Temperature and Heat

Problem 1
How much heat is required to change 1.0 kg of ice, originally at –20.0°C, into steam at 110.0°C? Assume 1.0 atm of pressure.

Useful constants:

$c_{water} = 4186$ J/(kg C°)
$c_{ice} = 2.00 \times 10^3$ J/(kg C°)
$c_{steam} = 2.00 \times 10^3$ J/(kg C°)

$L_f = 33.5 \times 10^4$ J/kg
$L_v = 22.6 \times 10^5$ J/kg

Note: There are 5 separate terms necessary to calculate the heat required to turn ice at $-20.0°C$ into steam at $110.0°C$

ice warming to 0°C $\longrightarrow$ $Q = m c_{ice} \Delta T$

ice melting $\longrightarrow$ $Q = m L_f$

water warming to 100°C $\longrightarrow$ $Q = m c_{water} \Delta T$

water turning to steam $\longrightarrow$ $Q = m L_v$

steam warming to 110°C $\longrightarrow$ $Q = m c_{steam} \Delta T$

$$Q = m_{ice} c_{ice} \underbrace{\Delta T}_{20.0 C°} + m_{ice} L_f + m_{water} c_{water} \underbrace{\Delta T}_{100.0 C°} + m_{water} L_v + m_{steam} c_{steam} \underbrace{\Delta T}_{10.0 C°}$$

Note: $m_{ice} = m_{water} = m_{steam} = 1.0\,\text{Kg} \equiv m$

$$Q = m\left[c_{ice}\Delta T + L_f + c_{water}\Delta T + L_v + c_{steam}\Delta T\right]$$

$$Q = (1.0\,\text{Kg})\Big[(2.00\times 10^3\,\text{J/KgC°})(20.0\,\text{C°}) + 33.5\times 10^4\,\text{J/Kg} +$$
$$(4186\,\text{J/KgC°})(100.0\,\text{C°}) + 22.6\times 10^5\,\text{J/Kg} + (2.00\times 10^3\,\text{J/KgC°})(10.0\,\text{C°})\Big]$$

$$\boxed{Q = 3.1\times 10^6\,\text{J}}$$

Problem 2

What mass of water at 95.0 °C must be mixed with 150.0 g of ice at –5.00 °C, in a thermally insulated container, to produce liquid water at 50.0 °C?

Useful constants:

$c_{water} = 4186$ J/(kg C°)
$c_{ice} = 2.00 \times 10^3$ J/(kg C°)
$L_f = 33.5 \times 10^4$ J/kg
$L_v = 22.6 \times 10^5$ J/kg

Note: using calorimetry, we know that

$$Q_{\text{lost by water}} = Q_{\text{gained by ice}}$$

$$Q_{\text{lost by water}} = m_{water} C_{water} \overset{45.0^\circ C}{\overset{=}{\Delta T_{water}}}$$

$$Q_{\text{gain by ice}} = m_{ice} C_{ice} \overset{5.00^\circ C}{\overset{=}{\Delta T_{ice}}} + m_{ice} L_f + m_{ice} C_{water} \overset{50.0^\circ C}{\overset{=}{\Delta T}}$$

ice warming to - 5.00°C

ice melting

water warming to 50.0° C

$$m_{water} C_{water} \Delta T_{water} = m_{ice} C_{ice} \Delta T_{ice} + m_{ice} L_f + m_{ice} C_{water} \Delta T$$

$$m_{water} = \frac{m_{ice} (C_{ice} \Delta T_{ice} + L_f + C_{water} \Delta T)}{C_{water} \Delta T_{water}}$$

$$m_{water} = \frac{(0.150\,Kg) \left[(2.00 \times 10^3\, J/Kg\,C^\circ)(5.00^\circ C) + 33.5 \times 10^4\, J/Kg + (4186 \tfrac{J}{Kg\,C^\circ})(50.0^\circ C) \right]}{(4186\, J/Kg\,C^\circ)(45.0^\circ C)}$$

$$\boxed{m_{water} = 0.441\,Kg = 441\,g}$$

Note: when using $Q_{gained} = Q_{lost}$ to solve calorimetry problems, Q is always > 0 so we must use $\Delta T > 0$, even if the temperature decreased

Problem 3

One end of an iron poker is placed in a fire where the temperature is 502 °C, and the other end is kept at a temperature of 26 °C. The poker is 1.2 m long and has a radius of 5.0×10^{-3} m. Ignoring the heat lost along the length of the poker, find the amount of heat conducted from one end of the poker to the other in 5.0 s.

Note: the heat Q conducted during a time t through a bar of length L and cross-sectional area A is given by:

$$Q = \frac{(KA\Delta T)t}{L}$$

where ΔT is temperature difference between the ends of the bar and K is thermal conductivity of the material.

$$\Delta T = 502\,°C - 26\,°C$$
$$= 476\,C°$$

$$L = 1.2\,m$$

$$A = \pi r^2 = \pi (5.0 \times 10^{-3}\,m)^2$$
$$= 7.85 \times 10^{-5}\,m^2$$

$$t = 5.0\,s$$

$$K_{iron} = 79\,J/s \cdot m \cdot C°$$

$$Q = \frac{(KA\Delta T)t}{L} = \frac{(79\,\frac{J}{s \cdot m \cdot C°})(7.85 \times 10^{-5}\,m^2)(476\,C°)(5.0\,s)}{(1.2\,m)}$$

$$\boxed{Q = 12.3\,J}$$