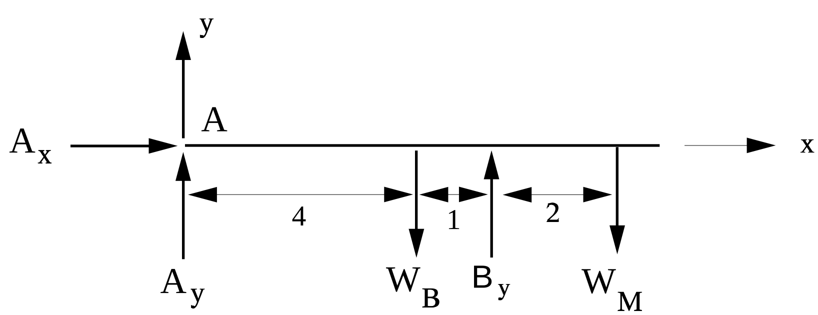

FIGURE S5.13

Taking moments about point A yields

$$\Sigma \mathbf{M}_A = \mathbf{0} = -(4)(441)\hat{\mathbf{k}} - (7)(882)\hat{\mathbf{k}} + 5B_y\hat{\mathbf{k}}$$

or

$$B_y = \underline{1588 \text{ N}}.$$

Now summing the forces in the y direction yields

$$\Sigma F_y = A_y - 441 + 1588 - 822 \text{ N}$$

or

$$\underline{A_y} = \underline{-265 \text{ N}}$$

which means that as drawn in the FBD the direction of A_y should be down along n negative y direction. Note that the only force in the x direction is A_x so that $A_x = 0$. ■

5.14 An air conditioning unit having a mass of 200 kg sits on a shelf supported by a pin and cable arrangement as illustrated. Assume the geometric center and center of gravity coincide and that the platform has a mass of 25 kg. Calculate the reactions at A and B.

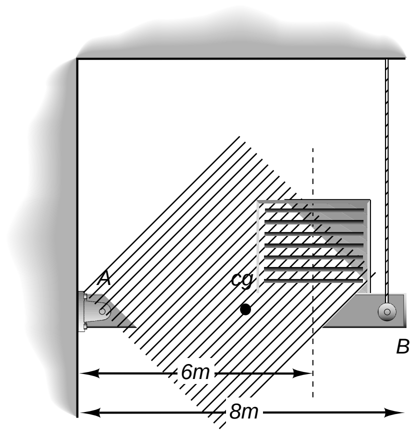

FIGURE P5.14

Solution: The free body diagram is

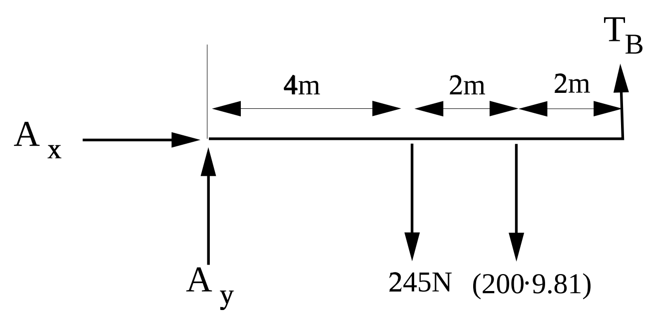

FIGURE S5.14

$$\Sigma M_A = 0 = -(4)(245)\hat{\mathbf{k}} - (6)(200)(9.81)\hat{\mathbf{k}} + (8)T\hat{\mathbf{k}}$$

or

$$T = (\frac{1}{8})(981 + 11772) = 1594.125 = \underline{1594 \text{ N}}.$$

Note T is a cable and must point up ($+y$ direction). Now

$$\Sigma F_y = 0 \text{ so that } 0 = A_y - 245 - 200(9.81) + 1594$$

or

$$A_y = 613.26 = \underline{613 \text{ N}}.$$

Again

$$\Sigma F_x = 0 \rightarrow \underline{A_x = 0}.$$

■

5.15 Consider the generic "two dimensional" automobile. In road test information provided in magazines and newspapers the c.g. is usually indicated as a percent of weight distribution, front to rear. With the wheel base given, the cars weight given and the weight distribution, calculate the distance to the front and rear tires from the center of mass for each car considered.

FIGURE P5.15

Car Make	Curb Weight	Wheel Base	Weight Distribution F/R
Lincoln Continental	3980 lb	109.0 in	62.4/37.6%
BMW 318ti	2778 lb	106.3 in	51.4/48.6
Porsche 911 Correra 4	3175 lb	89.4 in	50/50
Chevrolet Blazer LS	4218	107.0	56/44

Solution: The free body diagram for this coplane, parallel force system is

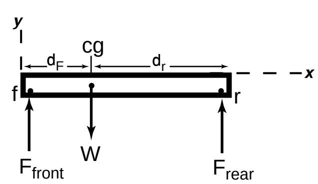

FIGURE S5.15

where

$$d = d_f + d_r = \text{ wheelbase.}$$

$$\Sigma F_y = 0$$

yields

$$F_f - W + F_r = 0$$

(continued)

or

$$F_f = W - F_r.$$

Next

$$\Sigma \mathbf{M}_f = \mathbf{0} = -d_f W \hat{k} + dF_r \hat{\mathbf{k}} = 0$$

or

$$d_f = d\left(\frac{F_r}{W}\right) = d(\%\ \text{Rear}).$$

Once d_f is known, $d_r = d - d_f$ which is tabulated in the following

Car	d_f (in)	d_r (in)
Lincoln	40.98	68.02
BMW	51.66	54.64
Porsche	49.7	44.7
Blazer	47.08	59.92

■

5.16 A sign fixture consists of a 40-kg beam pinned at point D and fixed by a cable at point A. The uniform beam is 1 m long with center of mass at its geometric center. The sign is centered and is 0.8 meters long having mass of 10 kg with center of mass at its geometric center and is held to the support beam by two cables at points B and C. Calculate the reaction at D and the tension in the cable at point A.

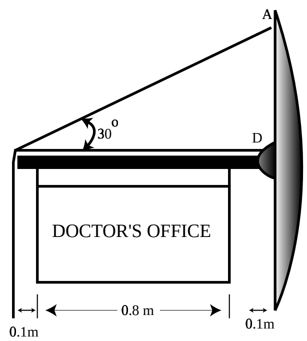

FIGURE P5.16

Solution: In this case there are two FBD's to work about and the forces are coplanar but not all parallel. Consider the sign itself. Equilibrium yields:

$$\Sigma M_B = 0 = (.8)T_C - (0.4)(98.1)$$

or

$$T_C = \frac{98.1}{2} = \underline{49.05 \text{ N}}.$$

$$\sum F_y = 0 = T_B + T_C - 98.1 = 0$$

or

$$T_B = 98.1 - 49.05 = \underline{49.05 \text{ N}}$$

which confirms the obvious.

Next consider a FBD of the beam

$$\Sigma F_x = 0 \rightarrow T_x - D_x = 0 \text{ or } T\cos 30 = D_x$$

$$\Sigma M_{pin} = -(1)T_y + (.9)(T_B) + (5)(W) + (.1)T_C \; (\hat{\mathbf{k}} \text{ component})$$

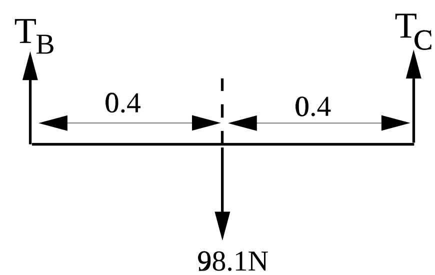

FIGURE S5.16a

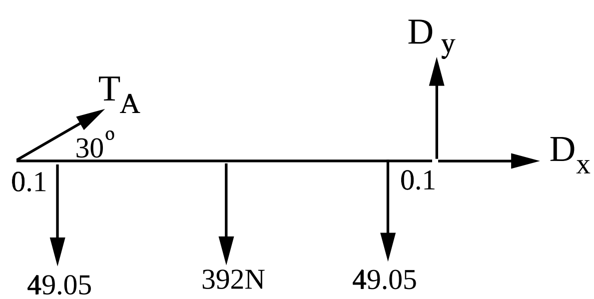

FIGURE S5.16b

or

$$T_y = T \sin 30^\circ = (.9)(49.01) + (.5)(382.4) + (.1)(99.01)$$

or

$$\underline{T = 490.42 \text{ N}} \text{ so from above } \underline{D_x = 424.72 \text{ N}}$$

Now to obtain the 3rd unknown D_y sum the force in the y direction to get

$$T_y - T_B - W - T_C + D_y = 0$$

or

$$D_y = T_C + T_B + W - T_y = 49.01 + 49.01 + 0392.4 - 245.21 = \underline{245.21 \text{ N}}.$$

∎

5.17 A man holds a 10-kg, 1 m long board at an angle of 60° by pushing with a force against the top of the board as illustrated. Calculate the force he must exert on the board and the reaction force at A.

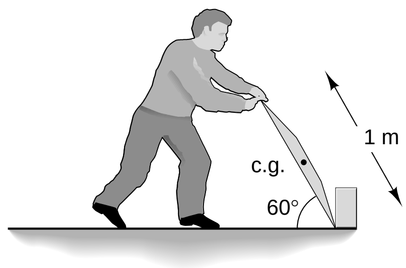

FIGURE P5.17

Solution: A FBD reveals

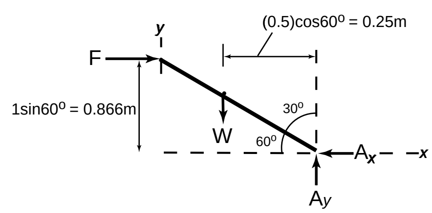

FIGURE S5.17

$\Sigma F_x = F - A_x = 0$ or $F = A_x$

$\Sigma F_y = W + A_y = 0$ or $\underline{A_y = W = 98.1 \text{ N}}$

$\Sigma M_A = -(.866)F + (.25)W = 0$ or $\underline{F} = \frac{.25}{.866}(98.1) = \underline{28.3 \text{ N} = A_x}$.

5.18 Calculate the reaction forces for the 100 kg cart illustrated in the figure. A, B, and the connections at C are all frictionless points of contact.

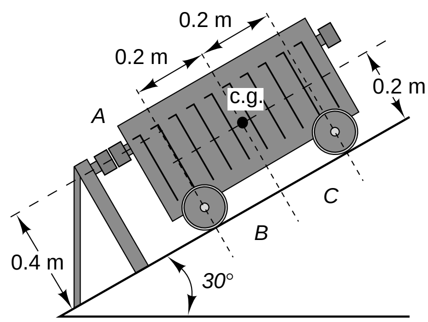

FIGURE P5.18

Solution: From the free-body diagram:

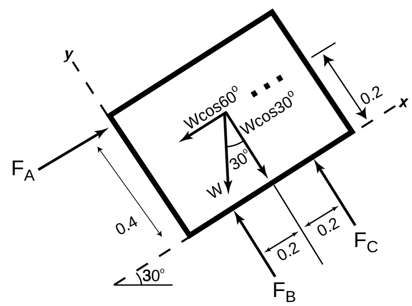

FIGURE S5.18

$$\Sigma F_x = 0 = F_A - W\cos 60^\circ = 0$$

or

$$F_A = (100)(9.81)\cos 60^\circ = \underline{491\ \text{N}}.$$

$$\Sigma M_B = 0 = -(.4)(F_A) + (.2)W\cos 60^\circ - (.2)W\cos 30^\circ + (.4)F_C = 0$$

or

$$F_C = \frac{1}{.4}[(.4)(491)+(.2)(9.81)(100)\cos 30 - .2(100)(9.81)\cos 60^\circ] = \underline{670\ \text{N}}.$$

$$\Sigma F_y = F_B + F_C - W\cos 30^\circ = 0$$

or

$$F_B = (100)(9.81)(\cos 30^\circ) - 670\ \text{N} = \underline{180\ \text{N}}.$$

5.19 A mother would like to balance her son on the teetor totter. Calculate the distance d so that the system is in equilibrium. Also calculate the reaction at point 0.

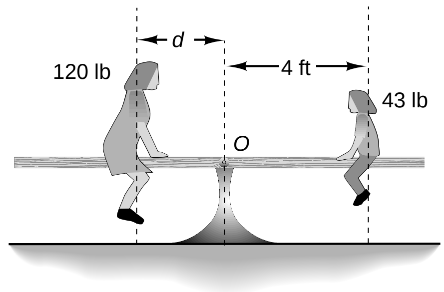

FIGURE P5.19

Solution: The appropriate free body diagram is given in S5.9. Summing the forces in the x and y direction and taking the moment about 0 yields

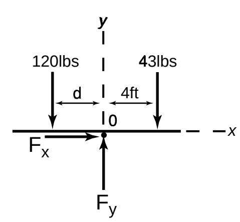

FIGURE S5.19

$$\sum \mathbf{M}_0 = \mathbf{0} = 4\hat{\mathbf{i}} \times -43\hat{\mathbf{j}} + (-d\hat{\mathbf{i}}) \times (-120\hat{\mathbf{j}}) = \mathbf{0}$$

or

$$-172\hat{\mathbf{k}} + 120d\hat{\mathbf{k}} = \mathbf{0}$$

or

$$d = \frac{172}{120} = \underline{1.4 \text{ ft}}$$

Next

$$\sum F_x = F_x + 0, \text{ so } F_x = 0, \text{ and } \sum F_y = F_y - 43 - 120$$

or

$$\underline{F_y = 163}$$

5.20 The bridge support structure has a mass of 101.94 kg with center of gravity located mid way between A and B. Calculate the reaction of the bridge supports if a 3 kN load is applied at the point indicated in the figure.

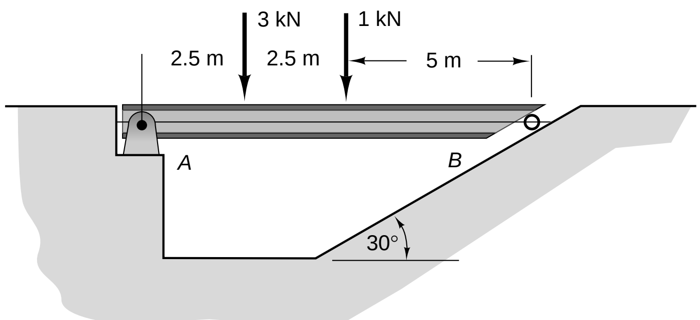

FIGURE P5.20

Solution: A free body diagram illustrating the geometry is given with the unknown reaction at B resolve into two components along the $x-y$ axis. Summing forces in the x and y directions and taking moments about the origin yields

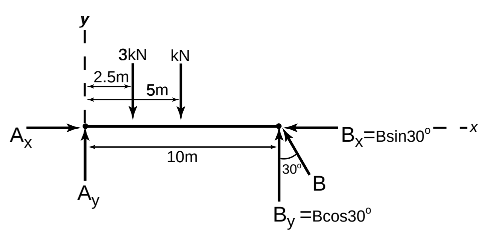

FIGURE S5.20

$$\sum F_x = 0 \text{ yields } A_x - B\sin 30° = 0$$

$$\sum F_y = 0 \text{ yields } A_y + B\cos 30° - 3kN - 1kN = 0$$

$$\sum \mathbf{M}_A = \mathbf{0} \text{ yields } 10\hat{\mathbf{i}} \times B\cos 30°\hat{\mathbf{j}} + 5\hat{\mathbf{i}} \times (-\hat{\mathbf{k}}) + 2.5\hat{\mathbf{i}} \times (-3\hat{\mathbf{k}}) = \mathbf{0}$$

or

$$A_x = B\sin 30°$$

$$A_y = -B\cos 30° + 4$$

(continued)

$$10B \cos 30^\circ = 5{+}7.5 \text{ or } B = \frac{12.5}{10\cos 30^\circ} = 1.433\text{kN},\ A_x = .722\text{kN},\ A_y = 2.75\text{kN}$$

Alternatively in matrix form the three equations of equilibrium can be written as

$$\begin{bmatrix} 1 & 0 & -\sin 30^\circ \\ 0 & 1 & \cos 30^\circ \\ 0 & 0 & 10\cos 30^\circ \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ B \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 12.5 \end{bmatrix}$$

so that

$$\begin{bmatrix} A_x \\ A_y \\ B \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin 30^\circ \\ 0 & 1 & \cos 30^\circ \\ 0 & 0 & 10\cos 30^\circ \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 4 \\ 12.5 \end{bmatrix} = \begin{bmatrix} 0.722 \\ -275 \\ 1.443 \end{bmatrix}$$

5.21 A storage door is held open by a cable. The manufacturer of the door needs to specify to the customer what type of cable must be used to hold the door open. Calculate the tension in the cable and the reactions at the hinge A for the case $\theta = 30°$. Assume that the cable is vertical at this angle and the mass of the door is 200 kg.

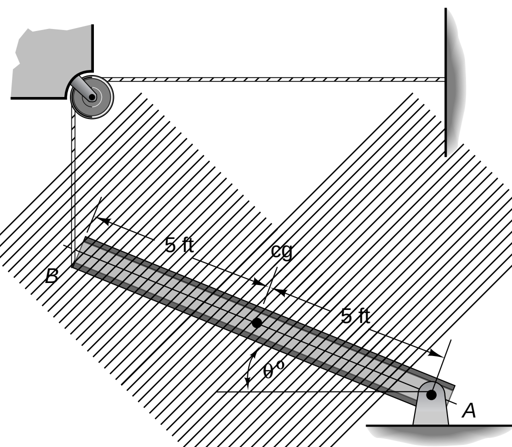

FIGURE S5.21

Solution: A free body diagram of the door is sketched below along with a coordinate system.

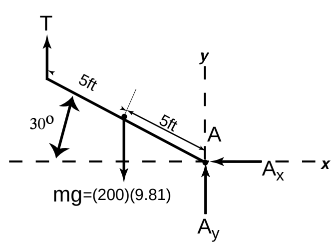

FIGURE S5.21

$$\sum F_x = 0 \text{ yields } \underline{A_x = 0}$$

$$\sum F_y = 0 \text{ yields } T - (mg) + A_y = 0$$

$$\sum \mathbf{M}_A = 0 \text{ yields } \mathbf{r}_1 \times -mg\hat{\mathbf{j}} + \mathbf{r}_2 \times T\hat{\mathbf{j}} = \mathbf{0}$$

(continued)

Here

$$\mathbf{r}_1 = -5\cos\theta\hat{\mathbf{i}}$$

and

$$\mathbf{r}_2 = -10\cos\theta\hat{\mathbf{i}},$$

so the moment equation becomes

$$5\cos\theta mg - 10\cos\theta T = 0$$

or

$$\underline{T} = \frac{mg}{2} = \underline{981 \text{ N}}$$

and

$$A_y = mg - T = (200)(9.81) - 981 = \underline{981 \text{ N}}$$

and θ doesn't matter! The manufacturer would then recommend a cable rated for tension of at least 981 N. Probably, the manufacturer would increase this recommendation by a "factor of safety" and about a 2000 N cable, just in case the assumptions are wrong, or in case someone leans on the door, etc. The next problem looks at what happens if the cable does not remain vertical.

5.22 The manufacturer of a storage bin uses a winding mechanism to open and close the door. If the uniform door weighs 100 lb, calculate the tension in the cable and the reaction force on the hinge for angles θ ranging from 5 to 40° in increments of 5°. Assume the center of mass of the door is its geometric center. This information is needed to properly design both the hinge and the cable.

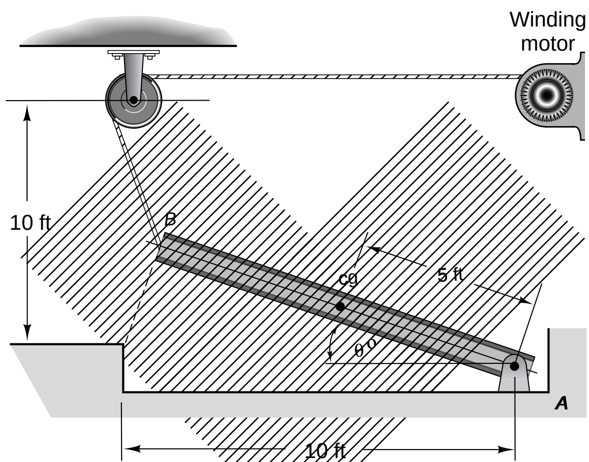

FIGURE P5.22

Solution: The first step is to make a free body diagram and to carefully note the geometry of the system. Note from the drawing that β, the angle T makes with the horizontal is a function of θ the angle the door forms with the horizontal. Specifically

$$\tan\beta = \frac{1-\sin\theta}{1-\cos\theta}$$

which can be seen from the sketch.

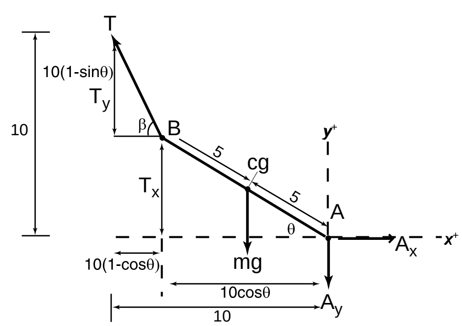

FIGURE S5.22

(continued)

The force and moment equilibrium yields (2 force equations, one moment equation)

$$\sum F_x = 0 : 0 = A_x - T\cos\beta \text{ or } A_x - (\cos\beta)T = 0 \tag{1}$$

$$\sum F_y = 0 : 0 = -mg - A_y + T\sin\beta \text{ or } -A_y + (\sin\beta)T = 100 \tag{2}$$

$$\sum \mathbf{M}_{cg} = \mathbf{0} : 0 = \mathbf{r}_1 \times (-T_x\hat{\mathbf{i}} + T_y\hat{\mathbf{j}}) + \mathbf{r}_2 \times (A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}})$$

or

$$\mathbf{0} = (-5\cos\theta\hat{\mathbf{i}} + 5\sin\theta\hat{\mathbf{j}}) \times (-T_x\hat{\mathbf{i}} + T_y\hat{\mathbf{j}}) + (5\cos\theta\hat{\mathbf{i}} - 5\sin\theta\hat{\mathbf{j}}) \times (A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}})$$

$$\mathbf{0} = (-5T_y\cos\theta + 5\sin\theta T_x + 5sin\theta A_x + 5\cos\theta A_y)\hat{\mathbf{k}}$$

Substitution of $T_x = T\cos\beta$ and $T_y = T\sin\beta$ results in

$$(-\cos\theta\sin\beta + \sin\theta\cos\beta)T - \cos\theta A_y + \sin\theta A_x = 0 \tag{3}$$

Writing equations (1), (2) and (3) as a matrix equation yields

$$\begin{pmatrix} 1 & 0 & -\cos\beta \\ 0 & -1 & \sin\beta \\ \sin\theta & -\cos\theta & \sin\theta\cos\beta - \cos\theta\sin\beta \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ T \end{pmatrix} = \begin{pmatrix} 0 \\ mg \\ 0 \end{pmatrix}$$

which can be solved by matrix inversion for each value of θ.

$$\begin{pmatrix} A_x \\ A_y \\ T \end{pmatrix} = \begin{bmatrix} 1 & 0 & -\cos\beta \\ 0 & -1 & \sin\beta \\ \sin\theta & -\cos\theta & \sin(\theta - \beta) \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ mg \\ 0 \end{bmatrix}$$

This formula can now be used to examine the effect on the tension in the reactions for varying values of the weight of the door and various angles of opening θ. Some solution for $mg = 100$lb, $\theta = 5, 10, 15, 20, 25, 30, 35, 40°$ are:

θ	5°	10°	15°	20°	25°	30°	35°	40°
A_x	.209 lb	.922	2.327	4.741	8.778	15.849	30.162	72.702
A_y	-49.982 lb	-49.837	-49.376	-48.274	-45.907	-40.849	-28.88	11.004
T	50.019 lb	50.171	50.677	51.942	54.801	61.237	77.251	132.693

Judging from this range of values, the rope must be rated to handle at least 133 lbs. and the hinge at least 75 lbs.

5.23 A truss structure is used to hold power lines as illustrated. Calculate the reaction forces at the base of the tower. Note that if the applied forces result in a net counter clockwise moment the tower will tip.

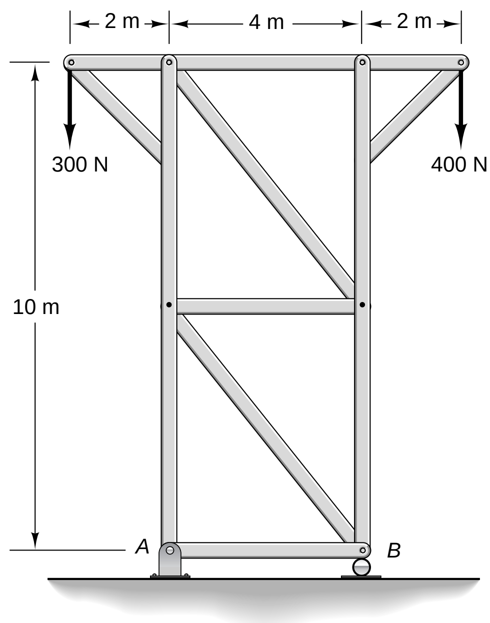

FIGURE P5.23

Solution: A free body diagram is illustrated

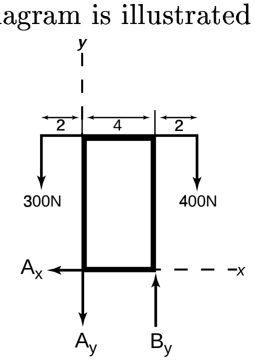

FIGURE S5.23

$$\sum F_x = \underline{-A_x = 0}, \ \sum F_y = -A_y + B_y - 700 = 0$$

$$\sum M_A = 4B_y - (6)(400) + 2(300) = 0, \ B_y = \frac{1}{4}(1800) = \underline{450\text{ N}}$$

$$\underline{A_y = -250\text{ N (up)}}$$

5.24 A truss structure is used to hold power lines as illustrated. Unlike the previous problem the power lines provide forces skewed to the tower. Calculate the reaction forces at the base of the tower.

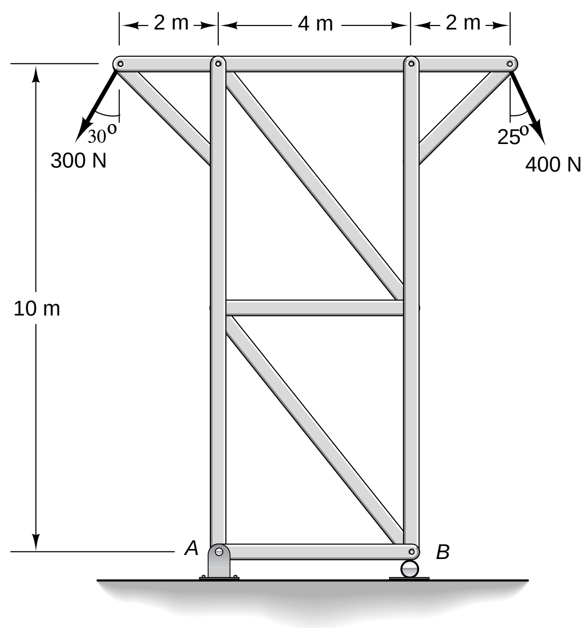

FIGURE P5.24

Solution: A free body diagram is illustrated in the figure

(1) $\sum F_x = 0$ yields $-A_x - 300\sin 30° + 400\sin 25° = 0$,

(2) $\sum F_y = 0$ yields $-A_y + B_y - 300\cos 30° - 400\cos 25° = 0$,

(3) $\sum M_A = 0$ yields $4B_y\hat{\mathbf{k}} + (6\hat{\mathbf{i}} + 10\hat{\mathbf{j}}) \times (400\sin 25\hat{\mathbf{i}} - 400\cos 25\hat{\mathbf{j}}) + (-2\hat{\mathbf{i}} + 10\hat{\mathbf{j}}) \times (-300\sin 30\hat{\mathbf{i}} - 300\cos 30\hat{\mathbf{j}}) = 4B_y - (10)(400)(\sin 25°) - (6)(400)(\cos 25) + (2)(300cos30°) + (10)(300\sin 30)$.

Thus from (1) $A_x = 400\sin 25° - 300\sin 30° = \underline{19\text{ N}}$.

From (3) $B_y = \underline{462\text{ N}}$.

From (2) $A_y = B_y - 300\cos 30° - 400\cos 25° = \underline{-161\text{ N}}$

(continued)

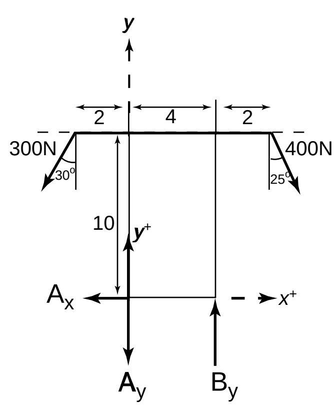

FIGURE S5.24

5.25 A light pole extends out of the ground forming a fixed support at its base. Ignore the weight of the light support. The light fixture weighs 50 lb and the pole weighs 250 lb. Calculate the reactions at A.

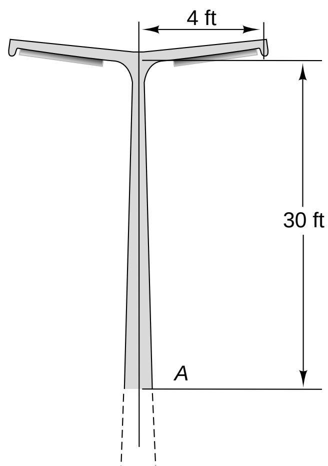

FIGURE P5.25

Solution: A free body diagram is illustrated in Figure S5.14. Summing forces and moments (about A) and writing the equilibrium equation yields

$$\sum F_x = 0 \text{ yields } A_x = 0 \text{ or } \underline{A_x = 0}$$

$$\sum F_y = 0 \text{ yields } A_y - 250 - 100 = 0 \text{ or } \underline{A_y = 350 \text{ lb}}$$

$$\sum M_A = 0 \text{ yields } M + (4) \text{ ft } (50) \text{ lbs } - 4.50 = 0, \text{ or } \underline{M = 0 \text{ lbft}}$$

or according to the coordinate system $\underline{\mathbf{M} = 0\hat{\mathbf{k}} \text{ counterclockwise}}$

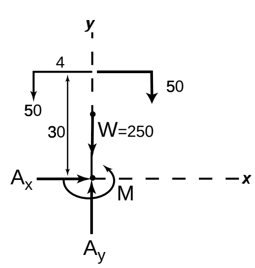

FIGURE S5.25

5.26 A one meter long bar of mass 10 kg is held up by three cables (F, T_1 and T_2). Determine the force in the three cables for $\alpha = 45°$ and $\beta = 30°$.

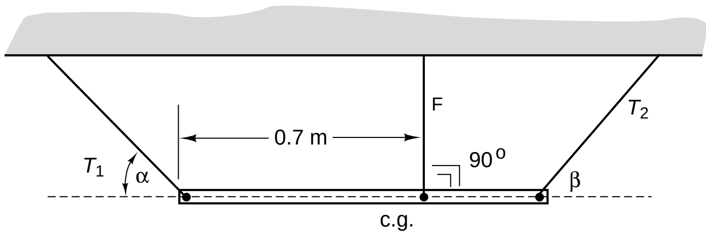

FIGURE P5.26

Solution: The free body diagram is given and results in the following equations of equilibrium

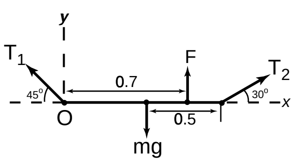

FIGURE S5.26

$\sum F = \mathbf{0}$ yields: $\sum F_x = +T_2 \cos 30 - T_1 \cos 45° = 0$ $\sum F_y = F + T_1 \sin 45° + T_2 \sin 30° - mg = 0$, $\sum \mathbf{M} = \mathbf{0}$ yields $\sum \mathbf{M}_0 = 0.7\hat{\mathbf{i}} \times F\hat{\mathbf{j}} + 1\hat{\mathbf{i}} \times T_2(\cos 30°\hat{\mathbf{i}} + \sin 30\hat{\mathbf{j}}) + .5\hat{\mathbf{i}} \times (-mg\hat{\mathbf{j}})$ or $(.7F + T_2 \sin 30 - .5mg)\hat{\mathbf{k}} = \mathbf{0}$ This yields 3 scalar equations. Writing these in matrix form yields ($mg = 98$N)

$$\begin{bmatrix} -\cos 45° & +\cos 30° & 0 \\ \sin 45° & \sin 30° & 1 \\ 0 & \sin 30° & 0.7 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ F \end{bmatrix} = \begin{bmatrix} 0 \\ 98 \\ 49 \end{bmatrix}$$

Using the matrix approach to solve these equations yields or two significant figures: $\underline{T_1 = 53\text{N}}$, $\underline{T_2 = 43\text{N}}$ and $\underline{F = 39\text{N}}$.

5.27 a) Repeat problem 5.26 for the case the the angles are $\alpha = 15°$ and $\beta = 10°$. b) Comment on the effect of changing the angle from the previous problem on the tension T_1 and T_2. From the point of view of smaller forces being better.

Solution: a) The solution is exactly as given in 5.26 where the angles are changed. The values are

$$T_1 = \underline{151},$$

$$T_2 = \underline{148\text{N}},$$

$$F = \underline{33\text{N}}.$$

b) The tension in the cables T_1 and T_2 is now <u>much larger</u>. Should give some hints to the students on design.

5.28 Suppose the cable F in problem 5.27 is moved to be directly over the center of mass and compute the magnitude of the forces in the three cables.

Solution: Again just repeat the solution of problem and change .7 to .5 to get

$$T_1 = T_2 = 0, \ \ F = 98\text{N}.$$

5.29 A uniform beam is pinned at one end and supported by a cable at the other end. The beam weighs 40 lb and is 10 ft long. Compute the reactions at the fixed end and the tension in the cable for the case $\theta = 30°$.

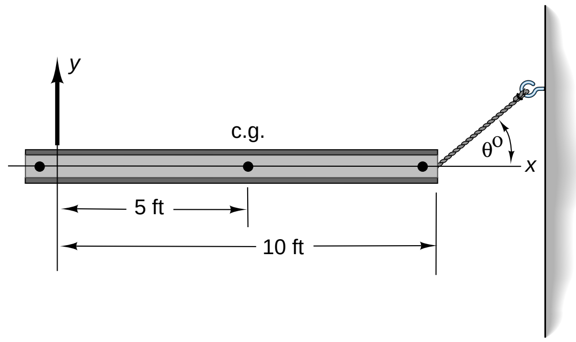

FIGURE P5.29

Solution: A free body diagram is drawn showing the center of mass at the middle. The equations of equilibrium then become

$$\sum F_x = F_x + T\cos 30° = 0$$

$$\sum F_y = T\sin 30° - mg + F_y = 0$$

$$\sum M_0 = -5(mg) + 10T\sin 30° = 0$$

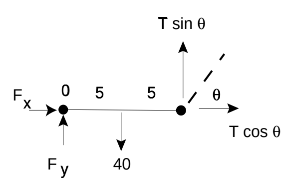

FIGURE S5.29

Solving simultaneously can be done by hand or by matrix inversion. Writing these as a matrix equation yields

$$\begin{bmatrix} 1 & 0 & \cos 30° \\ 0 & 1 & \sin 30° \\ 0 & 0 & 10\sin 30° \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ T \end{bmatrix} = \begin{bmatrix} 0 \\ 40 \\ 200 \end{bmatrix}$$

which upon multiplying by the inverse of the matrix yields

$$\underline{F_x = -34.6 \text{ lb}}, \underline{F_y = 20 \text{ lb}}, \ \underline{T = 40 \text{ lb} \cdot \text{ft}}$$

5.30 Consider the system of figure P5.29. Let the angle θ take on several values between 10° and 90°, say in increments of 10°, and compute the reaction forces and tension. Comment on what happens at $\theta = 0°$ and $\theta = 90°$.

Solution: The free body diagram and equations are given in the solution of the previous problem. The solutions using software presented in the following table

$\theta(°)$	10	20	30	40	50	60	70	80	90
F_x(lb)	113.4	55.0	34.6	23.8	16.8	11.5	7.3	3.57	0
F_y (lbs)	20	20	20	20	20	20	20	20	20
T lb-ft	115.2	58.5	40	31.1	26.1	23.1	21.3	20.3	20

These equations are also simple enough to be solved for analytically.

The solution is

$$F_x = 20 \cot \theta$$

$$F_y = 20$$

$$T = 20/ \sin \theta$$

At $\theta = 0°$, the matrix equation is singular, indicating more unknowns than equations and T cannot be determined. At $\theta = 90°$, the equation for F yields $F = 0$, because there is no component of T in the x direction, leaving F to be the only force in the x direction.

5.31 An over hanging roof for an outdoor cafe is consists of a beam 5-m long beam of mass 100 kg. Consider designing the support system. Design here refers to choosing where to attach the cable (pick ℓ) so that the tension in the cable and the reaction force at the pin are the smallest. Specifically compute values of the reaction forces for $\ell = .5,\ 1, 1.5 \ldots 5$.

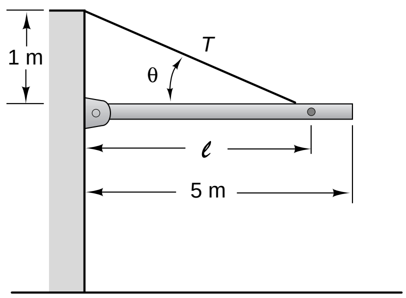

FIGURE P5.31

Solution:

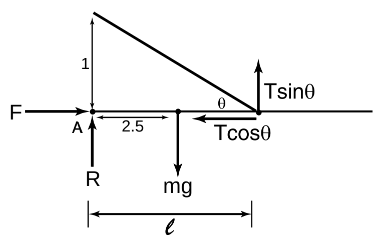

FIGURE S5.31

From the free-body diagram, the equilibrium equations are

$$(\text{mg} = (100)(9.81) = 981 \text{ N})$$

$$\sum F_x = 0: \quad F - T\cos\theta = 0$$

$$\sum F_y = 0: \quad R - mg + T\sin\theta = 0$$

$$\sum M_A = 0: (-2.5)(981) + \ell T\sin\theta = 0$$

Now ℓ and θ are related by

$$\tan\theta = \frac{1}{\ell}$$

(continued)

or

$$\theta = \tan^{-1}(\frac{1}{\ell}).$$

Writing the equations of equilibrium in matrix form yields (as a function of ℓ).

$$\begin{bmatrix} T(\ell) \\ R(\ell) \\ F(\ell) \end{bmatrix} = \begin{bmatrix} \ell \sin\theta(\ell) & 0 & 0 \\ -\cos\theta(\ell) & 0 & 1 \\ \sin\theta(\ell) & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} (2.5)(981) \\ 0 \\ 981 \end{bmatrix}$$

which has solution

$$T(L) = \frac{2453}{L\sin\theta(L)}, \quad R(L) = \frac{981L - 2453}{L}$$

$$F(L) = 2453\frac{\cos\theta(L)}{L\sin\theta(L)}$$

Let $Re = \sqrt{F^2(\ell) + R^2(\ell)}$

L	Re(**L**)	T(**L**)
0.5	932 N	55 N
1.0	957 N	35 N
1.5	965 N	29 N
2.0	969 N	27 N
2.5	969 N	26 N
3.0	972 N	26 N
3.5	974 N	25 N
4.0	975 N	25 N
4.5	976 N	25 N
5.0	976 N	25 N

The smallest tension is at $\mathbf{L} = 5$m, the smallest reaction is at $\mathbf{L} = 0.5$m.

5.32 A support is held in place by a collor on a frictionless rod and a pin. The center of mass of the 1000-kg support is at its geometric center. a) Determine the reactions at A and B if a 4000 kg mass is suspended from the support at C. b) Point C is attached to a moving track that runs between points E and F. Compute the reactions at A and B for each position of the hanging mass, starting at F, and placing it at 0.5 m increments until point E is reached.

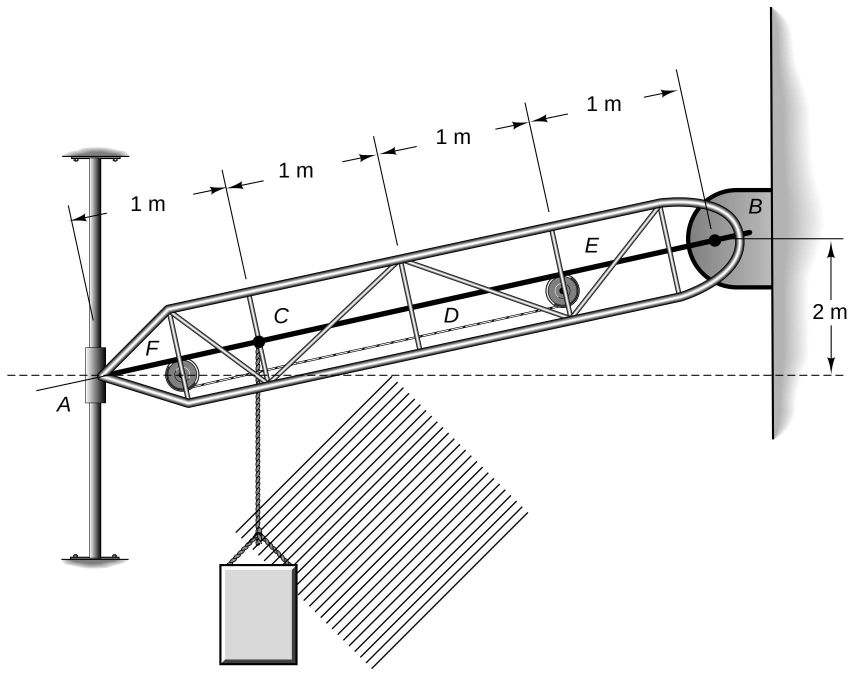

FIGURE P5.32

Solution: a) The geometry yields $\sin\theta = (2/4)$ or $\theta = 30°$. The free body diagram is given and yields 3 equations of equilibrium

$$\sum F_x = 0: \quad A - B_x = 0$$

$$\sum F_y = 0: \quad B_y - 49050 \text{ N} = 0$$

$$\sum M_A = -(39240)(\cos 30°) - (9810)(2)\cos 30° + 2B_x + 4\cos 30 B_y = 0$$

which has solution

$$\underline{A = B_x = -55,220 \text{ N}}$$

$$B_y = \underline{49050 \text{ N}}$$

b) Note from a) that B_y is unaffected by the position of the hanging mass, i.e., $\underline{B_y = 49050}$. Solving the moment equilibrium equation analytically yields

$$B_x = A = \frac{(L)(39240) + (2)(9810) - (4)(49050)\cos 30^\circ}{2}$$

L	0.5	1	1.5	2	2.5	3	3.5	4
$A(L)$	-67970	-59470	-50970	-42480	-33980	-25490	-16990	-8996

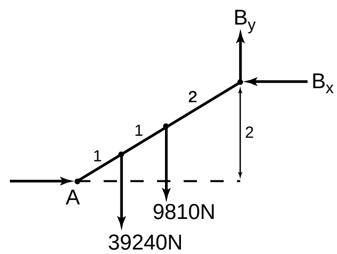

FIGURE S5.32

5.33 The lift bar (AC) of a tow truck is modeled as a pinned beam. The hoist cable (CD) is assumed to experience no friction as it goes over the top of the lift bar. Calculate the forces in the supporting cable (at B) and at the hinge of the lift bar (point A). Assume the lift bar to be 8ft in length, ignore its mass and assume that the support cable is attached two thirds of the way up the bar.

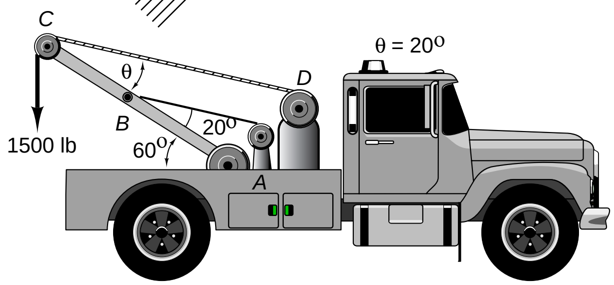

FIGURE P5.33

Solution: Construct a free body diagram of the bar with the x-axis alligned along the long axis of the bar as illustrated

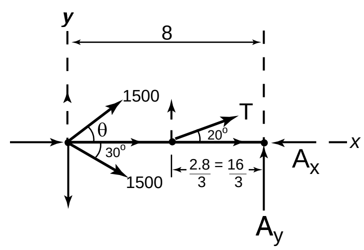

FIGURE S5.33

The equilibrium equations are

$$\sum F_x = 0: \quad 1500\cos\theta + 1500\cos 30^\circ + T\cos 20^\circ - A_x = 0 \qquad (1)$$

$$\sum F_y = 0: \quad 1500\sin\theta - 1500\sin 30^\circ + T\sin 20^\circ + A_y = 0 \qquad (2)$$

$$\sum M_A = 0: (8)(1500)\sin\theta - (8)(1500)\sin 30^\circ - \frac{16}{3}T\sin 20^\circ = 0 \qquad (3)$$

For $\theta = 20^\circ$ the solution is $\underline{T = 1039 \text{ lb}}$ $\underline{A_y = -118.5 \text{ lb}}$ (down), and $\underline{A_x = 3685 \text{ lb}}$.

5.34 Consider the tow truck mechanism illustrated in figure 5.33 and compute the reaction forces and the tension T for the case that $\theta = 30°$.

Solution: The free body diagram of figure S5.33 still holds only now $\theta = 30°$ and the solution is $\underline{T = 0 \text{ lb}}$, $\underline{A_x = 2598 \text{ lb}}$ and $\underline{A_y = 0 \text{ lb}}$.

5.35 Consider the tow truck mechanisms of figure P5.33 and calculate the value of the reaction forces and the tension in the cable for values of θ from 0 to 60° in increments of 5°. This corresponds to changing the position of the motor used to hoist an object (usually an automobile) relative to the truck frame. Such information is useful in the design of the hoist system. Is there a preferred position (θ), where the magnitude of the reaction force at A is a minimum?

Solution: The free body diagram in terms of θ is given in S5.33. The equations of equilibrium are the same. The solution for each value of θ is

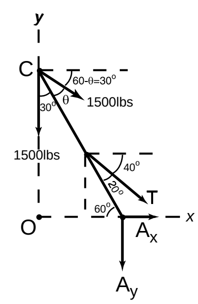

FIGURE S5.35

θ	A_x	A_y	T
0	5890 lb	-375 lb	3289 lb
5°	5345 lb	-309.6 lb	2716 lb
10°	4794 lb	-244.8 lb	2147 lb
15°	4239 lb	-180 lb	1587 lb
20°	3685 lb	-118.5 lb	1039 lb
25°	3137 lb	58.0 lb	509 lb
30°	2598 lb	0	0
35°	2073 lb	55.2	-484 lb

So for values above $\theta = 30°$, the tension in the cable changes to compression which the cable cannot provide and hence the values above $\theta = 30°$ are not viable. The minimizing value of the magnitude of the reaction force at A is at $\theta = 30°$.

5.36 A lifting mechanism consists of a uniform beam pinned at point A and positioned by a roller at point B. As hydraulic motion c moves the roller up, the point of control ℓ changes. a) Calculate the symbolic relationship for the reaction force at A and B in terms of the length ℓ, weight W, and the angle β. The center of mass is at the geometric center of the beam. b) Then evaluatetheseforcesfor the case $\beta = 30°$, $\ell = 4$ft and $W = 100$ lbs.

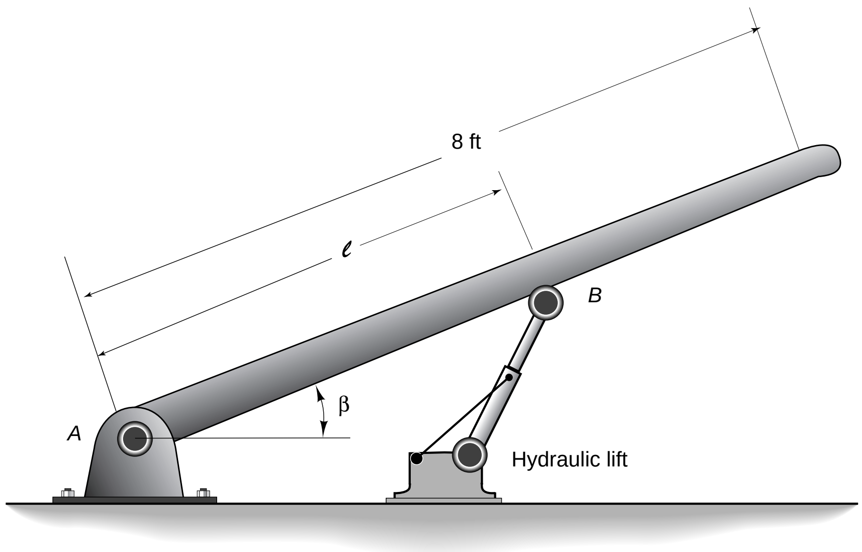

FIGURE P5.36

Solution: a) From the free body diagram:

$$\sum F_x = 0 : -B\sin\beta + A_x = 0 \tag{1}$$

$$\sum F_y = 0 : -mg + B\cos\beta + A_y = 0 \tag{2}$$

$$\sum \mathbf{M}_A = \mathbf{0} : (4\cos\beta\hat{\mathbf{i}} + 4\sin\beta\hat{\mathbf{j}}) \times (-mg\hat{\mathbf{j}}) + \ell(\cos\beta\hat{\mathbf{i}} + \sin\beta\hat{\mathbf{j}}) \times B(-\sin\beta\hat{\mathbf{i}} + \cos\beta\hat{\mathbf{j}}) = \mathbf{0}$$

or

$$(-4\text{mg}\cos\beta)\hat{\mathbf{k}} + \ell B(\cos^2\beta + \sin^2\beta)\hat{\mathbf{k}} = 0$$

or

$$\underline{B = \left(\frac{4}{\ell} mg\cos\beta\right)} \tag{3}$$

(continued)

Thus from (1)

$$\underline{A_x = B \sin\beta = \frac{4}{\ell} mg \cos\beta \sin\beta}$$

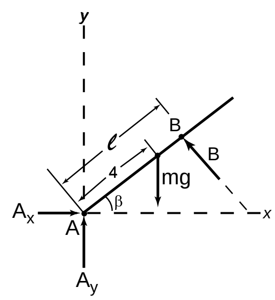

FIGURE S5.36

from (2)

$$A_y = mg - B\cos\beta = \underline{mg - \frac{4}{\ell} mg \cos^2\beta}$$

b)

$$B = \frac{4}{4}(100)\cos 30^\circ = \underline{86.6 \text{ lbs}}$$

$$A_x = (\frac{4}{4})(100)(\cos 30^\circ)(\sin 30^\circ) = B \sin 30 = \frac{1}{2}B = \underline{43.3 \text{ lbs}}$$

$$A_y = 100 - \frac{4}{4}(100)(\cos^2 30^\circ) = 100 - B\cos\beta = \underline{25.0 \text{ lbs}}$$

5.37 A lifting mechanism consists of a motor rotating a bar with a roller on its end. Calculate the reaction forces at points A and B as the angle θ changes from 0 to 90° in increments of 10 degrees. Assume $W = 100\text{lb}$ and that the center of mass of the bar is at its geometric center.

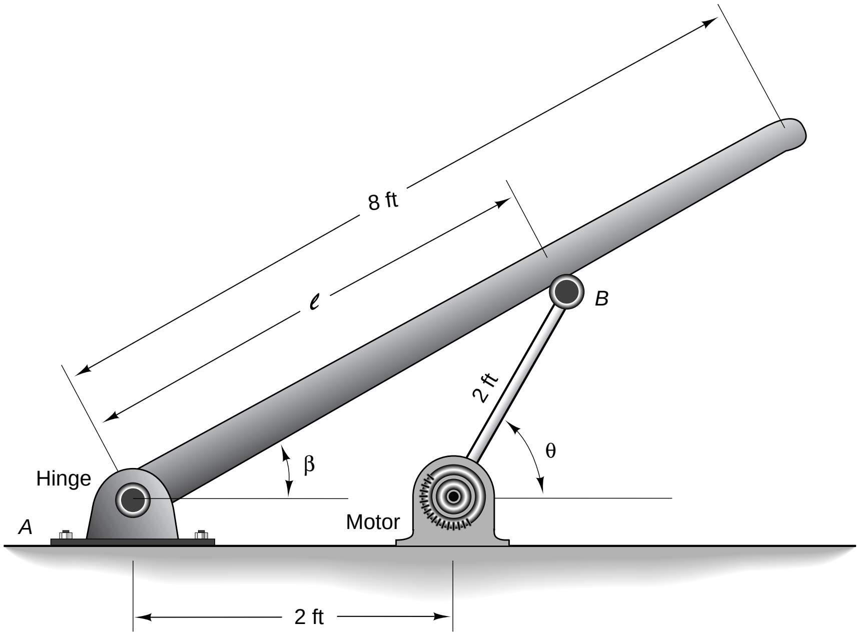

FIGURE P5.37

Solution: The free body diagram is given in figure S5.37.

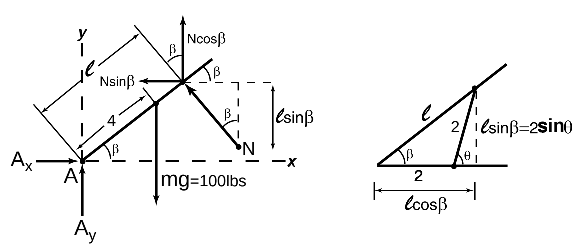

FIGURE S5.37

From the sketch of angles $\ell\cos\beta = 2 + 2\cos\theta = 2(\cos\theta + 1)$ and

$$\frac{\ell\sin\beta}{\ell\cos\beta} = \tan\beta = \frac{2\sin\theta}{2(\cos\theta + 1)}$$

(continued)

so the relationship between θ and β is

$$\beta = \tan^{-1}\frac{\sin\theta}{(\cos\theta + 1)}.$$

Also from the drawing,

$$\ell = 2\sin\theta/\sin\beta.$$

Thus for a given value of θ, both β and ℓ are determined. From the free body diagram, the equations of equlibrium are (same as 5.36)

$$\sum F_x = A_x - N\sin\beta = 0$$

$$\sum F_y = A_y + N\cos\beta - mg = 0$$

$$\sum M_A = -(4\cos\beta)mg + N\ell = 0$$

Solving yields

$$\underline{N = \frac{4mg}{\ell}\cos\beta}$$

$$\underline{A_x} = N\sin\beta = \underline{\frac{4}{\ell}mg\cos\beta\sin\beta}$$

$$\underline{A_y} = mg - N\cos\beta = \underline{mg - \frac{4}{\ell}mg\cos^2\beta}$$

When

$$mg = 100, \ell = 2\sin\theta/\sin\beta$$

and

$$\beta = \tan^{-1}\frac{2\sin\theta}{2(\cos\theta + 1)}.$$

This leads to the following values: (see computer solution section):

(continued)

θ	β	ℓ ft	A_x lbs	A_y lbs	N lbs
10°	5°	3.99	8.76	.381	100
20°	10°	3.94	17.365	1.519	100
30°	15°	3.86	25.882	3.407	100
40°	20°	3.76	34.202	6.031	100
50°	25°	3.63	42.262	9.369	100
60°	30°	3.4	50	13.397	100
70°	35°	3.28	57.358	18.085	100
80°	40°	3.06	64.279	23.396	100
90°	45°	2.83	70.711	29.289	100

5.38 A rear axle trailing arm weighs 10 lbs. The force at C is $400\hat{\mathbf{i}} + 400\hat{\mathbf{j}}$ lb, and the force at B is acting at a point 8 in from A. The center of mass is taken to be 7 in from A and the distance AC is 10 in. Compute the reactions at B and A assuming that $\theta = 5°$. Investigate the changes in the reaction forces at A and B for $\theta = 0,\ 10,\ 15°$.

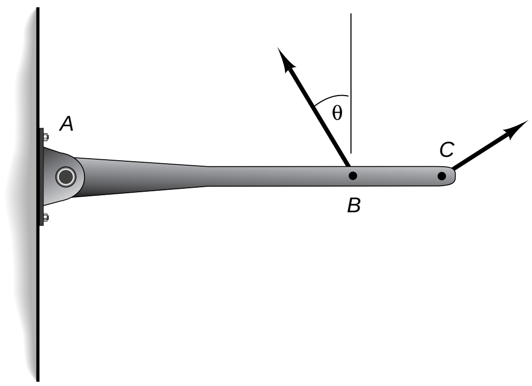

FIGURE P5.38

Solution: The free body diagram of sample 5.1 is repeated here with the given dimension labelled. Summing the forces and moment yields

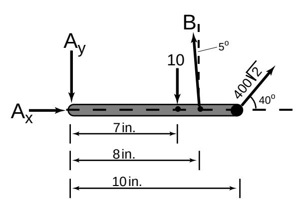

FIGURE S5.38

$$\sum F_x = 0;\ A_x - B\sin\theta + 400 = 0$$
$$\sum F_y = 0; -A_y - 10 + B\cos\theta + 400 = 0$$
$$M_A = 0; -(10)(7) + (B\cos\theta)(8) + (400)(10) = 0$$

Setting this up in matrix form yields ($A_x,\ A_y,\ B$ in lb)

$$\begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & -1 & \cos\theta \\ 0 & 0 & 8\cos\theta \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ B \end{bmatrix} = \begin{bmatrix} -400 \\ 10-400 \\ 70-4000 \end{bmatrix}$$

(continued)

Solving for $\theta = 0,\ 5,\ 10,\ 15°$ yields

$\theta°$	A_x	A_y	B
0	-400	-101	-491
5	-357	-101	-493
10	-313	-101	-499
15°	-268	-101	-509

So the change in mounting angle makes little difference in the fixed spring force but does change the reaction at A substantially.

5.39 A rock climber is held in equilibrium by a rope and her foot. Calculate the reaction forces R a force along the surface provided by the rock below the climbers body, N a force normal to the surface and T the tension in the rope for a climber with mass of 81.5 kg. Model the climber as a straight beam 1.9 m long, with cg 1.4 m up from the point of contact making an angle of 60° with the horizontal. T_1 makes an angle of 60° with the line through the climber's body modeled as being applied at a point 1 meter up from the point of contact. Explain why this configuration can be modeled as a particle.

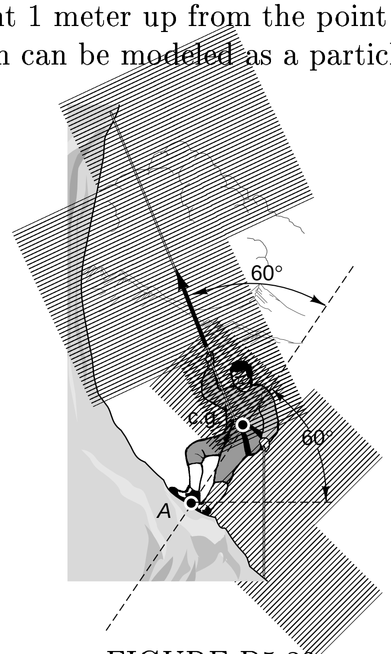

FIGURE P5.39

Solution: The free body diagram of sample 5.1 part d is repeated here with the dimensions given.

$$\sum F_x = N + T\cos 60° - 800\sin 60° = 0$$

$$\sum F_y = R + T\sin 60° - 800\cos 60° = 0$$

$$\sum M_A = T\sin 60° - (800)(1.4)\cos 60° = 0$$

Solving yields:

$$N = 369.5 \text{ N}$$

$$T = 646.6 \text{ N}$$

(continued)

$$R = -160 \text{ N}$$

The figure forms a 3 force member. Thus the forces must be concurrent or parallel. They are concurrent so it may be treated as a particle.

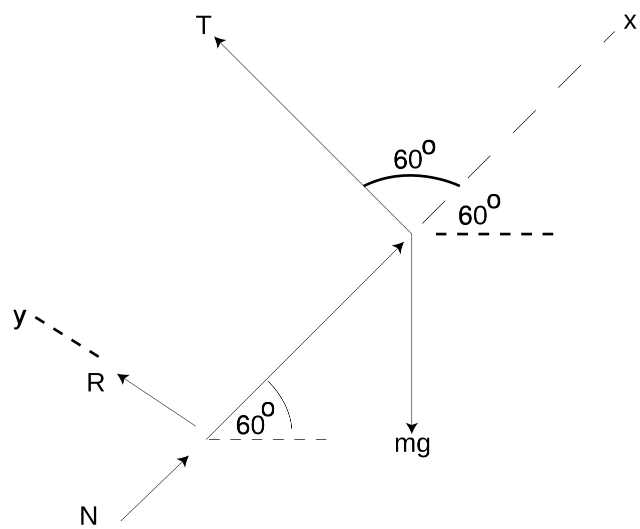

FIGURE S5.39

5.40 The tail rotor of a helicopter can be modeled as a beam fixed at the cabin. Draw a free body diagram and compute the reaction forces, in terms of the force T and the angle θ, at the point of connection of the tail section to the cabin. Assume that point A is a "fixed" connection.

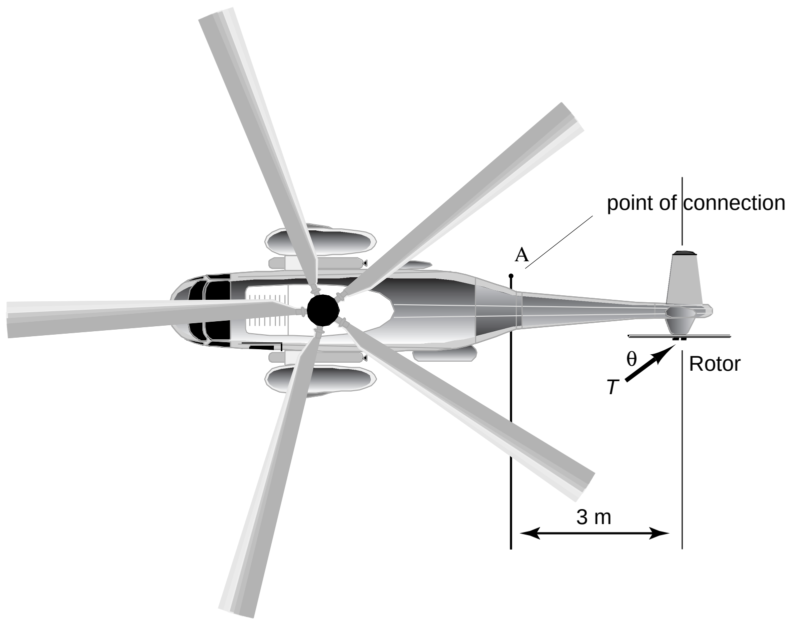

FIGURE P5.40

Solution: The free body diagram is a straight forward application of figure 5.3.7. The equations of equilibrium are

$$\sum F_x = 0 \text{ or } T\cos\theta - A_x = 0 \text{ or } \underline{A_x = T\cos\theta},$$

$$\sum F_y = 0 \text{ or } A_y + T\sin\theta = 0 \text{ or } \underline{A_y = -T\sin\theta},$$

$$\sum M_A = 0 \text{ or } M + T(\sin\theta)(3) = 0 \text{ or } \underline{M = -3T\sin\theta}$$

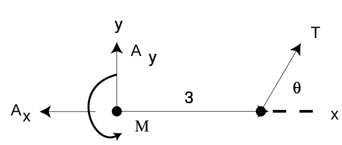

FIGURE S5.40

In problems 5.41 through 5.51 a figure, schematic or photograph is given. For each problem, construct a free-body diagram, write the equations of equilibrium and state if the system is properly constrained or not (i.e., geometrically stable) and whether it is statically determinate or statically indeterminate.

5.41 Construct a free-body diagram of a beam, 10 m long, fixed at each end, and with a load of 1000 N applied at its center of gravity.

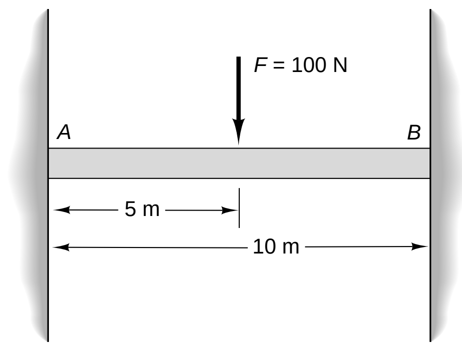

FIGURE P5.41

Solution: The free body diagram is given in Figure S5.41. The equations of equilibrium are

$$\sum F_x = 0 \text{ or } N_A - N_B = 0$$

$$\sum F_y = 0 \text{ or } R_A + R_B - 1000 = 0$$

$$\sum M_B = 0 \text{ or } M_A - M_B + (R_A)(10) - (1000)(5) = 0$$

There are 3 equations and 6 unknown reactions so that this system is statically indeterminant. The supports do constrain the motion in the $x-y$ and rotational direction so that the system is properly constrained (the forces do not form a concurrent force system nor a parallel force system).

(continued)

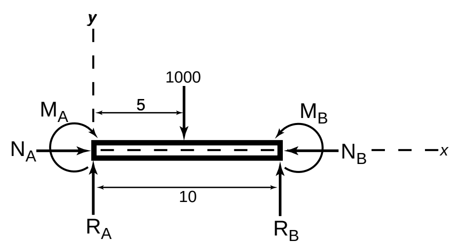

FIGURE S5.41

5.42 A radius arm for a suspension system is 18″ long, the cg is 10″ from A and that the spring is attached at 10°, 12″ from point A. The arm weighs 8 lb.

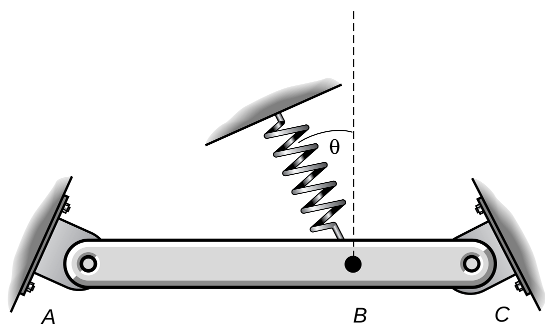

FIGURE P5.42

Solution: The free body diagram is given in the figure.

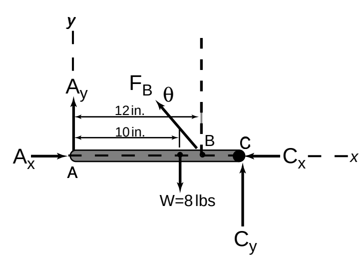

FIGURE S5.42

The equilibrium equations are

$$\sum F_x = 0 \text{ or } A_x - C_x - F_B \sin\theta = 0$$

$$\sum F_y = 0 \text{ or } A_y + C_y - 8 + F_B \cos\theta = 0$$

$$\sum M_A = 0 \text{ or } (-10)(8) + (F_B)\sin\theta(12) + (18)(C_y) = 0$$

This represents 3 equations in 5 unknowns and hence is Statically indeterminant. The forces are not concurrent, nor all parallel and motion is constrained in both the x and y directions as well as rotation so that the system is properly constrained.

5.43 A beam is supported by a pin at point A and a rope at point B. It has a mass of 25 kg and the force F at B is 140 N. a) Consider the system neglecting the weight and b) consider the system including the effects of the weight.

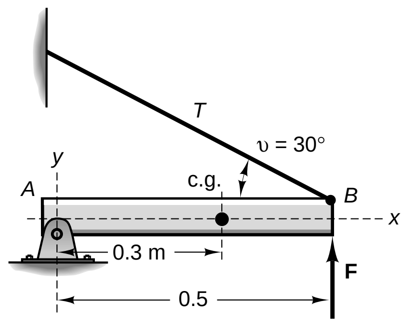

FIGURE P5.43

Solution: The free body diagram is given in figure S5.43 with the unknown force T broken up into its x and y components. a) Ignoring W, the equations of motion are

$$\sum F_x = 0 \text{ or } A_x - T\cos 30^\circ = 0$$

$$\sum F_y = 0 \text{ or } A_y + 140 + T\sin 30^\circ = 0$$

$$\sum M_A = 0 \text{ or } (T\sin 30)(.5) + (140)(.5) = 0$$

which are 3 independent equations in 3 unknowns; T, A_x and A_y hence this system are statically determinant. The system may look like it is properly constrained, but notice that solution of the moment equation yields $T = -280$N. The minus sign indicates that the rope is in compression, an impossibility for a rope, thus the system will tend to rotate up and is therefore improperly constrained b) if the weight $W = 250$N is now considered, the equations of equilibrium become

$$\sum F_x = 0 \text{ or } A_x - T\cos 30^\circ = 0$$

$$\sum F_y = 0 \text{ or } A_y - 250 + 140 + T\sin 30^\circ = 0$$

$$\sum M_A = 0 \text{ or } (-.3)(250) + (.5)[140 + T\sin 30^\circ] = 0$$

(continued)

Since we have only "added" the effect of a known force, these are still a set of 3 equations and 3 unkowns and hence the system is still statically determinant. Now note that including the weight in the moment equation renders the possibility that $T > 0$ and the system might now be statically determinant. To check this, the moment equation yields

$$T(.5) = -70 + 75 = 5 \text{ or } T = 10 \text{ N} > 0$$

and the system is now properly constrained.

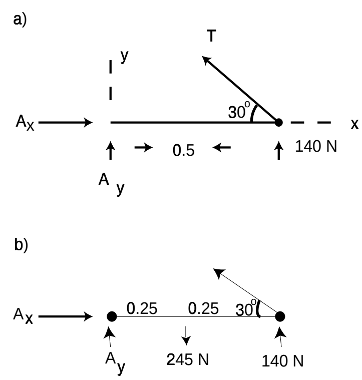

FIGURE S5.43a and b

5.44 This system consisting of a bar is held in place by a rigid massless link at C and two rollers, one at A and one at D. The mass of the bar is 100 kg and its center of gravity is at point B.

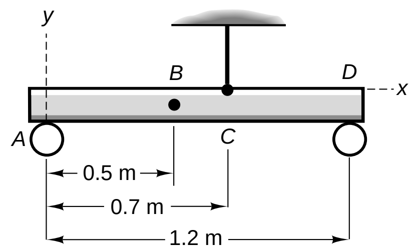

FIGURE P5.44

Solution: The free body diagram is given in figure S5.44. The equilibrium equations are

$$\sum F_x = 0, \text{ no forces in the } x\text{−direction}$$

$$\sum F_y = 0; \quad A_y + D_y + F - (100)(9.81) = 0$$

$$\sum M_A = 0; \quad (-.5)(981) + (.7)(F) + (1.2)D_y = 0$$

This is 2 equations in 3 unknowns and is thus statically indeterminant. The forces are all parallel and the motion along x is not constrained. Thus this system is improperly constrained.

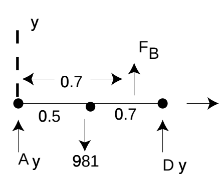

FIGURE S5.44

5.45 A shelf is held in place by a roller at A and a rope at B. The shelf has a mass of 20 kg. The center of mass is at point C.

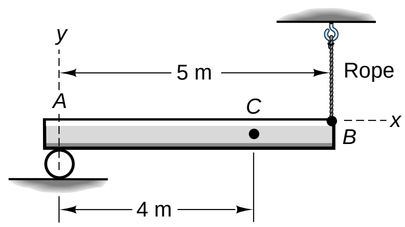

FIGURE P5.45

Solution: The free body diagram is given in figure S5.45. The equations of equilibrium are $\sum F_x = 0$; no forces in this direction.

$$\sum F_y = 0; \quad R_A - 196 + T = 0;$$

$$\sum M_A = 0; \ (-4)(196) + 5(T) = 0.$$

These are two equations in two unknowns so the system is statically determinant. The solution is $T = 156.8$ N, $R_A = 39.2$ N. However this is a parallel for system, no forces exist in the x direction to restrain motion and the system is improperly constrained.

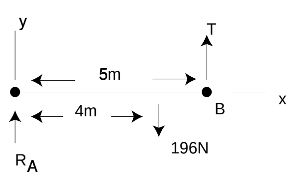

FIGURE S5.45

5.46 A 300-lb block is held in place by a pin at A and a rope at B. A 100-lb force is applied at point C. It is 2 ft by 1 ft and its center of gravity is located at its geometric center. Discuss the solution for the cases, θ not specified, $\theta = 0°$, and $\theta = 30°$ Does it matter whether or not θ is specified? Does the specific value of θ matter?

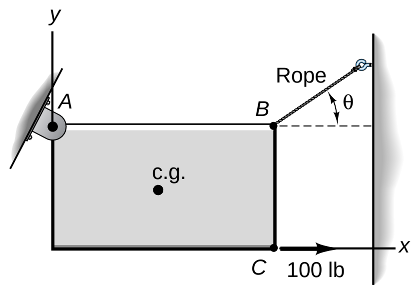

FIGURE P5.46

Solution: The free body diagram is given in figure S5.46 and gives rise to the following equations of motion:

$$\sum F_x = 0;\ \ A_x + T\cos\theta + 100 = 0$$

$$\sum F_y = 0;\ \ A_y - 300 + T\sin\theta = 0$$

$$\sum M_A = 0;\ \ (-1)(300) + (1)(100) + (2)T\sin\theta = 0$$

These are 3 equations and 3 unknowns as long as θ is specified, and the system is <u>statically determinant</u>. If θ is not specified the system of equations would contain 4 unkonwns (still only 3 equations) and a solution would not be possible. The solutions are: at $\theta = 0$, the moment equation cannot be satisfied and the system is <u>improperly constrained</u>. For the case $\theta = 30°$, the moment equation yields

$$T = \frac{100}{\sin 30°} = \underline{200\text{N}},$$

so that $A_y = 100$ N and $A_x = -273$ N and the system is <u>properly constrained</u>.

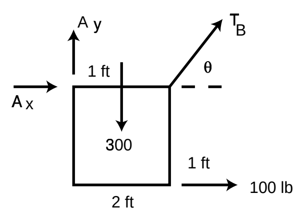

FIGURE S5.46

5.47 The prosthetic knee contains an actuator to assist movement. The actuator for the knee has the dimensions illustrated and weighs 2 lb. The connection at A is a pin while that at B is a frictionless roller.

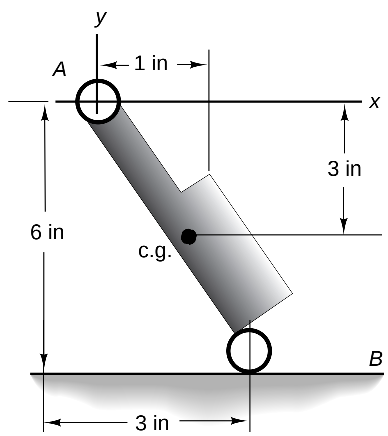

FIGURE P5.47

Solution: The free body diagram is given in figure S5.47. The equations of equilibrium become:

$$\sum F_x = 0; \ -A_x = 0$$

$$\sum F_y = 0; \ A_y - 2 + R_B = 0$$

$$\sum M_A = 0; \ (2)(R_B) - (2)(1) = 0$$

Which is a system of 3 equations in 3 unknowns so the system is statically determinant and properly constrained for the case illustrated. However if a moment of force were to be applied to the actuator, the possibility that R_B becomes negative would render the constraints improper.

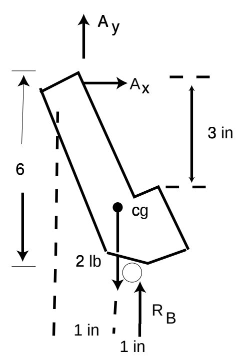

FIGURE S5.47

5.48 A steel brackett of mass 2 kg is mounted with a frictionless connection at C and a pin at A. The center of gravity is at point B. For what values of the horizontally applied force F will the system be properly constrained?

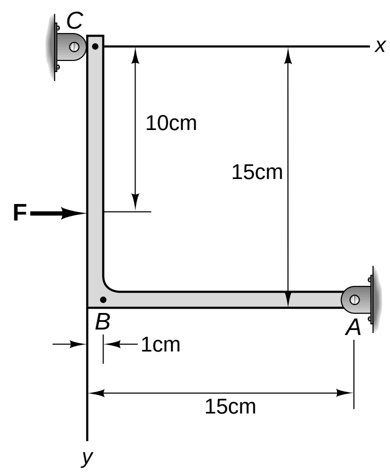

FIGURE P5.48

Solution: The free body diagram is given in figure S5.48. The equations of equilibrium are

$$\sum F_x = C - A_x + F = 0$$

$$\sum F_y = -19.6 + A_y = 0$$

$$\sum M_A = F(5 \text{ cm}) - (19.6)(14\text{cm}) + C(15\text{cm}) = 0$$

This represents a system of 3 equations in the 3 unknowns A_x, A_y, and C (considering F to be known, but not yet specified). Thus, given F the system is statically determinant. The question of proper constraints however depends on the solution of these equations. Specifically, will the clockwise moment produced by F be larger than the counter clockwise moment produced by the weight, 19.6 N? The solutions are

$$A_y = 19.6 \text{ N}$$

$$A_x = C + F$$

$$C = 18.3 - F/3$$

(continued)

The last equation reveals that the condition for a proper constraint, $C > 0$ becomes $18.3 > F/3$ or $\underline{F < 54.9\text{ N}}$. Thus as long as the applied force F remains less than 56 N and no other forces are applied, the system is properly constrained.

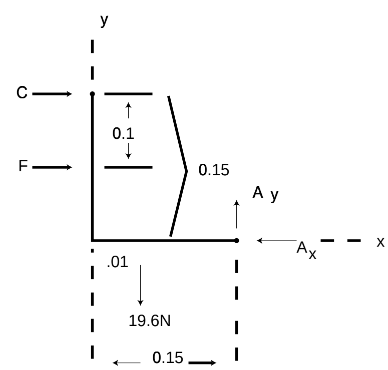

FIGURE S5.48

5.49 A pole has a moment M and force F applied to point C as the result of the wires. Consider these to be known, but not specified. The pole is held in place by a cable attached at A and fixed at B.

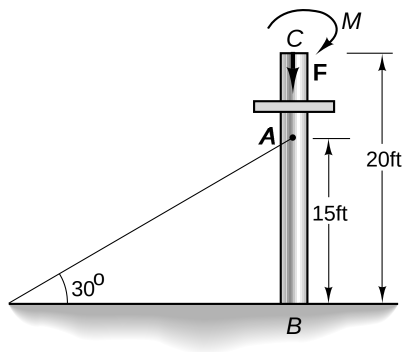

FIGURE P5.49

Solution: The free body diagram is illustrated in figure S5.49. The equations of equilibrium become:

$$\sum F_x = B_x - T \sin 60^\circ = 0$$

$$\sum F_y = B_y - T \cos 60^\circ - F = 0$$

$$\sum M_A = M_B - M + 15B_x = 0$$

With M and F known, this is a system of 3 equations in 4 unkowns: $M_B, B_x,\ B_y$ and T, so this system is statically indeterminad. The fixed connection at B constrains motion in any direction, and the system is properly constrained.

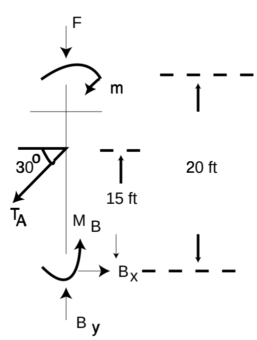

FIGURE S5.49

5.50 a) A 3500-lb sport utility vehicle is parked on a 10° incline (such as the rough surface of figure 5.8). With the parking brake set, only the rear wheels present a frictional force up the plane. b) With the driver pushing on the brake pedal both the front brake and rear wheels produce a friction force up the plane.

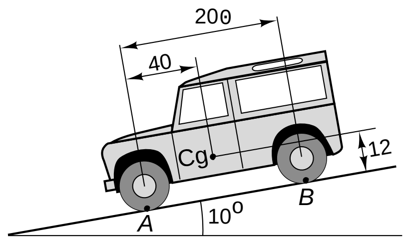

FIGURE P5.50

Solution: a) The free body for case a, with only the parking brake set, is given in figure S5.50a. The equations of equilibrium are:

$$\sum F_x = B_x - 3500 \sin 10° = 0$$

$$\sum F_y = A_y - 3500 \cos 10° + B_y = 0$$

$$\sum M_B = (3500 \cos 10°)(160 \text{ in}) - (A_y)(200 \text{ in}) + (3500) \sin 10° (12 \text{ in}) = 0$$

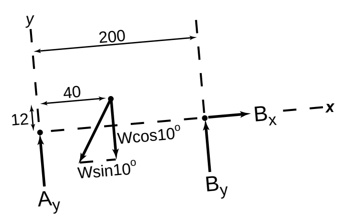

FIGURE S5.50a

This is a system of 3 equations in the 3 unknown reactions A_x, B_y and B_x. Thus this system is statically determinant. The forces are not all parallel or concurrent. With the only applied force (gravity) pushing against the ramp, the system is properly constrained.

b) The free body body diagram for the case that both wheels provide braking is illustrated in figure S5.50b. The equations of motion are now

$$\sum F_x = A_x + B_x - 3500 \sin 10° = 0$$

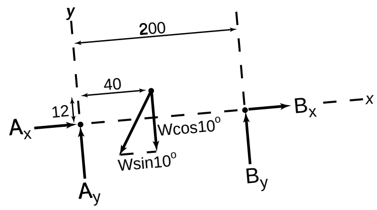

FIGURE S5.50b

$$\sum F_y = A_y + B_y - 3500\cos 10^\circ = 0$$

$$\sum M_B = (3500\cos 10^\circ)(160 \text{ in}) - (A_y)(200 \text{ in}) + (3500\sin 10^\circ)(1.2 \text{ in}) = 0$$

This is a system in 3 equations in 4 unkowns; A_x, A_y, B_x, B_y and is thus statically indeterminant. Again, however, the system remains properly constrained because the forces are neither parallel nor concourrent.

5.51 A schematic of a human arm is sketched in the figure. Here the muscle is modeled as a cable attached and point B, 3 cm from point A, a load F, unspecified but treated as known, is applied 25 cm from A and the weight is ignored. a) model the connection at A as a pin, b) with certain muscles fired, the connection at A would be considered as fixed.

FIGURE P5.51

Solution a) The free body diagram for the forearm modeled as having a pinned end is given in figure S5.51a. The equations of equilibrium are

$$\sum F_x = A_x - T\cos\theta = 0$$

$$\sum F_y = A_y - F - T\sin\theta = 0$$

$$\sum M_A = (3)(T\sin\theta) - (25)F = 0$$

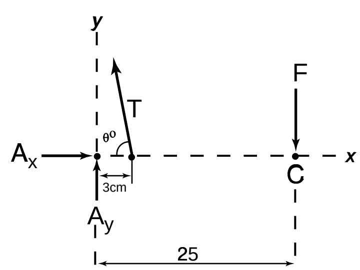

FIGURE S5.51a

This is a set of 3 equations in the three unknowns A_x, A_y and T. Hence, for a specified F this set of 3 reaction forces is satisfied and the system is statically determinant. The constraints are proper as long as F points down and improper if F points up because it is modeled as a cable which cannot provide compression.

b) For another configuration of muscles the elbow is fixed supplying an additional moment to the moment equation. The free body diagram is given in Figure S5.51b and the equations of equilibrium become

$$\sum F_x = A_x - T\cos\theta = 0$$

(continued)

$$\sum F_y = A_y - F + T\sin\theta = 0$$
$$\sum M_A = M - 25F + 3T\sin\theta = 0$$

which is now 3 equations in 4 unknowns (A_x, A_y, T and M). Thus these reaction forces cannot be determined and the system is statically indeterminant. A fixed connection constrains all 3 directions so this system is properly constrained.

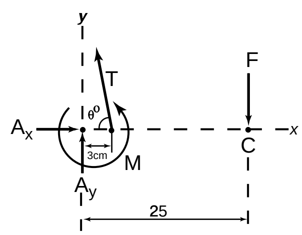

FIGURE S5.51b

5.52 Construct a free-body diagram of the hanging walkway. The two men weigh 180 lb each which can be modeled as acting at points A and B. The weight of the walkway is 350 lb acting at the geometric center of the floor. Is this system properly constrained? Is it statically determinate?

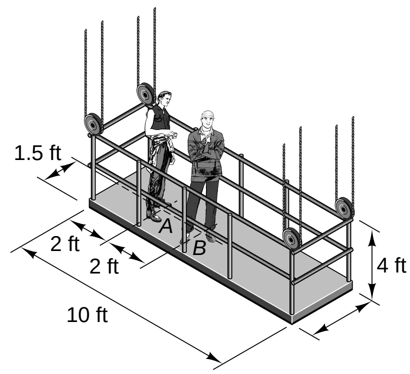

FIGURE P5.52

Solution:

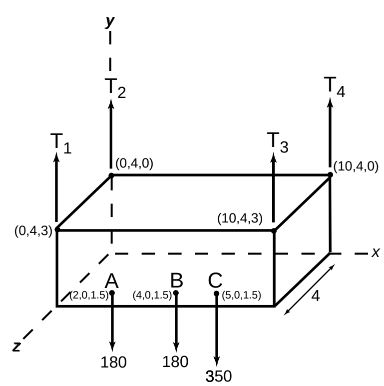

FIGURE S5.52

The free body diagram is illustrated. There are 4 unknown forces which are labeled T_1, T_2, T_3 and T_4 each of the cable type. The system is improperly constrained, as all the reactive forces are parallel, so that nothing resists motion in the x or z directions. However if no external forces act on the system with components in these directions

(continued)

(such as a strong wind) the system will remain in equilibrium for the forces shown. The system is also statically indeterminant because there are 4 unknowns (T_i) and only 3 equations of equilibrium ($\sum F_y = 0$ and $\sum M_z = 0$, $\sum M_x = 0$). The other 3 equations of equilibrium ($\sum M_y = 0$, $\sum F_x = 0$, $\sum F_z = 0$) do not contain any of the unknowns, so there are not enough equations to determine the support forces.

5.53 A 100-lb light pole is fixed at its base. The light fixture is modeled as a 50-lb weight acting at the extremity of the support arm. Draw a free-body diagram of the light pole system. Is this system properly constrained? Is it staically determinate?

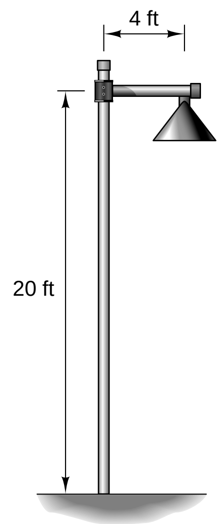

FIGURE P5.53

Solution:

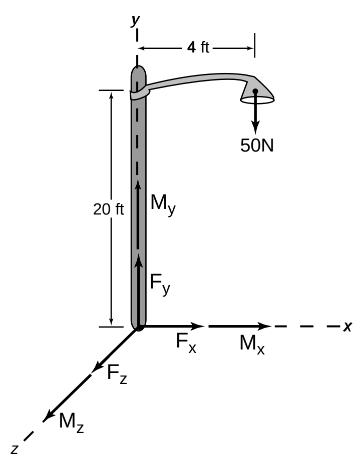

FIGURE S5.53

There are 6 unknown components, three forces and three moments as illustrated. The system is thus properly constrained as motion in all 3 translations and all three rotations is resisted. There are six equations and six unknowns, so this system is statically determinant.

5.54 A crane is used to lift a pile of logs with center of mass at point D. Make a free-body diagram of the logs and discuss whether or not the system is statically determinate and/or properly constrained.

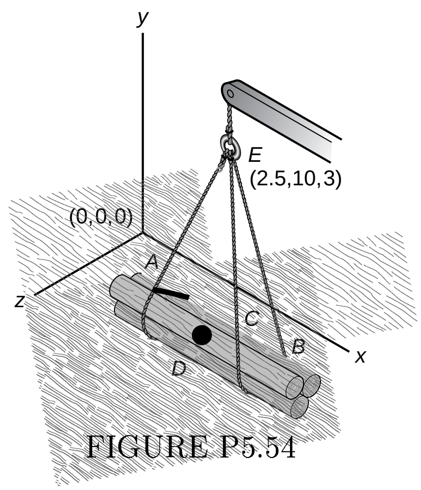

FIGURE P5.54

Solution:

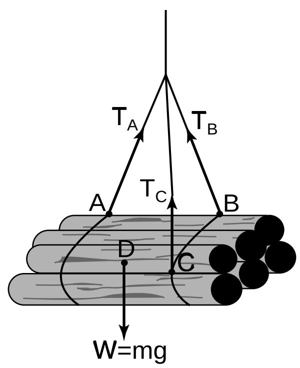

FIGURE S5.54

There are 3 unknown scalar forces T_A, T_B, and T_C, provided the geometry is given. Thus there will be 3 unknowns and potentially six equations. However, the 3 forces pass through a common point so the moment equations yield no information. Thus there are 3 equations in 3 unknowns and the system is <u>statically determinant</u> but <u>improperly constrained</u> as rotation could occur about the point of intersection of the three forces.

5.55 A drum hopper, used to sort parts is supported at A by a bearing offering no thrust resistance and a frictionless support at B providing a restraint perpendicular to the shaft (like a roller) but no restraint of moments. Construct the free body diagram with C indicating the center of gravity of the shaft drum system. State whether this system is statically determinate or not and whether or not it is properly constrained.

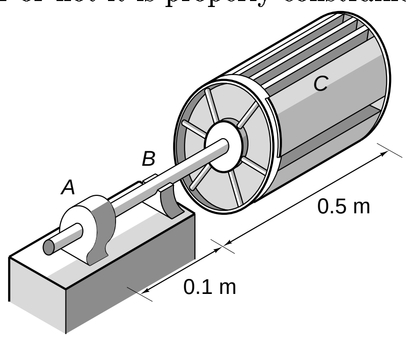

FIGURE P5.55

Solution:

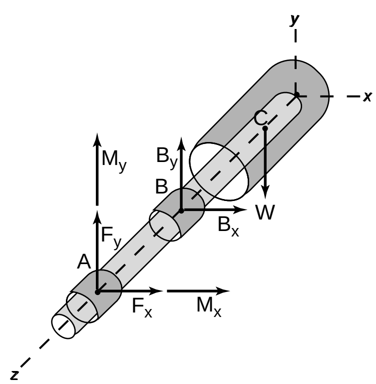

FIGURE S5.55

The free body diagram reveals 6 unknown forces and moments (F_y, F_x, M_y, M_x, B_y and B_x). There are however only 2 force equations ($\sum F_z = 0$) and two moment equations because all the forces act on the same line (and $M_z = 0$ yields no information). Thus there are only 4 equations in 6 unknowns and the system is statically indeterminant. All of the forces intersect a common axis (the z axis) so the system is improperly constrained it is free to rotate about the z axis.

5.56 Sketch the free body diagram of the circus tent stake. The stake is mounted on a hard surface by a ball and socket arrangement at point C. Is this system properly constrained? Is it statically indeterminate?

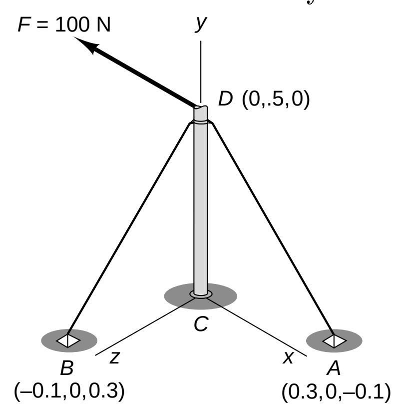

FIGURE P5.56

Solution: From the free body diagram there are 5 unknown magnitudes: T_1, T_2, C_x, C_y and C_z, which, with geometry assumed given, can be determined from the 5 equations of equilibrium

$$\sum F_x = 0,\ \sum F_y = 0,\ \sum F_z = 0,\ \sum M_x = 0, \sum M_z = 0.$$

Thus the system is statically determinant. The sixth equation, $\sum M_y = 0$ is automatically satisfied because all the forces intersect the y-axis, implying that the system is improperly constrained as nothing prevents rotation about the y-axis. You may want to point out that this is a two force member.

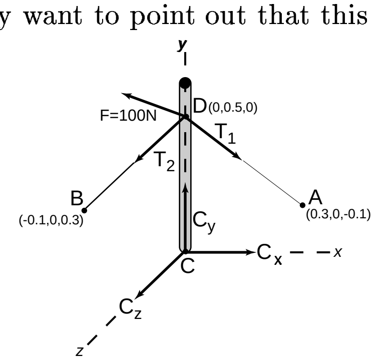

FIGURE S5.56

5.57 Compute the forces and moments exerted by the ground on the end of the pole of problem 5.53. Ignore the mass of the pole.

Solution: From the free body diagram of figure S5.53, there are 6 unknowns. Summing the forces yield

$$\sum F_x = 0 = F_x \text{ so } \underline{F_x = 0}, \; \sum F_y = F_y - 50 = 0$$

$$\text{or } \underline{F_y = 50}$$

and

$$\sum F_z = 0 = F_z \text{ so } \underline{F_z = 0}.$$

Let $\mathbf{M}$ be the moment about the origin. Then $\sum \mathbf{M}_0 = \mathbf{0}$ requires

$$\mathbf{M} + \mathbf{r}_0 \times (-50\hat{\mathbf{j}}) = 0$$

when

$$\mathbf{r}_0 = 4\hat{\mathbf{i}} + 20\hat{\mathbf{j}}(\text{m})$$

so that

$$\mathbf{M} = (4\hat{\mathbf{i}} + 20\hat{\mathbf{j}}) \times (50\hat{\mathbf{j}})$$

or

$$\mathbf{M} = 200\hat{\mathbf{k}} \text{ ft lb}$$

so that

$$\underline{M_x = 0},$$

$$\underline{M_y = 0}$$

and

$$\underline{M_z = 200 \text{ ftlb.}}$$

5.58 Compute the tensions in the cables holding up the logs of problem 5.54. The point A is at (1,1,3) m, B is at (4,1,2), C is at (4,1,4), and the center of mass D is at (2.5,0,3). The logs have a mass of 815.5 kg.

Solution: From Figure S5.54, the free body diagram yields 3 unknown force magnitudes T_A, T_C and T_B. First write these as vectors by defining the unit vector $\hat{a}$, $\hat{b}$, $\hat{c}$ along the lines AE, BE and CE respectively. They are

$$A = (2.5-1)\hat{\mathbf{i}} + (10-1)\hat{\mathbf{j}} + (3-3)\hat{\mathbf{k}} = 1.5\hat{\mathbf{i}} + 9\hat{\mathbf{j}} \text{ so that } \hat{a} = A/|A|$$

$$B = (2.5-4)\hat{\mathbf{i}} + (10-1)\hat{\mathbf{j}} + (3-2)\hat{\mathbf{k}} = -1.5\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + \hat{\mathbf{k}} \text{ so that } \hat{b} = B/|B|$$

$$C = (2.5-4)\hat{\mathbf{i}} + (10-1)\hat{\mathbf{j}} + (3-4)\hat{\mathbf{k}} = -1.5i + 9\hat{\mathbf{j}} - 1\hat{\mathbf{k}} \text{ so that } \hat{c} = C/|C|$$

and $W = -8000\hat{\mathbf{j}}$. So the vector equations of equalibrium is

$$T_A\hat{\mathbf{a}} + T_B\hat{\mathbf{b}} + T_C\hat{\mathbf{c}} - 8000\hat{\mathbf{j}} = 0$$

or from each component:

$$T_A(0.164) + T_B(-0.163) + T_c(-.163) = 0,$$

$$T_A(.986) + T_B(.981) + T_c(.981) = 8000,$$

$$T_A(0) + T_B(.109) + T_c(-.109) = 0$$

which yields (Mathcad)

$$\underline{T_A = 4055\text{N}, T_B = 2040 \text{ N},\ T_c = 2040 \text{ N}}.$$

5.59 A drum hopper is supported by two thrust-less bearings at point A and B. Calculate the reaction forces at A and B if the hopper has a mass of 15.29 kg. Treat the bearings as simple supports. The force F caused by the contents of the hopper is modeled as $F = 30$ N along x.

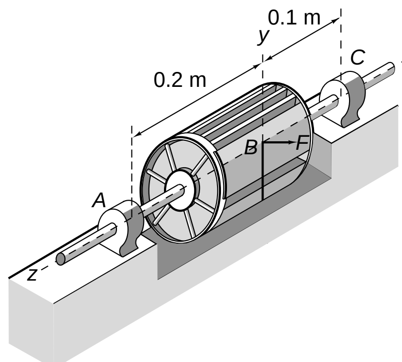

FIGURE P5.59

Solution: The force body diagram yields 4 unknown magnitudes C_x, C_y, A_x and A_y. Note that the system is statically determinant, but is not properly constrained as it may rotate about z and slide along z. From equilibrium

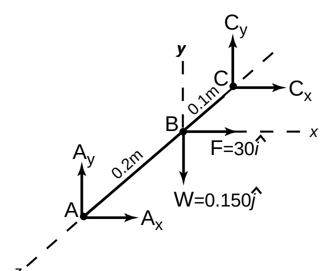

FIGURE S5.59

$\sum F_x = A_x + 30 + C_x = 0, \ \sum M_{y_c} = 0 \rightarrow 0.3A_x + (30)(.1) = 0$, or $\underline{A_x = -10 \text{ N}}$

$\sum F_y = A_y - 150 + C_y = 0 \ \sum M_{x_c} = 0 \rightarrow 0.3A_y - (150)(.1) = 0 \ \underline{A_y = 50 \text{ N}}$

which yields: $\underline{A_x = -10 \text{ N}}$, $\underline{A_y = 50 \text{ N}}$, $\underline{C_x = -20 \text{ N}}$, $\underline{C_y = 100 \text{ N}}$.

5.60 The force applied by the tent wire in problem 5.56 is along the vector $\mathbf{r}_F = -\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$. Calculate the tension in the support wires at A and B as well as the reaction forces at the ball and socket connection at point C.

Solution: The free body diagram is given in Figure S5.56. To determine the 5 unknown magnitudes T_1, T_2, C_x, C_y and C_z the directions of each of the vectors $\mathbf{T}_1$, $\mathbf{T}_2$ and $\mathbf{F}$ must first be determined. A unit vector along $\mathbf{F}$ is

$$\hat{\mathbf{r}} = \frac{\mathbf{r}_f}{|\mathbf{r}_f|} = -.316\hat{\mathbf{i}} + .949\hat{\mathbf{j}}$$

so that

$$\mathbf{F} = -31.623\hat{\mathbf{i}} + 94.868\hat{\mathbf{j}}.$$

A vector along DA is

$$\mathbf{A} = .3\hat{\imath} - .5\hat{\mathbf{j}} - 0.1\hat{\mathbf{k}}$$

so that a unit vector along $\mathbf{A}$ becomes

$$\tilde{\mathbf{a}} = \frac{\mathbf{A}}{|\mathbf{A}|} = 0.507\hat{\mathbf{i}} - 0.845\hat{\mathbf{j}} - 0.169\hat{\mathbf{k}}.$$

Likewise

$$\mathbf{B} = -0.1\hat{\mathbf{i}} - .5\hat{\mathbf{j}} + 0.3\hat{\mathbf{k}}$$

so that

$$\hat{\mathbf{b}} = \frac{\mathbf{B}}{|\mathbf{B}|} = -0.169\hat{\mathbf{i}} - 0.845\hat{\mathbf{j}} + 0.507\hat{\mathbf{k}}.$$

The equilibrium equations can now be written from the free body diagram by summing forces and moments.

$$\sum F = 0 = T_1\hat{\mathbf{a}} + T_2\hat{\mathbf{b}} + C_x\hat{\mathbf{i}} + C_y\hat{\mathbf{j}} + C_z\hat{\mathbf{k}} + \mathbf{F} = 0.$$

Taking moments about the origin yields

$$.5\hat{\mathbf{j}} \times (\mathbf{F} + T_1\hat{\mathbf{a}} + T_2\hat{\mathbf{b}}) = 0.$$

Writing these two vector equations in component form results in the five equations

$$.507T_1 - .169T_2 + C_x = 31.623$$

(continued)

$$-.845T_1 - .845T_2 + C_y = -94.868$$

$$-.169T_1 + .507T_2 + C_z = 0$$

from the force summation. From $\hat{\mathbf{i}}$ component of moment equation

$$-(.5)(.168)T_1 + (.5)(.502)T_2 = 0$$

and from the $\hat{\mathbf{k}}$ component

$$-(.5)(.507)T_1 + (.5)(.169)T_2 = -(.5)(31.623)$$

Combining the 5 equations together as a matrix equation yields

$$\begin{bmatrix} .507 & -.169 & 1 & 0 & 0 \\ -.845 & -.845 & 0 & 1 & 0 \\ -.169 & .507 & 0 & 0 & 1 \\ -.168 & .502 & 0 & 0 & 0 \\ -.507 & .169 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ C_x \\ C_y \\ C_z \end{bmatrix} = \begin{bmatrix} 31.623 \\ -94.168 \\ 0 \\ 0 \\ -31.623 \end{bmatrix}$$

which solved numericaly yields

$$T_1 = 70 \text{ N} \quad T_2 = 23 \text{ N} \quad C_x = 0 \quad C_z = 70 \text{ N} \quad C_y = -156 \text{ N}.$$

Note that this is a two force member so that

$$C_x = C_z = 0.$$

5.61 Consider again the circus tent stake of problem 5.56, however move the top of the stake point D, so that it is now at the location (0.5,0.5,0.5). With F lying along the line $\mathbf{r} = -\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ and points A and B as illustrated in figure P5.56, compute the reaction forces at point C and the two cable tensions.

Solution: The free-body diagram of S5.56 is redrawn here to illustrate the new geometry imposed by driving the stake in at an angle. While $\mathbf{F}$ remains the same

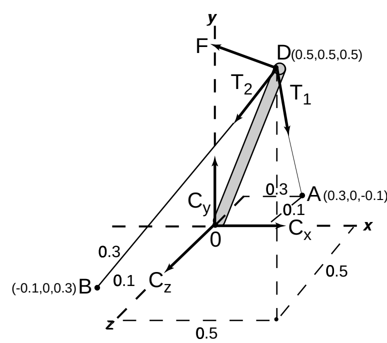

FIGURE S5.61

as calculated in the previous problem, the orientation of T_1 at T_2 has changed. Now

$$\mathbf{A} = (.3 - .5)\hat{\imath} + (0 - .5)\hat{\mathbf{j}} + (-.1 - .5)\hat{\mathbf{k}} = -.2\hat{\mathbf{i}} - .5\hat{\mathbf{j}} - .6\hat{\mathbf{k}}$$

and

$$\mathbf{B} = (-.1 - .5)\hat{\mathbf{i}} + (0 - .5)\hat{\mathbf{j}} + (.3 - 5)\hat{\mathbf{k}} = -.6\hat{\mathbf{i}} - .5\hat{\mathbf{j}} - .2\hat{\mathbf{k}}.$$

The corresponding unit vectors become

$$\hat{\mathbf{a}} = -.248\hat{\mathbf{i}} - .62\hat{\mathbf{j}} - .744\hat{\mathbf{k}}$$

and

$$\hat{\mathbf{b}} = -.744\hat{\mathbf{i}} - .62\hat{\mathbf{j}} - .248\hat{\mathbf{k}}.$$

The force equilibrium equations then become

$$-.248T_1 - .744T_2 + C_x = 31.623 \qquad (1)$$

(continued)

$$-.62T_1 - .62T_2 + C_y = -94.868 \tag{2}$$

$$-.744T_1 - .248T_2 + C_z = 0 \tag{3}$$

Now the vector from the origin to the point D becomes

$$\mathbf{D} = .5(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$$

so that the moment equations now become

$$\mathbf{D} \times (\mathbf{T}_1 + \mathbf{T}_2 + \mathbf{F}) = 0.$$

or

$$(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) \times [(T_1)(-.248\hat{\mathbf{i}} - .62\hat{\mathbf{j}} - .744\hat{\mathbf{k}}) + T_2(-.744\hat{\mathbf{i}} - .62\hat{\mathbf{j}} - .248\hat{\mathbf{k}})$$

$$= -(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) \times (-31.623\hat{\mathbf{i}} + 94.868\hat{\mathbf{j}}).$$

In component form this becomes

$$(M_x) \quad - .124T_1 + .372T_2 = 94.868 \tag{4}$$

$$(M_y) \quad .496T_1 - .496T_2 = 31.623 \tag{5}$$

$$(M_z) \quad - .372T_1 + 124T_2 = -126.491 \tag{6}$$

Only two of these equations are independent. Combining (1-5) into a matrix equation and solving yields

$$\underline{T_1 = 478 \text{ N}, \ T_2 = 414 \text{ N and } C_x = C_y = C_z = 459 \text{ N}}$$

The third moment equation does not yield any new information.

5.62 A winch system consists of a 0.1-m diameter drum, shaft and motor. Compute the reactions at A and C, and the motor torque M_z required to keep the 100-kg mass in equilibrium. The spindle supports at A and C are thrustless bearings. Neglect any moments at A and C.

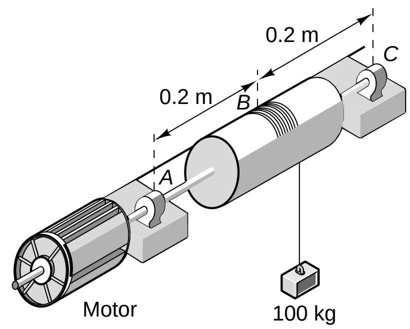

FIGURE P5.62

Solution: The free-body is given and the equilibrium equations are (unrestrained in z direction)

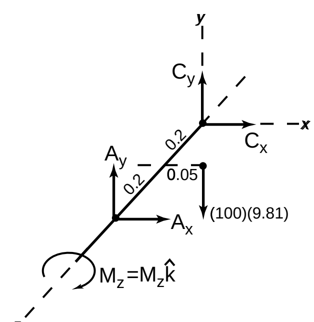

FIGURE S5.62

$$\sum F_x = 0 : A_x + C_x = 0 \tag{1}$$

$$\sum F_y = 0 : A_y + C_y = 981 \tag{2}$$

$$\sum M_c = 0 = M_z\hat{\mathbf{k}} + (.05\hat{\mathbf{i}} + 0.2\hat{\mathbf{k}}) \times (-981\hat{\mathbf{j}}) + 0.4\hat{\mathbf{k}} \times (A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}) = 0$$

Further manipulation of the moment equation yields

(continued)

$$M_z\hat{\mathbf{k}} - (.05)(981)\hat{\mathbf{k}} + (0.2)(981)\hat{\mathbf{i}} + (0.4A_x)\hat{\mathbf{j}} - 0.4A_y\hat{\mathbf{i}} = 0$$

or in scalar form

$$k: \quad M_z = 49.05 \text{ Nm} \tag{3}$$

$$j: \quad 0.4A_x = 0 \tag{4}$$

$$i: 196.2 - 0.4A_y = 0 \tag{5}$$

These 5 equations yield $A_x = C_x = 0$, $A_y = 490.5$ N, $C_y = 490.5$ m.

5.63 A window washing scaffold system has a mass of 152.9 kg. Compute the tensions in the ropes assuming that the weight acts at the geometric center and that the three pulleys offer no friction to the rope. Also assume that the tension in the stabilizing rope labeled T_7 is zero.

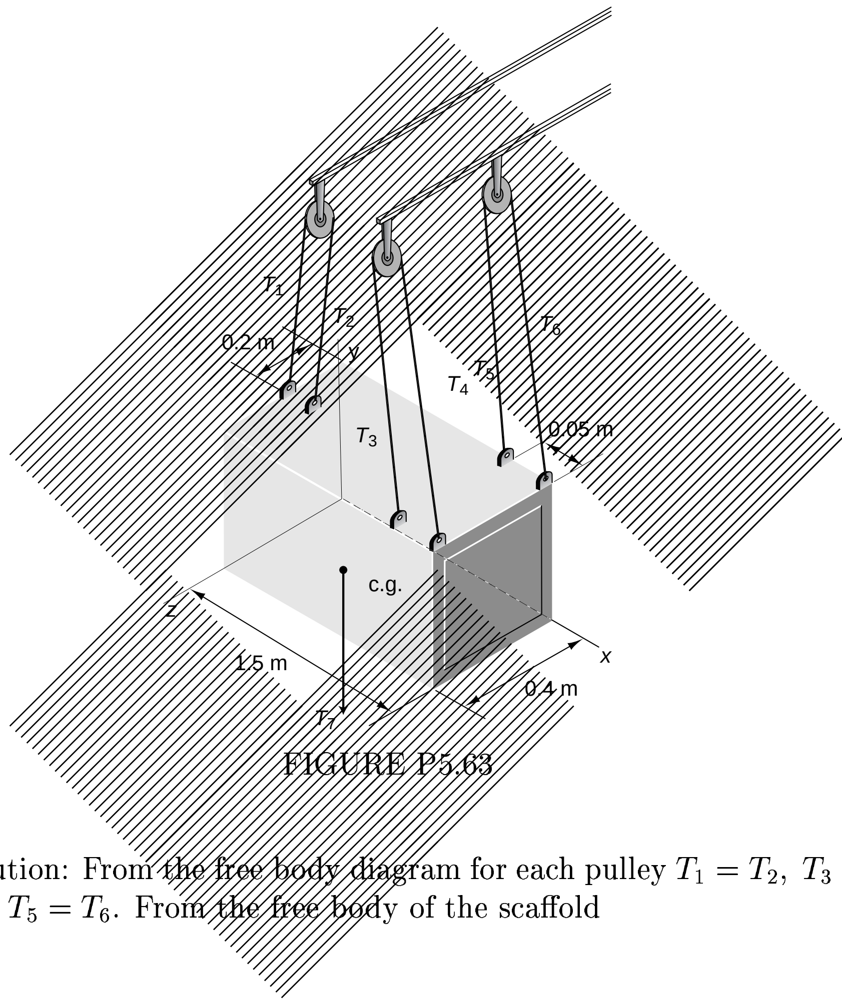

FIGURE P5.63

Solution: From the free body diagram for each pulley $T_1 = T_2,\ T_3 = T_4$ and $T_5 = T_6$. From the free body of the scaffold

$$\sum F_y = -1500 + 2T_1 + 2T_3 + 2T_5 = 0$$

or

$$T_1 + T_3 + T_5 = 750 \qquad (1)$$

From the moment

(continued)

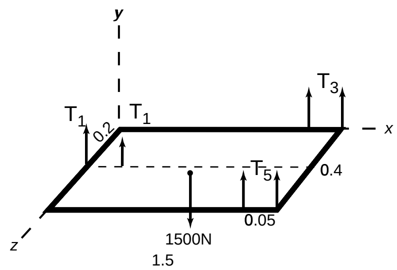

FIGURE S5.63

$$\sum M_0 = 0; (0.2\hat{\mathbf{k}} \times T_1\hat{\mathbf{j}}) + (0.05\hat{\mathbf{i}} + 0.2\hat{\mathbf{k}}) \times T_1\hat{\mathbf{j}}$$
$$+(1.45\hat{\mathbf{i}} \times T_5\hat{\mathbf{j}}) + (1.5\hat{\mathbf{i}} \times T_5\hat{\mathbf{j}}) + (1.45\hat{\mathbf{i}} + .4\hat{\mathbf{k}})$$
$$\times(T_3\hat{\mathbf{j}}) + (1.5\hat{\mathbf{i}} + .4\hat{\mathbf{k}}) \times (T_3\hat{\mathbf{j}}) + 0.75\hat{\mathbf{i}} + .2\hat{\mathbf{k}} \times (-1500\hat{\mathbf{j}} = 0$$

or

$$-0.4T_1 - 0.8T_3 = -300 \qquad (2),$$
$$0.05T_1 + 2.95T_5 + 2.95T_3 = 1125 \qquad (3).$$

Solving 1, 2, and 3 via matrix methods yields

$$\underline{T_1 = 375 \text{ N},\ T_2 = 187.5 \text{ N},\ T_3 = 187.5 \text{ N}}.$$

5.64 A mounting platform is secured in place by a frictionless support at A, a ball and socket at B and a rope at C. The A 100 N gravitational force acts at its geometric center and two boxes sit on the platform modeled by the force $F_1 = 500$ N and $F_2 = 50$ N. Calculate the components of the reaction forces at the supports.

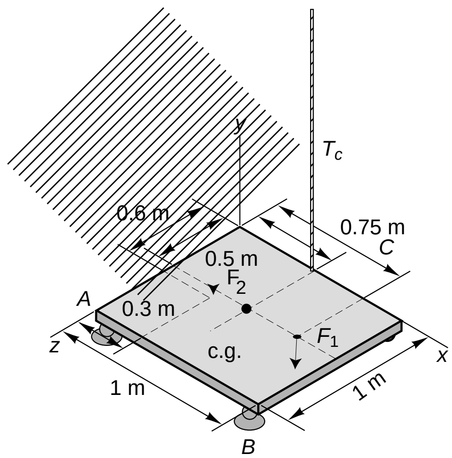

FIGURE P5.64

Solution: A free body diagram yields the following equilibrium equations

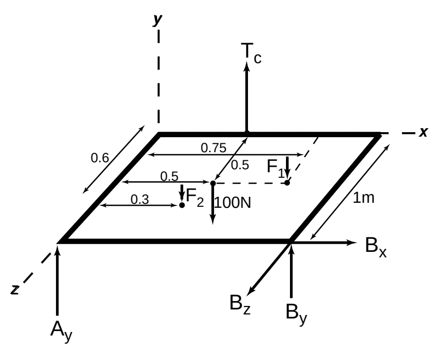

FIGURE S5.64

$$\sum F_x : \quad B_x = 0 \tag{1}$$

(continued)

$$\sum F_y : \quad T_c + B_y + A_y - 650 = 0 \tag{2}$$

$$\sum F_z = \underline{B_z = 0} \tag{3}$$

$$\sum \mathbf{M}_0 = (0.5\hat{\mathbf{i}}) \times T_c\hat{\mathbf{j}} + (\hat{\mathbf{k}} \times A_y\hat{\mathbf{j}}) + \hat{\mathbf{i}} \times (B_y\hat{\mathbf{j}}) + \hat{\mathbf{k}} \times (B_y\hat{\mathbf{j}}) + .5(\hat{\mathbf{i}} + \hat{\mathbf{k}}) \times (-100\hat{\mathbf{j}})$$

$$+ (.75\hat{\mathbf{i}} + .5\hat{\mathbf{k}}) \times (-500\hat{\mathbf{j}}) + (0.3\hat{\mathbf{i}} + 0.6\hat{\mathbf{k}}) \times (-50\hat{\mathbf{j}}) = \mathbf{0}$$

multiplying out the moment terms (with $B_x = B_z = 0$) yields from

$$\hat{\mathbf{i}} : -A_y - B_y + 330 = 0$$

or

$$A_y + B_y = 330 \tag{4}.$$

From

$$\hat{\mathbf{k}} : .5T_c + B_y = 440 \tag{5}$$

Solving 1-5 yields

$$\underline{T_C = 320 \text{ N}}, \ \underline{B_x = 0}, \ \underline{B_y = 280 \text{ N}}, \ \underline{B_z = 0}, \ \underline{A_y = 50 \text{ N}}$$

Note that while stable for the given load, there is no support against twisting about the y axis so this is *improperly constrained*.

5.65 The support arm for a lifting mechanism has a mass of 10.2 kg and supports a load at point D of $\mathbf{T} = 1000\hat{\mathbf{i}} - 1000\hat{\mathbf{k}}$ (N). The sleeve support at point C acts as simple support. The support at A is a ball and socket and that at B is a ball or roller. Compute the reaction forces at these connections.

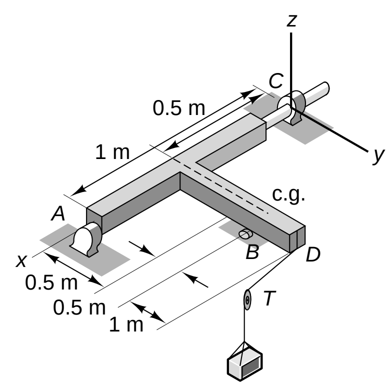

FIGURE P5.65

Solution: The free body diagram of the support arm is

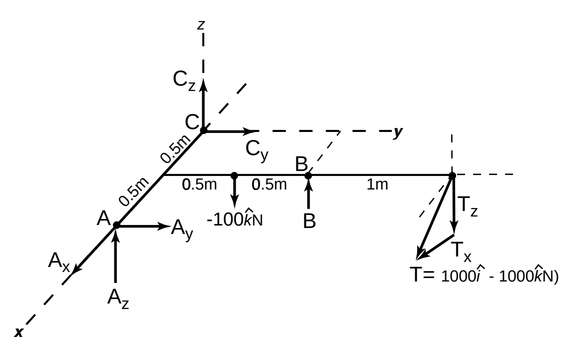

FIGURE S5.65

From equilibrium of the free body

$$\sum F_x = 0 \text{ or } A_x + 1000 = 0 \tag{1}$$

$$\sum F_y = 0 \text{ or } A_y + C_y = 0 \tag{2}$$

$$\sum F_z = 0 \text{ or } A_z + C_z - 1000 - 100 + B = 0 \tag{3}$$

(continued)

$$\sum \mathbf{M}_0 = \text{ or } \hat{\mathbf{i}} \times (A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}) + 0.5(\hat{\mathbf{i}} + \hat{\mathbf{j}}) \times (-100\hat{\mathbf{k}})$$

$$+(0.5\hat{\mathbf{i}} + \hat{\mathbf{j}}) \times (B\hat{\mathbf{k}}) + (0.5\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) \times (1000\hat{\mathbf{i}} - 1000\hat{\mathbf{k}}) = \mathbf{0}$$

evaluating the various cross products yields

$$A_y\hat{\mathbf{k}} - A_z\hat{\mathbf{j}} + 50\hat{\mathbf{j}} - 50\hat{\mathbf{i}} - \frac{B}{2}\hat{\mathbf{j}} + B\hat{\mathbf{i}} + 500\hat{\mathbf{j}} - 2000\hat{\mathbf{k}} - 2000\hat{\mathbf{i}} = \mathbf{0}$$

which in component form becomes

$\mathbf{i}$: $-50 + B - 2000 = 0$ (4)

$\mathbf{j}$: $-A_z + 50 - \frac{B}{2} + 500 = 0$ (5)

$\mathbf{k}$: $A_y - 2000 = 0$ (6)

Solving equations (1)-(6) yields,

$\underline{A_y = 2000 \text{ N}}$, $\underline{B = 2050 \text{ N}}$, $\underline{A_z = 475 \text{ N}}$, $\underline{A_x = -1000 \text{ N}}$, $\underline{C_y = -2000 \text{ N}}$

and $\underline{C_z = -1425 \text{ N}}$

5.66 A harbor barge and crane is used to lift a 1019 kg crate. Model the joint at A as a ball and socket and the joint at B as a pin providing only an x and y force component. The rope T is used to rotate the crane about the AB axis. Compute the tension T and the reactions at A and B. Ignore the radius of the pulley and assume that W and T act at the same point on the cross bar.

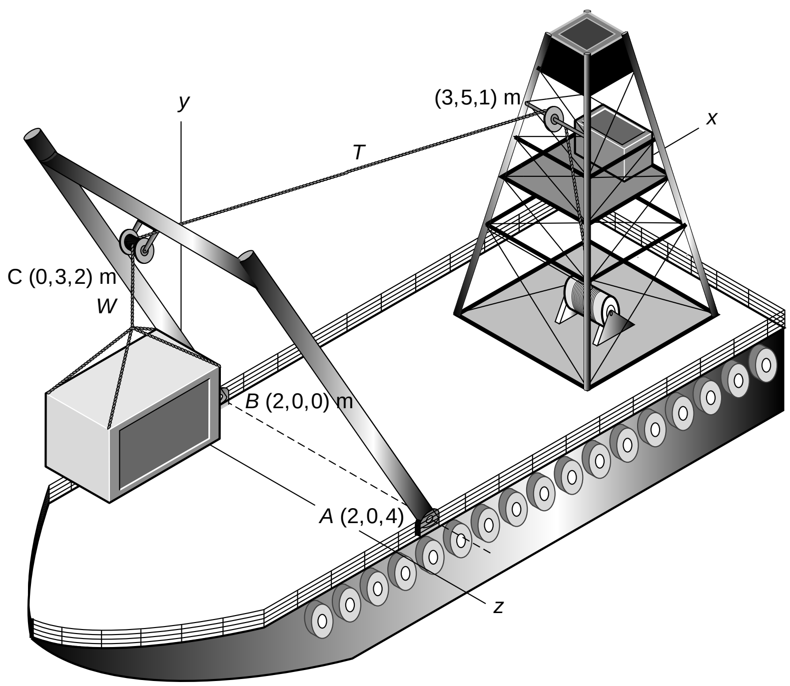

FIGURE P5.66

Solution. A free body diagram is given in the figure. Summing forces and moments yields

$$\sum F_x = 0 : \quad A_x + B_x + 0.802T = 0 \tag{1}$$

$$\sum F_y = 0 : \quad A_y + B_y - 9996 + 0.535T = 0 \tag{2}$$

$$\sum F_z = 0 : \quad A_z - .267T = 0 \tag{3}$$

(continued)

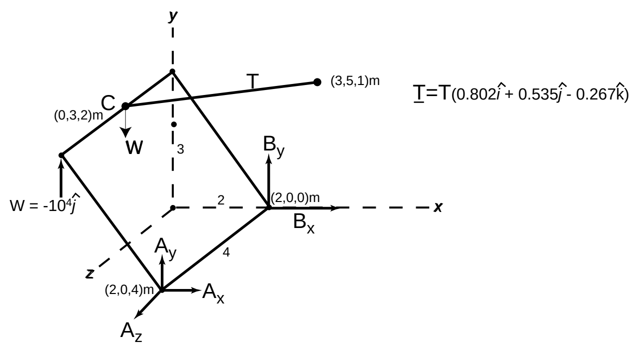

FIGURE S5.66

$$\sum M_0 = 0$$

yields

$$(3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) \times (-9996\hat{\mathbf{j}}) + (2\hat{\mathbf{i}} + 4\hat{\mathbf{k}}) \times (A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}) + 2\hat{\mathbf{i}}$$

$$\times (B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}}) + (3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) \times T(.802\hat{\mathbf{i}} + .535\hat{\mathbf{j}} - 0.267\hat{\mathbf{k}}) = \mathbf{0}$$

or using a symbolic evaluation the components are

$$\hat{\mathbf{i}}: \quad -4A_y - 1.871T = -19992 \tag{4}$$

$$\hat{\mathbf{j}}: \quad 4A_x - 2A_z + 1.604T = -0 \tag{5}$$

$$\hat{\mathbf{k}}: \quad 2B_y + 2A_y - 2.406T = 0 \tag{6}$$

which has solution (via Mathcad matrix inversion)

$$\underline{T = 5751 \text{ N}}, \underline{A_x = -1539 \text{ N}}, \underline{A_y = 2308 \text{ N}}, \underline{A_z = 1536 \text{ N}}$$

$$\underline{B_x = -3075 \text{ N}}, \underline{B_y = 4611 \text{ N}}$$

5.67 A harbor barge equipped with a crane and a winch is used to hold a 1000-N crate. Model the joint at A as a ball and socket and the joint at B is a pin providing only x and y force components. The rope made by W and the line CD is continuous and runs over a pulley at C without friction. Ignore the dimension of the pulley and lift mechanism and compute the tension in rope T as well as the reactions at A and B.

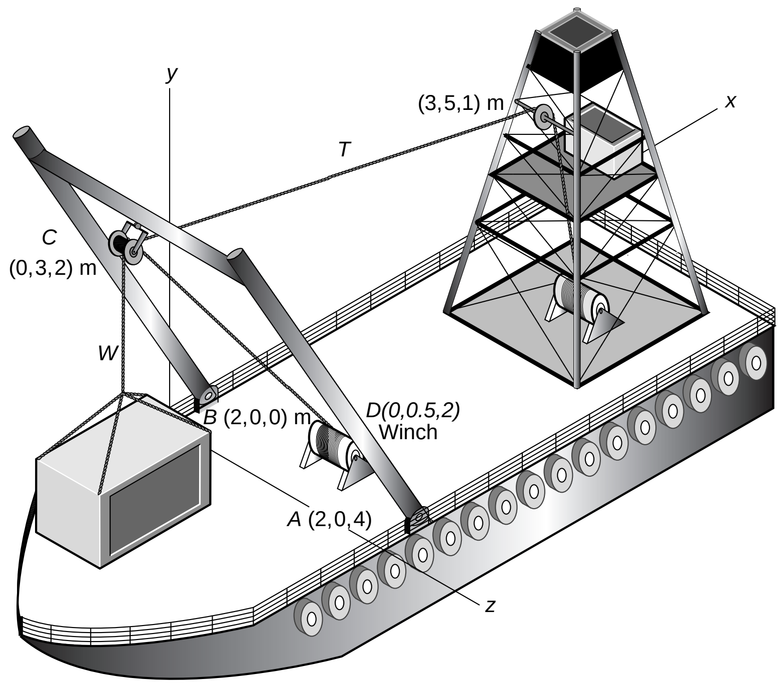

FIGURE P5.67

Solution: The weight of 1000 N is the tension in the rope CD from a free body of the weight and pulley. Since the pulley is frictionless, this must be the magnitude of CD. The vector form of this tension, we will call $W2$ and comes from the unit vector

$$\overline{CD}/|\overline{CD}|$$

or

$$W2 = 10^3 \frac{\overline{CD}}{|\overline{CD}|} = 371.4\hat{\mathbf{i}} - 928.5\hat{\mathbf{j}}.$$

The free body diagram becomes

(continued)

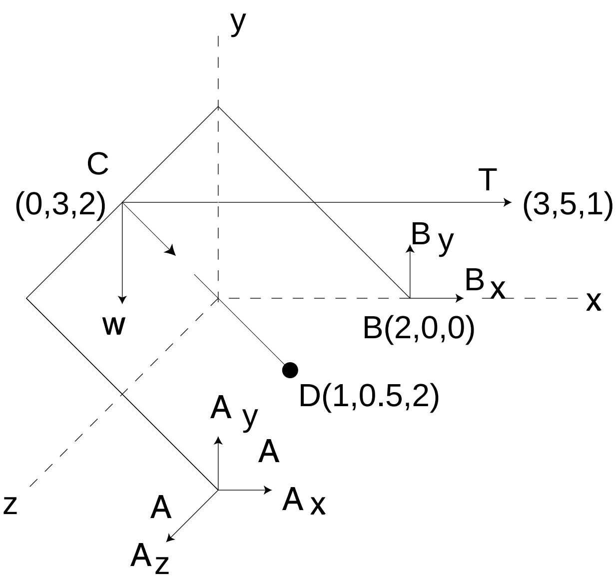

FIGURE S5.67

The unit vector along TC is found from $TC = (3-9, 5-3, 1-2)$ or $\hat{t} = TC/TC| = 0.802\hat{\mathbf{i}} + 0.535\hat{\mathbf{j}} - 0.267\hat{\mathbf{k}}$. Then the vector expression for T is $\mathbf{T} = T(0.802\hat{\mathbf{i}} + 0.535\hat{\mathbf{j}} - 0.267\hat{\mathbf{k}})$. The equilibrium equations become

$$\sum F_x = 0: \quad A_x + B_x + 0.802T = -371.4 \tag{1}$$

$$\sum F_y = 0: \quad A_y + B_y + 0.535T = 1928.5 \tag{2}$$

$$\sum F_z = 0: \quad A_z - .267T = 0 \tag{3}$$

The moment equation about the origin is

$$\sum M_0 = 0$$

or

$$(3\hat{\mathbf{j}}+2\hat{\mathbf{k}})\times(371.4\hat{\mathbf{i}}-1928.5\hat{\mathbf{j}})+(3\hat{\mathbf{j}}+2\hat{\mathbf{k}})\times T(0.802\hat{\mathbf{i}}+0.535\hat{\mathbf{j}}-0.267\hat{\mathbf{k}})+(2\hat{\mathbf{i}})$$

$$\times(B_x\hat{\mathbf{i}}+B_y\hat{\mathbf{j}})+(2\hat{\mathbf{i}}+4\hat{\mathbf{k}})\times(A_x\hat{\mathbf{i}}+A_y\hat{\mathbf{j}}+A_z\hat{\mathbf{k}})+(3\hat{\mathbf{j}}+2\hat{\mathbf{k}})\times(-1000)\hat{\mathbf{j}} = 0$$

evaluating symbolically yields the following 3 equations

$$\hat{\mathbf{i}}: \quad -4A_y - 1.871T = -3857.0 \tag{4}$$

$$\hat{\mathbf{j}}: \quad 4A_x - 2A_z + 1.604T = -742.8 \tag{5}$$

$$\hat{\mathbf{k}}: \quad 2A_y + 2B_y - 2.406T = 1114.2 \tag{6}$$

Writing these 6 equations as a matrix equation and inverting yields

$$\underline{A_x = -397 \text{ N}}, \underline{A_y = 595 \text{ N}}, \underline{A_z = 211 \text{ N}}, \underline{B_x = -608 \text{ N}}$$

$$\underline{B_y = 911 \text{ N}}, \underline{T = 789 \text{ N}}$$

5.68 A container barge and crane holds a ship's cargo container at rest in the position shown. The joint at A is a ball and socket which allows the ropes T_1 and T_2 to manipulate the container. (Several other control ropes are not shown in the drawing.) Calculate the tensions T_1, T_2 and the reaction forces at A. Assume the container causes a tension $W = 4 \times 10^4$ N.

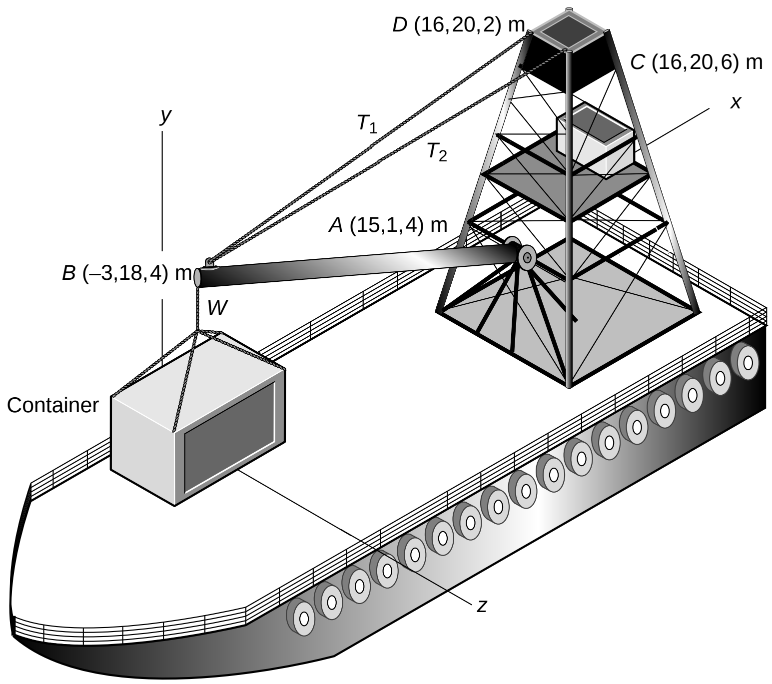

FIGURE P5.68

Solution: A free body diagram of member AB is shown. Unit vectors along BC and BD are required in order to write the force vector for T_1 and T_2.

$$\mathbf{T}_1 = T_1 \frac{\overline{BD}}{|\overline{BD}|} = T_1(0.989\hat{\mathbf{i}} + 0.104\hat{\mathbf{j}} - 0.104\hat{\mathbf{k}})$$

$$\mathbf{T_2} = T_2 \frac{\overline{BC}}{|\overline{BC}|} = T_2(0.989\hat{\mathbf{i}} + 0.104\hat{\mathbf{j}} + 0.104\hat{\mathbf{k}})$$

Summing forces yields

(continued)

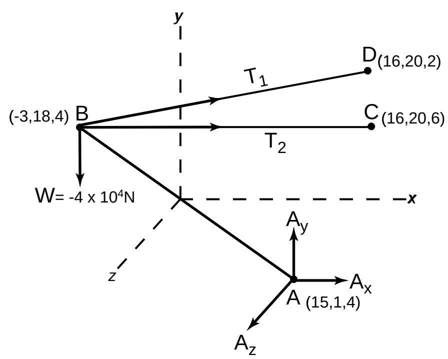

FIGURE S5.68

$$\sum F_x = 0: \quad A_x + 0.989T_1 + 0.989T_2 = 0 \tag{1}$$

$$\sum F_y = 0: \quad A_y + 0.104T_1 + 0.104T_2 = 4 \times 10^4 \tag{2}$$

$$\sum F_z = 0: \quad A_z - 0.104T_1 + 0.104T_2 = 0 \tag{3}$$

The moment equation about the origin is

$$\sum M_0 = 0 : (-3\hat{\mathbf{i}} + 18\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) \times [(T_1)(.989\hat{\mathbf{i}} + 0.104\hat{\mathbf{j}} - 0.104\hat{\mathbf{k}})$$

$$+(T_2(.989\hat{\mathbf{i}}+.104\hat{\mathbf{j}}+0.104\hat{\mathbf{k}})-4\times10^4\hat{\mathbf{j}}]+(15\hat{\mathbf{i}}+\hat{\mathbf{j}}+4\hat{\mathbf{k}})\times(A_x\hat{\mathbf{i}}+A_y\hat{\mathbf{j}}+A_z\hat{\mathbf{k}}) = 0$$

Evaluating symbolically and equating the components to zero yield

$$\mathbf{i}: \quad -2.288T_1 + 1.456T_2 + A_z - 4A_y = -160,000 \tag{4}$$

$$\mathbf{j}: \quad 3.644T_1 + 4.268T_2 + 4A_x - 15A_z = 0 \tag{5}$$

$$\hat{\mathbf{k}}: \quad -18.114T_2 - A_x + 15A_y = -120,000 \tag{6}$$

Now only 5 of these equations are independent. Choosing (1) (2) (3) (4) and (6) and solving by matrix yields

$$\underline{A_x = -38,110 \text{ N}}, \ \underline{A_y = 35,990 \text{ N}}, \underline{A_z = 0}, \underline{T_1 = T_2 = 1.927 \times 10^4 \text{ N}}$$

5.69 Compute the reactions at the ball and socket support at point D and the tensions in the support ropes (T_1 and T_2) for the sign support system. The weight of the sign exerts a force of 300 N in the down direction ($-y$) at point E, 0.25 m from D and at point F, 1.75 m from D. Note that DC is not constrained from rotation about its axis.

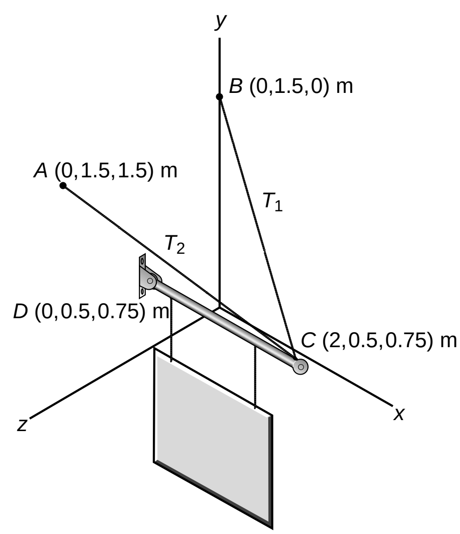

FIGURE P5.69

Solution: A free body diagram of the sign support is given in the figure

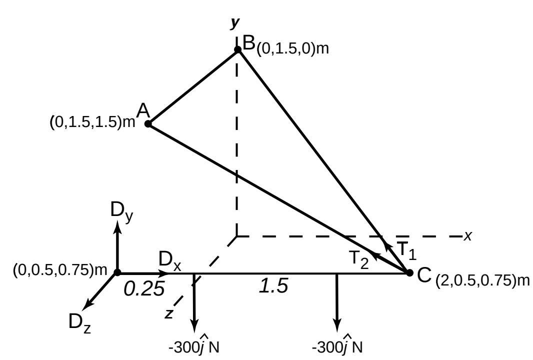

FIGURE S5.69

First note that all the forces intersect a common axis (that of the

(continued)

line $\overline{DC}$), thus there will be only 5 independent equations and only 5 variables can be determined (D_x, D_y, D_z, T_1 and T_2). The vectors $\mathbf{T}_1$ and $\mathbf{T}_2$ must first be written in terms of unit vectors along CB and CA respectively:

$$\mathbf{T}_1 = T_1 \frac{\overline{CB}}{|\overline{CB}|} = T_1(-0.848\hat{\mathbf{i}} + 0.424\hat{\mathbf{j}} - 0.318\hat{\mathbf{k}}),$$

$$\mathbf{T}_2 = T_2 \frac{\overline{CA}}{|\overline{CA}|} = T_2(-0.848\hat{\mathbf{i}} + 0.424\hat{\mathbf{j}} + 0.318\hat{\mathbf{k}}).$$

Equilibrium in the coordinate directions becomes

$$\sum F_x = 0: \quad D_x - 0.848T_1 - 0.848T_2 = 0 \tag{1}$$

$$\sum F_y = 0: \quad D_y + 0.424T_1 + 0.424T_2 = 600 \tag{2}$$

$$\sum F_z = 0: \quad D_z - 0.318T_1 + 0.318T_2 = 0 \tag{3}$$

The moments about point C yield

$$(-0.25\hat{\mathbf{i}}) \times (-300\hat{\jmath}) + (-1.75\hat{\mathbf{i}}) \times (-300\hat{\mathbf{j}}) + (-2\hat{\mathbf{i}}) \times (D_x\hat{\mathbf{i}} + D_y\hat{\mathbf{j}} + D_z\hat{\mathbf{k}}) = \mathbf{0}$$

or by component

$$\hat{\mathbf{i}}: \quad 0 = 0 \tag{4}$$

$$\hat{\mathbf{j}}: \quad 2D_z = 0 \tag{5}$$

$$\hat{\mathbf{k}}: \quad D_y = 300 \tag{6}$$

Solving equations 1, 2, 3 with

$$\underline{D_z = 0} \text{ and } \underline{D_y = 300 \text{ N}}$$

yields

$$\underline{D_x = 600 \text{ N}} \quad \underline{T_1 = 354 \text{ N}} \quad \underline{T_2 = 354 \text{ N}}$$

5.70 A crate of mass 200 kg is suspended by a crane. The joint at A is modeled as a ball and socket. Compute the tensions in the three supporting ropes and the reaction at point A.

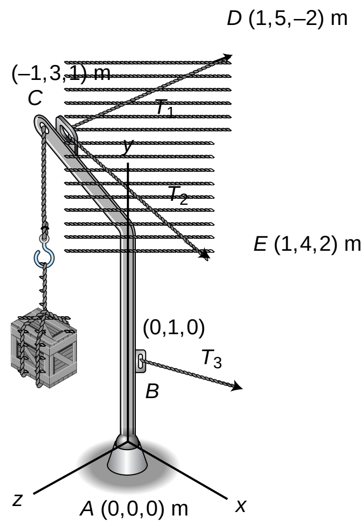

FIGURE P5.70

Solution: The free-body diagram is given in figure S5.70

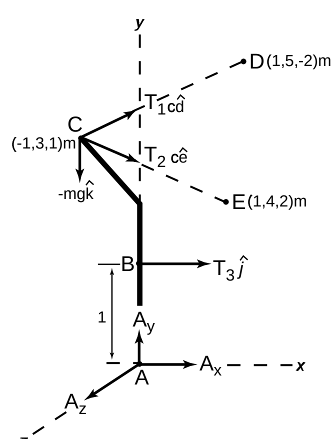

FIGURE S5.70

The unit vectors require to define the tensions T_1 and T_2 are

(continued)

$$\overline{CD} = (1 - -1)\hat{\mathbf{i}} + (5-3)\hat{\mathbf{j}} + (-2-1)\hat{\mathbf{k}} = 2\hat{\imath} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$$

$$\hat{cd} = \overline{CD}/|\overline{CD}| = 0.485\hat{\mathbf{i}} + 0.485\hat{\mathbf{j}} - 0.728\hat{\mathbf{k}}$$

and

$$\mathbf{T}_1 = T_1\hat{cd} = T_1(0.485\hat{\mathbf{i}} + 0.485\jmath - 0.728\hat{\mathbf{k}})$$

Likewise,

$$\mathbf{T}_2 = T_2(0.816\hat{\mathbf{i}} + 0.408\hat{\mathbf{j}} + 0.408\hat{\mathbf{k}}).$$

The force equilibrium becomes

$$\sum F_x = 0: \quad A_x + 0.485T_1 + 0.816T_2 + T_3 = 0 \tag{1}$$

$$\sum F_y = 0: \quad A_y + 0.485T_1 + 0.408T_2 - (9.81)\times(200) = 0 \tag{2}$$

$$\sum F_z = 0: \quad A_z - 0.728T_1 + 0.408T_2 = 0 \tag{3}$$

$$\sum M_A = \hat{\mathbf{j}} \times T_3\hat{\mathbf{i}} + (-\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}) \times (\mathbf{T}_1 + \mathbf{T}_2 - (9.81)(200)\hat{\mathbf{j}}) = \mathbf{0}$$

or

$$-2.6698T_1 + 0.816T_2 = -(9.81)(200) \tag{4}$$

$$-0.234T_1 + 1.224T_2 = 0 \tag{5}$$

$$1.9408T_1 + 2.856T_2 + T_3 = (9.81)(200) \tag{6}$$

Which has solution (via matrix methods)

$$\underline{A_x = -521\text{ N}},\ \underline{A_y = 1523\text{ N}},$$

$$\underline{A_z = 507\text{ N}},\ \underline{T_1 = 781\text{ N}},\ \underline{T_2 = 149\text{ N}}$$

and

$$\underline{T_3 = 21\text{ N}}.$$

5.71 The total force acting on a telephone pole due to the wires attached to it is computed to be $\mathbf{F} = 100\hat{\mathbf{i}} - 50\hat{\mathbf{j}} + 10\hat{\mathbf{k}}N$. Compute the reaction at the fixed connection at point$A$.

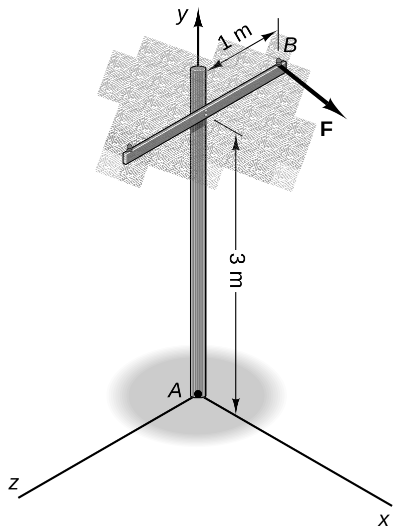

FIGURE P5.71

Solution: The free body diagram is given in the figure. The equations of equilibrium become simply

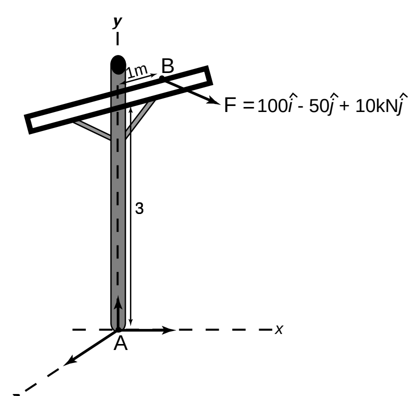

FIGURE S5.71

(continued)

$$\sum F_x = 0 : A_x + 100 = 0 \text{ or } \underline{A_x = -100 \text{ N}}$$

$$\sum F_z = 0 : A_y - 50 = 0 \text{ or } \underline{A_y = 50 \text{ N}}$$

$$\sum F_z = 0 : A_z + 10 = 0 \text{ or } \underline{A_z = -10 \text{ N}}$$

$$\sum \mathbf{M}_B = \mathbf{0} : M_x\hat{\mathbf{i}} + M_y\hat{\mathbf{j}} + M_z\hat{\mathbf{k}} + (3\hat{\mathbf{j}} - \hat{\mathbf{k}})\text{m } \times (100\hat{\mathbf{i}} - 50\hat{\mathbf{j}} + 10\hat{\mathbf{k}})\text{N} = \mathbf{0}$$

$$= (M_x + 30 - 50)\hat{\mathbf{i}} + (M_y - 100)\hat{\mathbf{j}} + (M_z - 300)\hat{\mathbf{k}} = 0$$

or

$$\underline{M_x = 20 \text{ N} \cdot \text{m}}, \ \underline{M_y = 100 \text{ N} \cdot \text{m}}, \ \underline{M_z = 300 \text{ N} \cdot \text{m}}$$

5.72 A section of piping for a nuclear power plant is held in place by three sleeve connections which act somewhat like bearing supports. The flow in the pipe causes the forces and a moment to be applied to the pipe. Neglect any reaction moments at the supports and compute the resulting unknown reactions at A, B, and C.

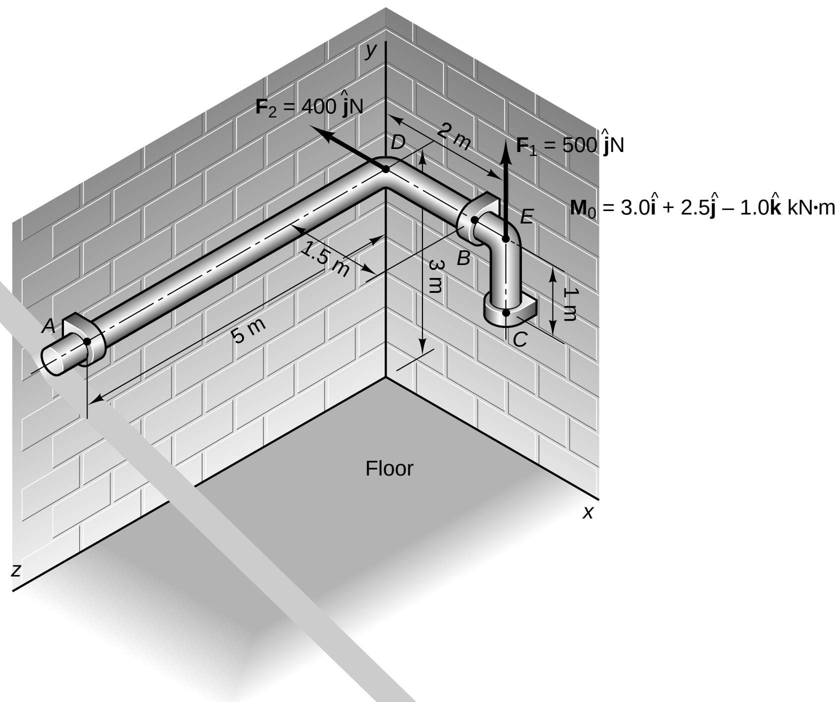

FIGURE P5.72

Solution: A free body diagram is given in the figure.

The force equlibrium yields:

$\sum F_x = 0 : A_x + C_x - 400 = 0$ (1)

$\sum F_y = 0 : A_y + B_y + 500 = 0$ (2)

$\sum F_z = 0 : B_z + C_z = 0$ (3)

The moment equilibrium about D becomes

$$\mathbf{M}_D = 0 : 3000\hat{\imath} + 2500\hat{\mathbf{j}} - 1000\hat{\mathbf{k}} + 1.5\hat{\mathbf{i}} \times (-B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}})$$

(continued)

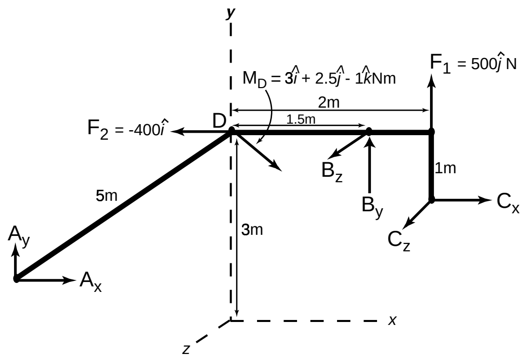

FIGURE S5.72

$$+(2\hat{\mathbf{i}} - \hat{\mathbf{j}}) \times (C_x\hat{\mathbf{i}} + C_z\hat{\mathbf{k}}) + 2\hat{\mathbf{i}} \times 500\hat{\mathbf{j}} + 5\hat{\mathbf{k}} \times (A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}) = 0$$

from the $\hat{\mathbf{i}}$ component $3000 - 5A_y - C_z = 0$ (4)

from $\hat{\mathbf{j}}$: $2500 - 1.5B_z - 2C_z + 5A_x = 0$ (5)

from $\hat{\mathbf{k}}$: $1.5B_y + C_x = 0$ (6)

Solving these 6 equations for the 6 unknowns A_x, A_y. B_y, B_z, C_x and C_z yields

$$\underline{A_x = -125 \text{ N}},\ \underline{A_y = -150 \text{ N}},\ \underline{B_y = -350 \text{ N}},\ \underline{B_z = -3750\text{N}},\ \underline{C_x = 525 \text{ N}},$$

$$\underline{C_z = 3750 \text{ N}}.$$

The two negative signs mean, of course, that A_x A_y and B_z are in directions opposite those sketched in the figure.

5.73 A folding platform is used to hold parts as well as conserve floor space when not in use. The platform is supported by a hinge at C which is assumed to support negligible moments, a leg, at B modeled as a frictionless support and a removable pin at A modeled as a thrustless bearing again with negligible moments. If the platform is loaded as illustrated compute the reaction forces at A, B and C. Ignore the thickness of the platform.

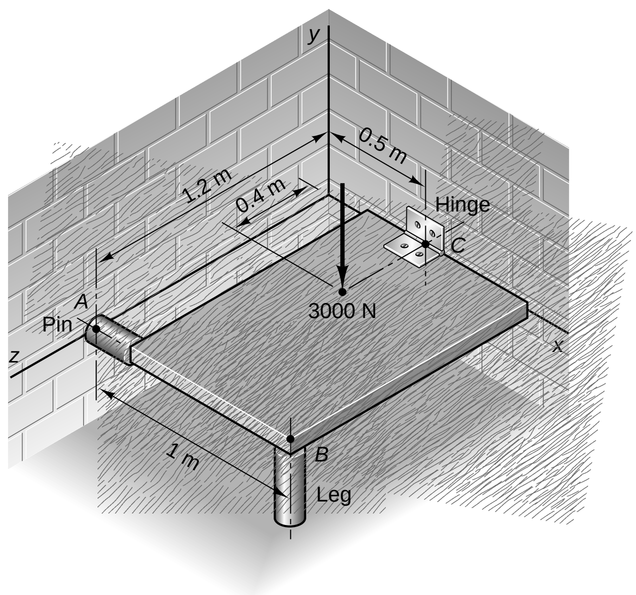

FIGURE P5.73

Solution: The free body diagram is given in figure S5.73.

The equations of equilibrium become: (continued)

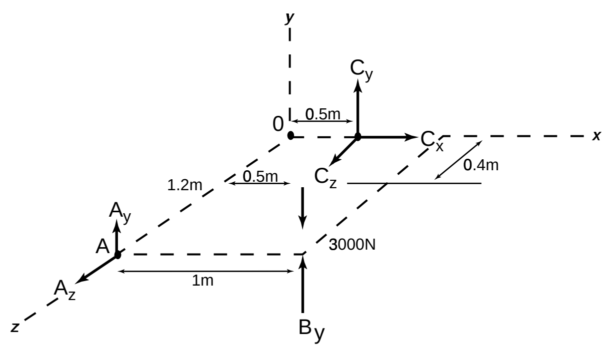

FIGURE S5.73

Force Summation:

$$\sum F_x = 0 : C_x = 0 \quad (1)$$

$$\sum F_y = 0 : A_y + C_y + B_y = 3000 \quad (2)$$

$$\sum F_z = 0 : A_z + C_z = 0 \quad (3)$$

The moment equation is

$$\sum \mathbf{M}_0 = (.5\hat{\mathbf{i}} + .4\hat{\mathbf{k}}) \times (-3000\hat{\mathbf{j}}) + (.5\hat{\mathbf{i}}) \times (C_x\hat{\mathbf{i}} + C_y\hat{\mathbf{j}} + C_z\hat{k}) + (\hat{\mathbf{i}} + 1.2\hat{\mathbf{k}}) \times B_y\hat{\mathbf{j}}$$
$$+1.2\hat{\mathbf{k}} \times (A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}) = 0$$

Which yields the 3 component equations

$$\hat{\mathbf{i}} : 1200 - 1.2B_y - 1.2A_y = 0 \quad (4)$$

$$\hat{\mathbf{j}} : -0.5C_z = 0 \quad (5)$$

$$\hat{\mathbf{k}} : -1500 + .5C_y + B_y = 0 \quad (6)$$

Equations (1), (3) and (5) yield

$$\underline{C_z = A_z = C_x = 0}$$

by inspection. The remaining 3 equations (2, 4, 6) (solved in Mathcad) yield

$$\underline{A_y = 500 \text{ N}},$$
$$\underline{B_y = 500 \text{ N}}$$

and

$$\underline{C_y = 2000 \text{ N}}.$$

5.74 The exhaust pipe on the tractor (of a tractor/trailer semi) vibrates because of the gas moving through the muffler system and because of the wind rushing past it. To stabilize this vibration a cord is fixed between the pipe and the cab. If the support at A is modeled as a thrust bearing unable to provide a moment about the x-axis, calculate the tension, T, in the wire and the reactions at A.

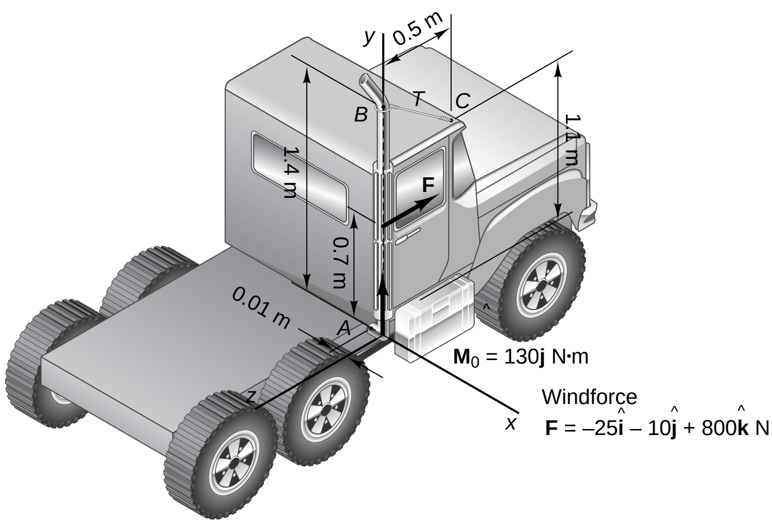

FIGURE P5.74

Solution: The free body diagram is illustrated and yields the following equations of equilibrium. First the tension T must be expressed as a vector by using the geometry of the directed line from B to

$$C : \overline{BC} = -.01\hat{\mathbf{i}} - .3\hat{\mathbf{j}} - .5\hat{\mathbf{k}} \quad \hat{b} = \frac{\overline{BC}}{|\overline{BC}|} = -0.017\hat{\mathbf{i}} - 0.514\hat{\mathbf{j}} - 0.857\hat{k}$$

so that

$$\mathbf{T} = T(\hat{b}) = T(-.017\hat{\mathbf{i}} - .514\hat{\mathbf{j}} - .857\hat{\mathbf{k}}).$$

Then force equilibrium becomes

$$\sum F_x = 0 : A_x - 0.017T - 25 = 0 \qquad (1)$$

$$\sum F_y = 0 : A_y - 0.514T - 10 = 0 \qquad (2)$$

$$\sum F_z = 0 : A_z - 0.857T + 800 = 0 \qquad (3)$$

(continued)

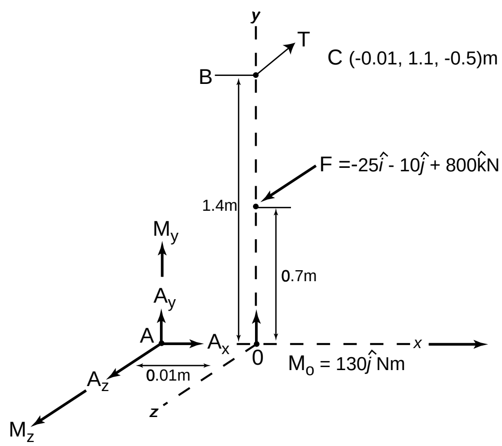

FIGURE S5.74

The moment equation about z is

$$M_y\hat{\mathbf{j}} + M_z\hat{\mathbf{k}} + (-.01\hat{\mathbf{i}}) \times (A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}) + (0.7\hat{\mathbf{j}}) \times (-25\hat{\mathbf{i}} - 10\hat{\mathbf{j}} + 800\hat{\mathbf{k}})$$

$$+130\hat{\mathbf{j}} + 1.4\hat{\mathbf{j}} \times T(-.017\hat{\mathbf{i}} - .514\hat{\mathbf{j}} - .857\hat{\mathbf{k}})$$

or in component form

$$\mathbf{i} : 560 - 1.1998T = 0 \tag{4}$$

$$\mathbf{j} : .01A_z + 130 + M_y = 0 \tag{5}$$

$$\hat{\mathbf{k}} : -.01A_y + M_z + .0238T + 17.5 = 0 \tag{6}$$

Solving these six equations in six unknowns yield

$$\underline{A_x = 32.935 \text{ N}}, \underline{A_y = 249.9 \text{ N}},$$

$$\underline{A_z = -400 \text{ N}},\ \underline{T = 466.744 \text{ N}},\ \underline{M_y = -126 \text{ N} \cdot \text{m}},$$

and

$$\underline{M_z = -26.11 \text{ N} \cdot \text{m}}$$

5.75 A lifting mechanism can be modeled as a pin and cable arrangement. Compute the reactions at A and the tension in the cable assuming the system is in equilibrium.

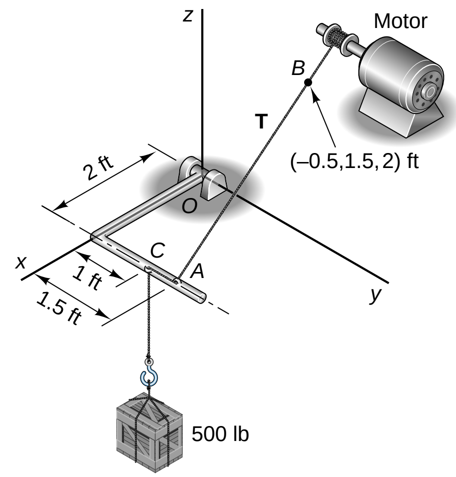

FIGURE P5.75

Solution: Consider the free body diagram with a pin at 0.

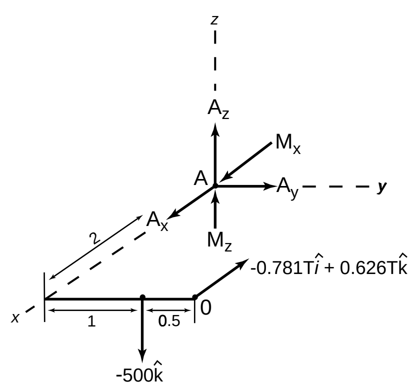

FIGURE S5.75

First calculate the direction cosines for the tension along

$$\overline{AB} = (-.5-2)\hat{\mathbf{i}} + (1.5-1.5)\hat{\mathbf{j}} + (2-0)\hat{\mathbf{k}}, \ \hat{a} = \frac{\overline{AB}}{\overline{AB}} = -0.781\hat{\mathbf{i}} + 0.625\hat{\mathbf{k}}$$

so that

(continued)

$$\mathbf{T} = -0.781T\hat{\mathbf{i}} + 0.625T\hat{\mathbf{k}}.$$

The equilibrium of forces yields

$$\sum F_x = 0 : A_x - 0.781T = 0 \qquad (1)$$

$$\sum F_y = 0 : A_y = 0 \qquad (2)$$

$$\sum F_z = 0 : A_z + .625T - 500 = 0 \qquad (3)$$

The moment equation about A yields

$$\sum \mathbf{M}_A = M_x\hat{\mathbf{i}} + M_z\hat{\mathbf{k}} + (2\hat{\mathbf{i}} + \hat{\mathbf{j}}) \times (-500\hat{\mathbf{k}}) + (2\hat{\mathbf{i}} + 1.5\hat{\mathbf{j}}) \times (-0.781T\hat{\mathbf{i}} + 0.625T\hat{\mathbf{k}}) = 0.$$

By components this becomes

$$\hat{\mathbf{i}} : M_x + 0.9375T = 500 \qquad (4)$$

$$\hat{\mathbf{j}} : -1.25T = -1000 \qquad (5)$$

$$\hat{\mathbf{k}} : M_z + 1.1715T = 0 \qquad (6)$$

These 6 equations in 6 unknowns have the solution

$$\underline{A_x = 625 \text{ lb}},\ \underline{A_y = 0},\ \underline{A_z = 0 \text{ lb}},\ \underline{T = 800 \text{ lb}},\ \underline{M_y = -250 \text{ lb} \cdot \text{ft}}$$

and

$$\underline{M_z = -937 \text{ lb} \cdot \text{ft}}.$$

5.76 The bar AE forms one component of a lifting mechanism. If the connection at A is modeled as a ball and socket and the slider connection at E is modeled as providing only a single force reaction along the z axis compute the tensions T_1 and T_2 as well as the reaction forces at A and E

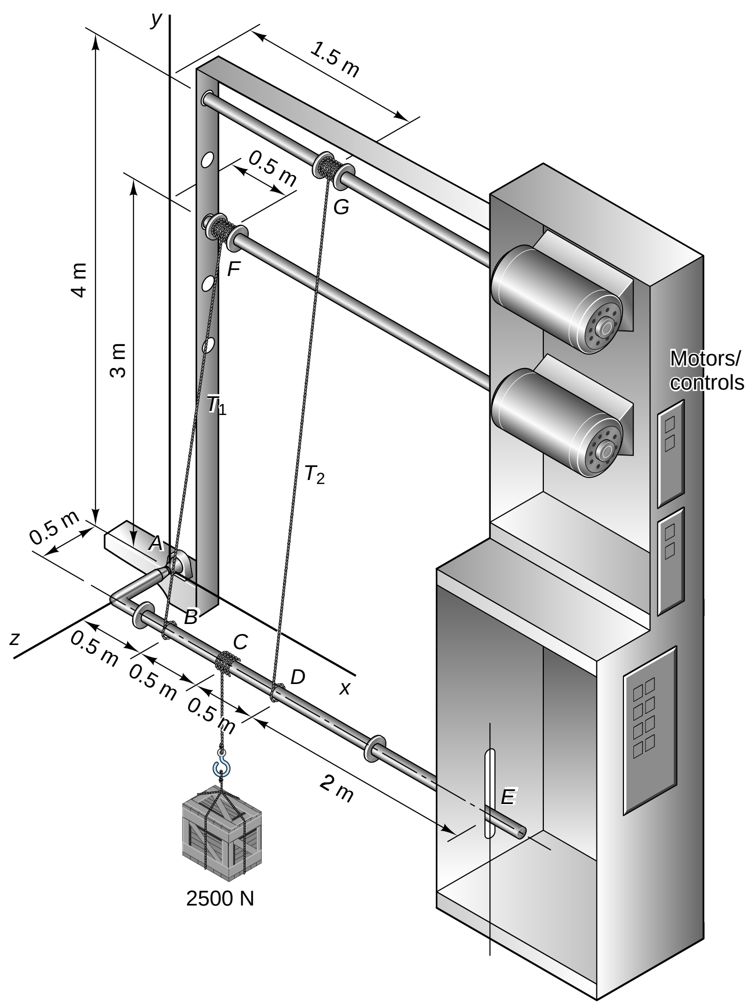

FIGURE P5.76

Solution: The free body diagram is given in the figure. From the geometry the ropes lie along the lines

(continued)

$$\overline{BF} = (.5 - .5)\hat{\mathbf{i}} + (3 - 0)\hat{\mathbf{j}} + (0 - .5)\hat{\mathbf{k}}$$

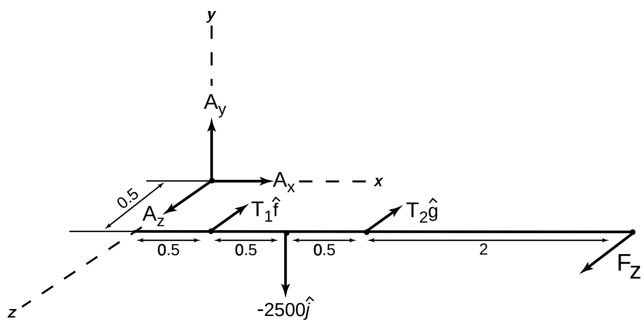

FIGURE S5.76

and

$$\underline{DG} = (1.5 - 1.5)\hat{\mathbf{i}} + (4 - 0)\hat{\mathbf{j}} + (0 - .5)\hat{\mathbf{k}}$$

so that a unit vector along BF is

$$\hat{f} = .986\hat{\mathbf{j}} - .164\hat{\mathbf{k}}$$

and along DG is

$$\hat{g} \equiv .992\hat{\mathbf{j}} - .724\hat{\mathbf{k}}.$$

The two tensions can then be written as

$$\mathbf{T}_1 = .986T_1\hat{\mathbf{j}} - 0.164T_1\hat{\mathbf{k}}$$

and

$$\mathbf{T}_2 = .992T_2\hat{\mathbf{j}} - .724T_2\hat{\mathbf{k}}.$$

The force equilibrium equation becomes

$$\sum F_x = 0: \quad A_x = 0 \tag{1}$$

$$\sum F_y = 0: \quad A_y + .986T_1 + .992T_2 = 2500 \tag{2}$$

$$\sum F_z = 0: \quad A_z - .164T_1 - .124T_2 + F_z = 0 \tag{3}$$

Taking moments about the origin yields

$$\mathbf{M}_A = (.5\hat{\mathbf{i}} + .5\hat{\mathbf{k}}) \times \mathbf{T}_1 + (\hat{\mathbf{i}} + .5\hat{\mathbf{k}}) \times (-2500\hat{\mathbf{j}})$$

$$+(1.5\hat{\mathbf{i}} + .5\hat{\mathbf{k}}) \times \mathbf{T}_2 + (3.5\hat{\mathbf{i}} + .5\hat{\mathbf{k}}) \times F_z\hat{\mathbf{k}} = 0$$

$$\mathbf{i}: -.493T_1 + 1250 - .496T_2 = 0 \tag{4}$$

$$\hat{\mathbf{j}}: .082T_1 + 0.86T_2 - 3.5F_z = 0 \tag{5}$$

$$\hat{\mathbf{k}}: .493T_1 - 2500 + 1.488T_2 = 0 \tag{6}$$

$\underline{A_x = A_y = 0}$, $\underline{A_z = 267.5 \text{ N}}$, $\underline{T_1 = 1268 \text{ N}}$, $\underline{T_2 = 1260 \text{ N}}$, $\underline{F_z = 96.7 \text{ N}}$, 5.77 Repeat problem 5.76 with point G placed at 2.0 m from the y axis.

Solution: The free body diagram is similar to that of the previous problem except the T_2 now has an x component. The line BF and the unit vector $\hat{f}$ and the form of T_1 are as given in the solution to 5.76.

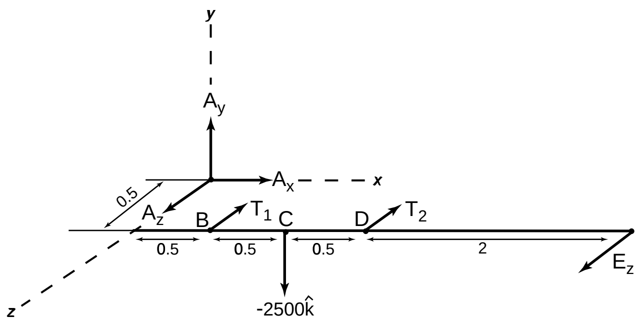

FIGURE S5.77

The vector $\overline{DG}$ becomes

$$\overline{DG} = (2 - 1.5)\hat{\mathbf{i}} + (4 - 0)\hat{\mathbf{j}} + (0 - .5)\hat{\mathbf{k}}$$

and

$$T_2\hat{g} = T_2\overline{DG}/|\overline{DG}| = 0.123T_2\hat{\mathbf{i}} + 0.985T_2\hat{\mathbf{j}} - 0.123T_2\hat{\mathbf{k}}.$$

Equilibrium yields

$\sum F_x = 0 : A_x + 0.123T_2 = 0$ (1)

$\sum F_y = 0 : A_y + .986T_1 + .985T_2 = 2500$ (2)

$\sum F_z = 0 : A_z - 0.164T_1 - 0.123T_2 + F_z = 0$ (3)

The moment equation is

$$\sum \mathbf{M}_0 = (.5\hat{\mathbf{i}}+.5\hat{\mathbf{k}})\times\mathbf{T}_1+(\hat{\mathbf{i}}+.5\hat{\mathbf{k}})\times(-2500\hat{\mathbf{j}})+(1.5\hat{\mathbf{i}}+.5\hat{\mathbf{k}})\times\mathbf{T}_2+(3.5\hat{\mathbf{i}}+.5\hat{\mathbf{k}})\times(F_z\hat{\mathbf{k}}) = 0$$

or in component form

$\hat{\mathbf{i}} : -.493T_1 - .4925T_2 = -1250$ (4)

$\hat{\mathbf{j}} : .082T_1 + .246T_2 - 3.5F_z = 0$ (5)

(continued)

$$\hat{\mathbf{k}} : .493T_1 + 1.4775T_2 = 2500 \tag{6}$$

which has solution

$$\underline{A_x = -156 \text{ N}},\ \underline{A_y = 0 \text{ N}},\ \underline{A_z = 245 \text{ N}},\ \underline{T_1 = 1268 \text{ N}},\ \underline{T_2 = 1269 \text{ N}}$$

and

$$\underline{F_z = 119 \text{ N}}.$$

5.78 A sign is fixed in place by two cables, a ball and socket at point A and a frictionless support or ball at B. Compute the reaction forces at A and B as well as the tension in the two cables given the mass of the sign is 15.29 kg and a wind load of $W = 50\hat{\mathbf{j}} - 25\hat{\mathbf{k}}$ N acts on the sign.

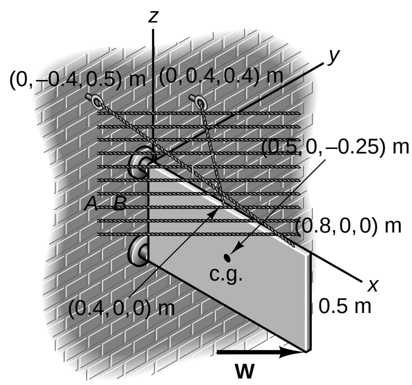

FIGURE P5.78

Solution: Note there are 6 unknowns and 6 equations. First the tensions $\mathbf{T}_1$ and $\mathbf{T}_2$ need to be written as vectors

$$\mathbf{t}_1 = (-.8)\hat{\mathbf{i}} - .4\hat{\mathbf{j}} + .5\hat{\mathbf{k}}_1$$

so that

$$\mathbf{T}_1 = T_1(\mathbf{t}_1/|\mathbf{t}_1|) = T_1(-0.781\hat{\mathbf{i}} - 0.39\hat{\mathbf{j}} + 0.488\hat{\mathbf{k}})$$

$$\mathbf{t}_2 = (-.4\hat{\mathbf{i}} + .4\hat{\mathbf{j}} + .4\hat{\mathbf{k}})$$

so that

$$\mathbf{T}_2 = T_2(\mathbf{t}_2/|\mathbf{t_2}|) = T_2(.577)(-\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}).$$

From the free body diagram illustrated, the force equilibrium equations are

$$\sum F_x = 0 : A_x + B_x - 0.781T_1 - 0.577T_2 = 0 \qquad (1)$$

$$\sum F_y = 0 : A_y - 0.39T_1 + .577T_2 + 50 = 0 \qquad (2)$$

$$\sum F_z = 0 : A_z + .488T_1 + .577T_2 = 175 \qquad (3)$$

(continued)

The moment equilibrium equation becomes

$$\sum \mathbf{M}_B = 0$$

or

$$(.4\hat{\mathbf{i}}) \times \mathbf{T}_2 + (.8\hat{\mathbf{i}} \times \mathbf{T}_1) + (-.5\hat{\mathbf{k}}) \times (A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}) + (.5\hat{\mathbf{i}} - .25\hat{\mathbf{k}})$$

$$\times(-150\hat{\mathbf{k}}) + (\hat{\mathbf{i}} - .5\hat{\mathbf{k}}) \times (50\hat{\mathbf{j}} - 25\hat{\mathbf{k}}) = 0$$

or in component form

$$\hat{\mathbf{i}} : 0.5A_y = -25 \quad (4)$$

$$\hat{\mathbf{j}} : -.5A_x - 0.3904T_1 - 0.2308T_2 = -100 \quad (5)$$

$$\hat{\mathbf{k}} : .2340T_2 - .312T_1 = -50 \quad (6)$$

These 6 equations have the solution:

$$\underline{A_x = -150N},\ \underline{A_y = -50 \text{ N}},\ \underline{A_z = -106 \text{ N}},$$

$$\underline{B_x = 526 \text{ N}},\ \underline{T_1 = 321 \text{ N}},\ \underline{T_2 = 217 \text{ N}}$$

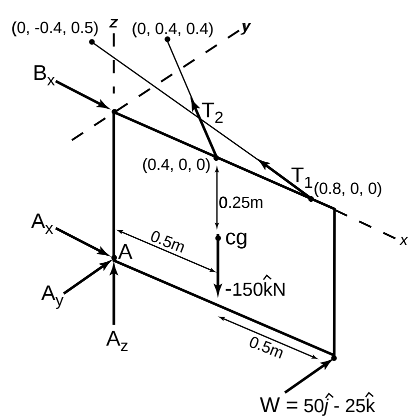

FIGURE S5.78

5.79 A piping system is held in place by a ball and socket at point A and a loose fitting ring at B (supporting negligible moments) as it protrudes from the wall. The rod (rigid link) connection is used to prevent movement of the pipe. The fluid rushing through the pipe is modeled as a force acting at point C of

$$\mathbf{F} = 50\hat{\mathbf{i}} + 20\hat{\mathbf{k}} \text{ N}$$

and a moment of

$$\mathbf{M}_c = 100\hat{\mathbf{i}} + 200\hat{\mathbf{k}} \text{ Nm.}$$

Calculate the reaction forces at A and B, as well as the force T.

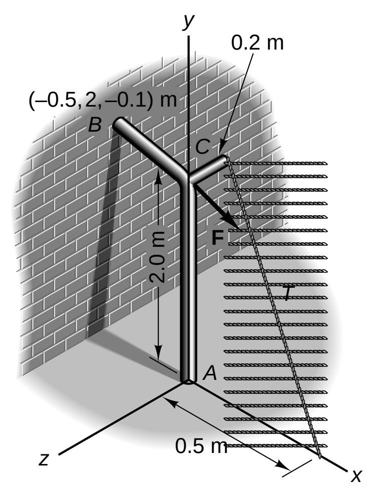

FIGURE P5.79

Solution: First write $\mathbf{T}$ as a vector

$$\mathbf{rt} = (.5\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 0.2\hat{\mathbf{k}})$$

so that

$$\mathbf{T} = T(\mathbf{rt}/|\mathbf{rt}|) = T(0.241\hat{\mathbf{i}} - 0.966\hat{\mathbf{j}} + 0.097\hat{\mathbf{k}}).$$

Then the free body diagram reveals the following force equilibrium

(continued)

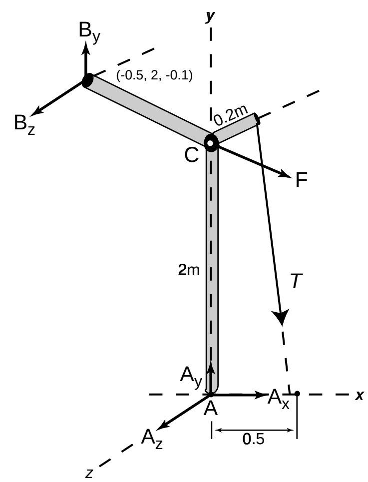

FIGURE S5.79

$$\sum F_x = 0 : A_x + 0.241T = -50 \tag{1}$$

$$\sum F_y = 0 : A_y + B_y - 0.966T = 0 \tag{2}$$

$$\sum F_z = 0 : A_z + B_z + 0.097T = -20 \tag{3}$$

Taking the moment about A yields:

$$100\hat{\mathbf{i}} + 200\hat{\mathbf{k}} + (-0.5\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 0.1\hat{\mathbf{k}}) \times (B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}) + 2\hat{\mathbf{j}} \times (50\hat{\mathbf{i}} + 20\hat{\mathbf{k}})$$

$$+(2\hat{\mathbf{j}} - 0.2\hat{\mathbf{k}}) \times T(0.241\hat{\mathbf{i}} - 0.966\hat{\mathbf{j}} + 0.097\hat{\mathbf{k}}) = 0$$

$\underline{A_x = 350\text{ N}}$, $\underline{A_y = -3400\text{ N}}$,
$\underline{A_z = 300\text{ N}}$, $\underline{B_y = 1800\text{ N}}$,
$\underline{B_z = -160\text{ N}}$, $\underline{T = -1657\text{ N}}$

5.80 A power line pole is held in place by a ball and socket connection at A, two cables at C and B and a rod at D. Ignoring the mass of the pole compute the reaction forces at A, D, C and B for the load indicated.

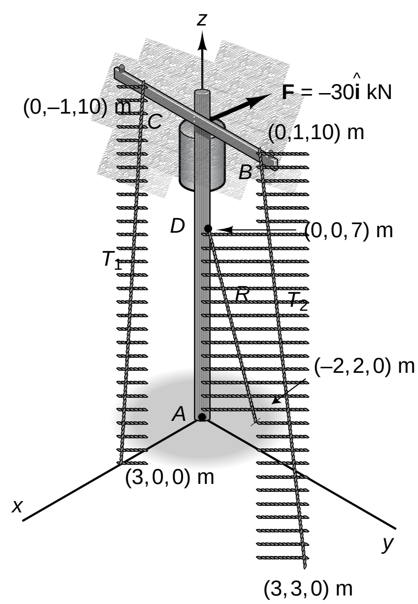

FIGURE P5.80

Solution: First establish unit vectors along the lines T_1, T_2 and R:

$$\mathbf{C} = (3\hat{\mathbf{i}} + \hat{\mathbf{j}} - 10\hat{\mathbf{k}})$$

and

$$\hat{\mathbf{c}} = \frac{\mathbf{c}}{|\mathbf{c}|} = 0.286\hat{\mathbf{i}} + 0.095\hat{\mathbf{j}} - 0.953\hat{k}$$

and

$$\mathbf{T}_1 = T_1\hat{\mathbf{c}}.$$

Likewise

$$\mathbf{T}_2 = T_2(0.282\hat{\mathbf{i}} + 0.188\hat{\mathbf{j}} - 0.941\hat{\mathbf{k}}).$$

And

$$\mathbf{R} = R(0.265\hat{\mathbf{i}} - 0.265\hat{\mathbf{j}} + 0.927\hat{\mathbf{k}}).$$

The free body diagram can then be used to write the equations of equilibrium. For forces

(continued)

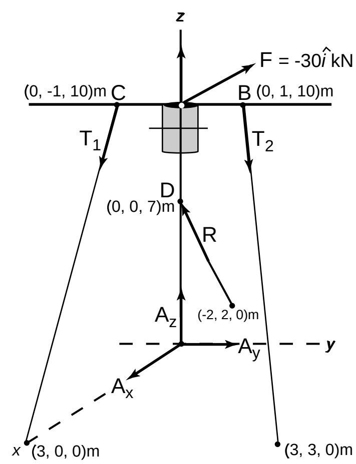

FIGURE S5.80

$$\sum F_x = 0 : A_x + 0.286T_1 + 0.282T_2 + 0.265R = 30 \quad (1)$$

$$\sum F_y = 0 : A_y + 0.095T_1 + 0.188T_2 - 0.265R = 0 \quad (2)$$

$$\sum F_z = 0 : A_z - 0.953T_1 - 0.941T_2 + 0.927R = 0 \quad (3)$$

The moment equation about A is

$$(3\hat{\mathbf{i}} \times \mathbf{T}_1) + (3\mathbf{i} + 3\hat{\mathbf{j}}) \times \mathbf{T}_2 + (-2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) \times \mathbf{R} + (10\hat{\mathbf{k}}) \times (-30\hat{\mathbf{i}}) = 0$$

or in compoment form

$$\mathbf{i} : -2.823T_2 + 1.854R = 0 \quad (4)$$

$$\hat{\mathbf{j}} : 2.859T_1 + 2.823T_2 + 1.854R - 300 = 0 \quad (5)$$

$$\hat{\mathbf{k}} : .285T_1 - .282T_2 = 0 \quad (6)$$

$$\underline{A_x = -6.616 \text{ kN}},\ \underline{A_y = 4.594 \text{ kN}}, \underline{A_z = 17.863 \text{ kN}},$$

$$\underline{T_1 = 37.41 \text{ kN}}, \underline{T_2 = 37.808 \text{ kN}},\ \underline{R = 57.568\text{kN}}$$