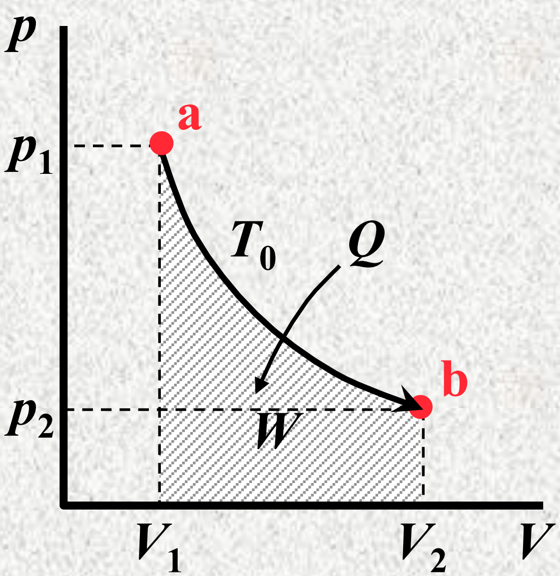
Isothermal process on *p-V*, *T-V*, and *p-T* diagrams

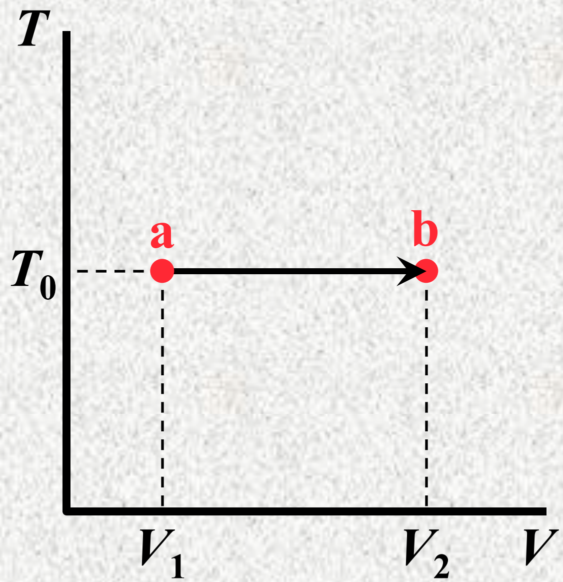
$$\text{isothermal} \Rightarrow T = T_0 = \text{constant}$$

$$\mathbf{a} = (p_1, V_1, T_0) \qquad \mathbf{b} = (p_2, V_2, T_0)$$

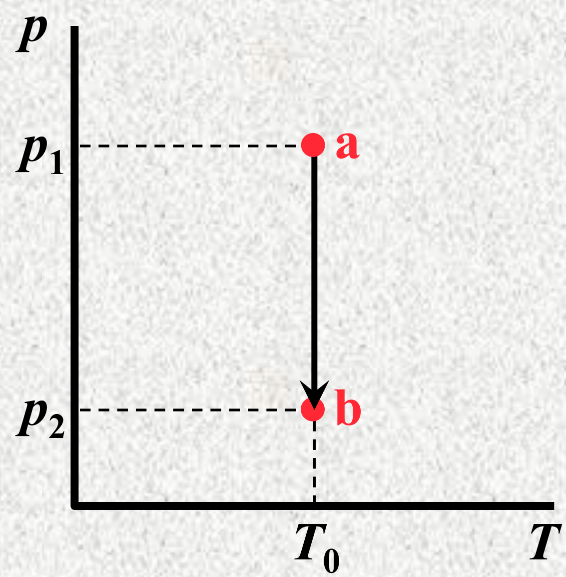
$$pV = nRT_0$$



$$p(V) = \frac{nRT_0}{V}$$



$$T(V) = T_0$$



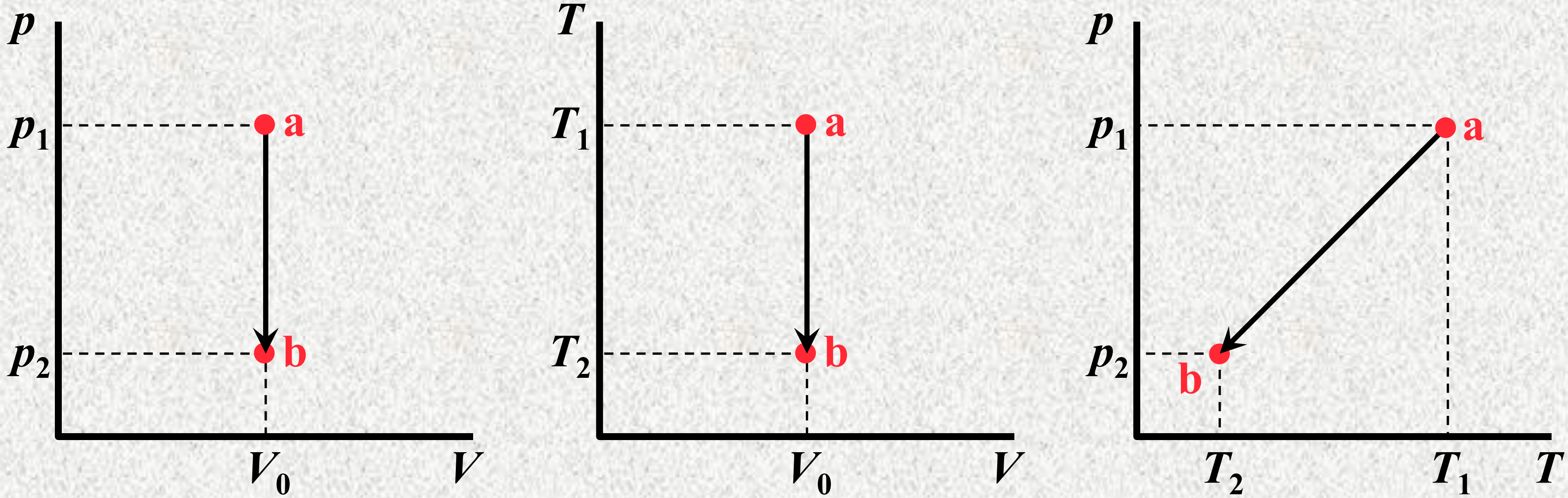
$$p(T) = \text{multivalued}$$

Isochoric process on *p-V*, *T-V*, and *p-T* diagrams

$$\text{isochoric} \Rightarrow V = V_0 = \text{constant}$$

$$\mathbf{a} = (p_1, V_0, T_1) \qquad \mathbf{b} = (p_2, V_0, T_2)$$

$$pV_0 = nRT$$



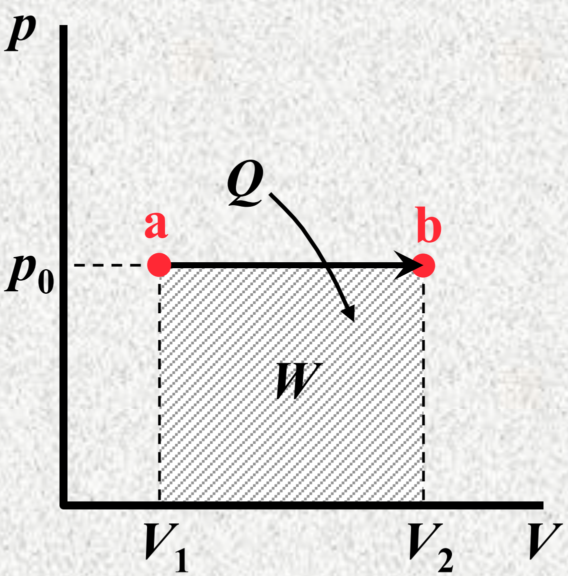
$p(V)$ = multivalued $\qquad T(V)$ = multivalued $\qquad p(T) = \dfrac{nRT}{V_0}$

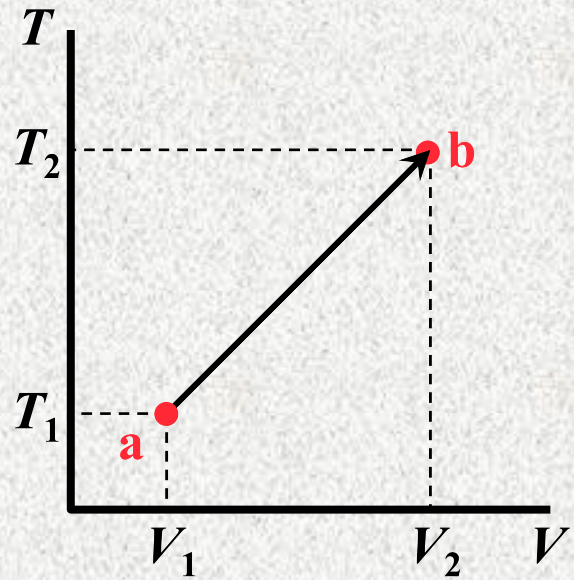
Isobaric process on *p-V*, *T-V*, and *p-T* diagrams

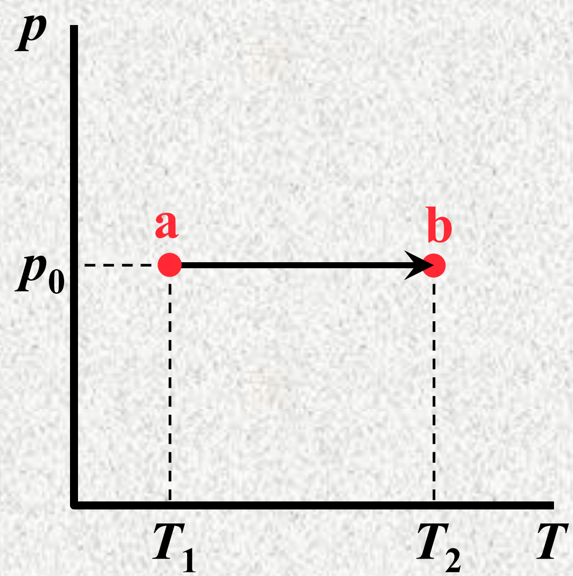
$$\text{isobaric} \Rightarrow p = p_0 = \text{constant}$$

$$\mathbf{a} = (p_0, V_1, T_1) \qquad \mathbf{b} = (p_0, V_2, T_2)$$

$$p_0 V = nRT$$



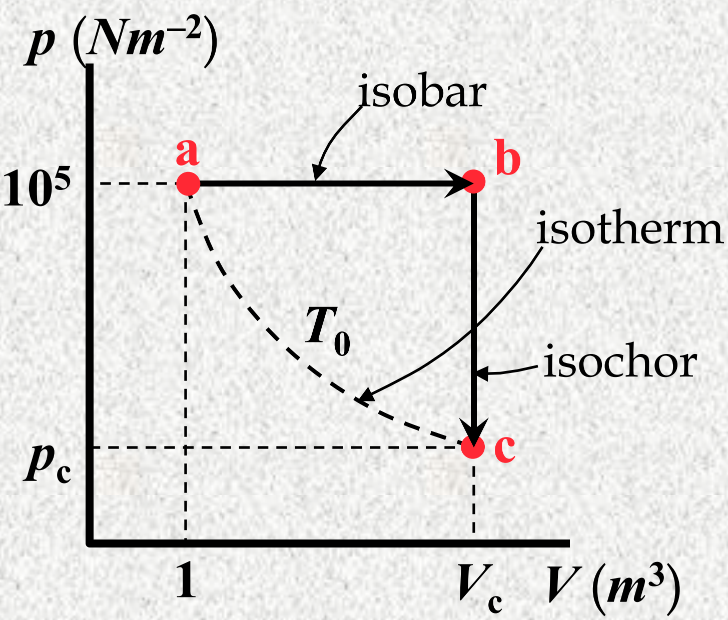




$$p(V) = p_0 \qquad T(V) = \frac{p_0 V}{nR} \qquad p(T) = p_0$$

Clicker question 1

Consider the *p-V* diagram below in which the system evolves from **a → b → c**. If T_0 ~ 240K (and thus RT_0 = 2,000 J mol^{-1}), how many moles of gas, n, are in the system?



a) **5**

b) **10⁵**

c) **50**

d) **1,000**

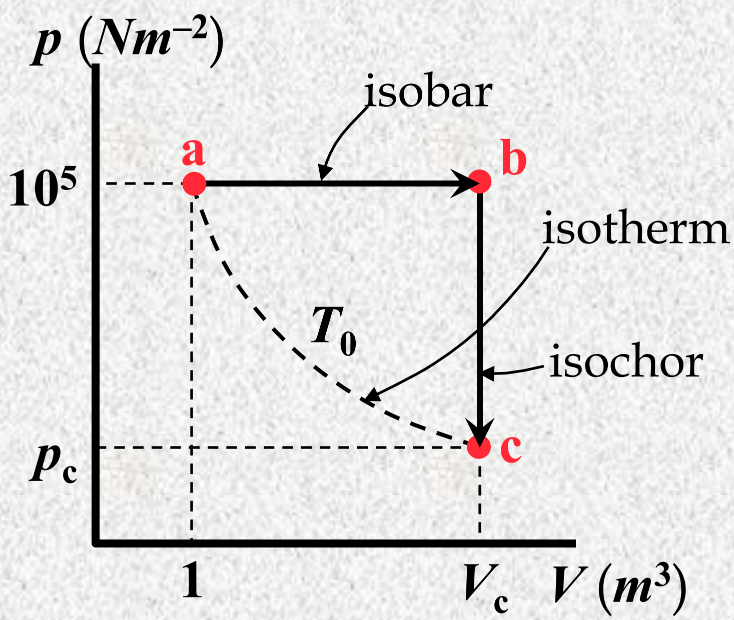
e) Not enough information to tell

first law of thermodynamics: $\Delta E_{int} = Q - W \; (= nC_V \Delta T)$

ideal gas law: $pV = nRT$

Clicker question 1

Consider the *p-V* diagram below in which the system evolves from **a → b → c**. If T_0 ~ 240K (and thus RT_0 = 2,000 J mol^{-1}), how many moles of gas, n, are in the system?



a) 5

b) 10^5

c) 50 ✔

d) 1,000

e) Not enough information to tell

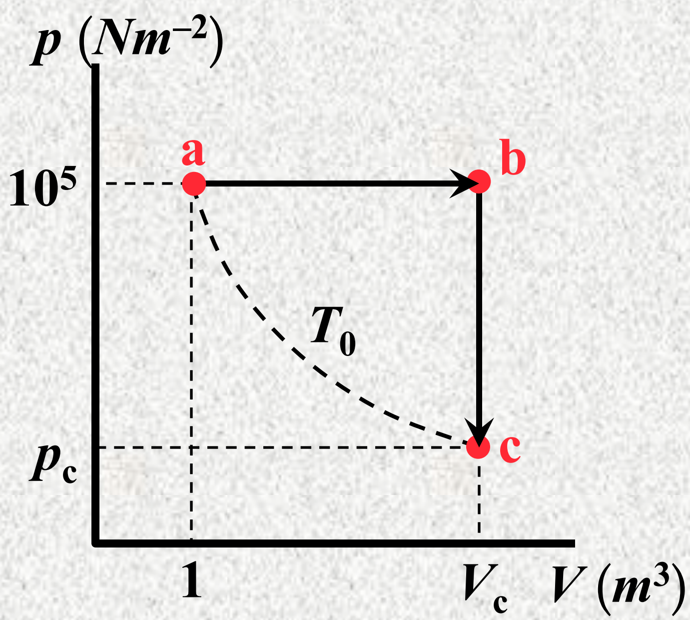
$$n = \frac{pV}{RT_0} = \frac{100{,}000}{2{,}000} = 50$$

first law of thermodynamics: $\Delta E_{int} = Q - W \;(= nC_V\Delta T)$

ideal gas law: $pV = nRT$

Clicker question 2

Consider the *p*-*V* diagram below in which the system evolves from **a** → **b** → **c**. What is V_c, the volume at state **c**?



a) $0.5\ m^3$

b) $2.0\ m^3$

c) $4.0\ m^3$

d) $8.0\ m^3$

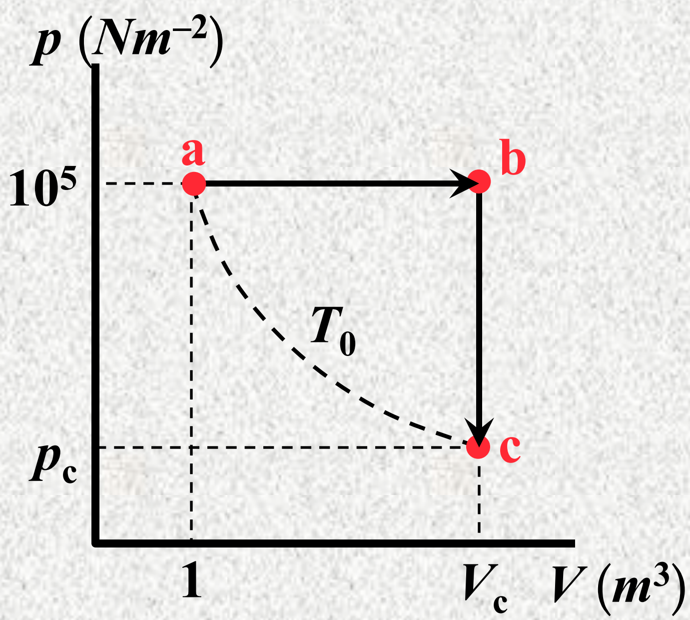
e) Not enough information to tell

first law of thermodynamics: $\Delta E_{int} = Q - W\ (= nC_V \Delta T\ )$

ideal gas law: $pV = nRT$

Clicker question 2

Consider the p-V diagram below in which the system evolves from **a** $\rightarrow$ **b** $\rightarrow$ **c**. What is V_c, the volume at state **c**?



a) $0.5\ m^3$

b) $2.0\ m^3$

c) $4.0\ m^3$

d) $8.0\ m^3$

e) Not enough information to tell

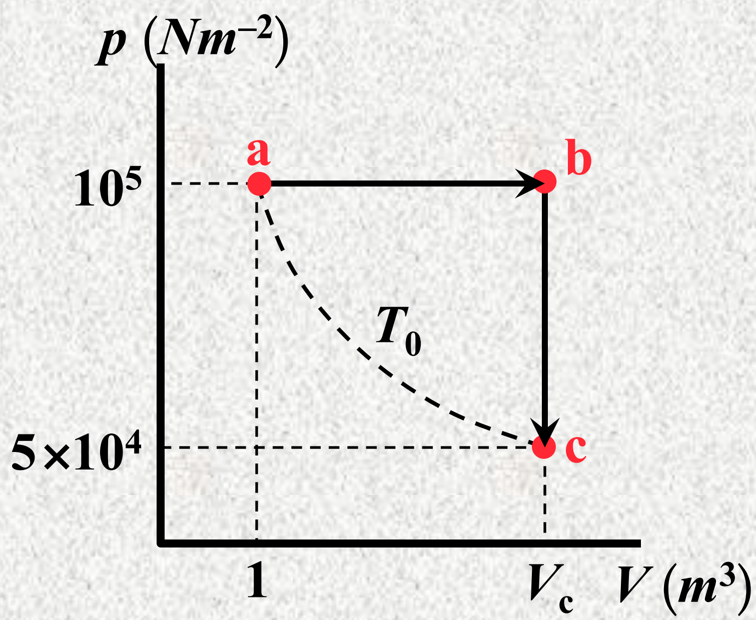
need to know p_c

first law of thermodynamics: $\Delta E_{int} = Q - W \ (= nC_V\Delta T)$

ideal gas law: $pV = nRT$

Clicker question 3

Consider the *p-V* diagram below in which the system evolves from **a → b → c**. What is V_c, the volume at state **c**?



a) $0.5\ m^3$

b) $2.0\ m^3$

c) $4.0\ m^3$

d) $8.0\ m^3$

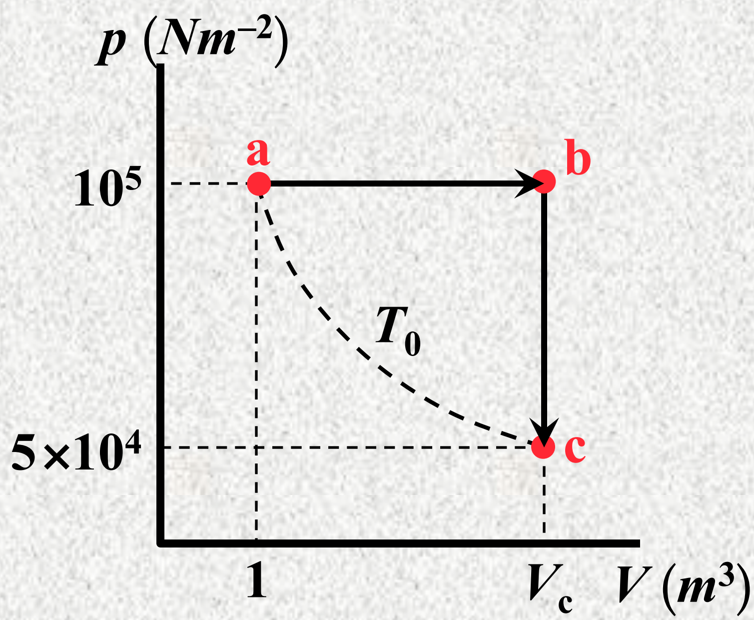
e) Not enough information to tell

first law of thermodynamics: $\Delta E_{int} = Q - W\ (= nC_V\Delta T)$

ideal gas law: $pV = nRT$

Clicker question 3

Consider the *p-V* diagram below in which the system evolves from **a** → **b** → **c**. What is V_c, the volume at state **c**?



a) $0.5\ m^3$

b) $2.0\ m^3$ ✔

c) $4.0\ m^3$

d) $8.0\ m^3$

e) Not enough information to tell

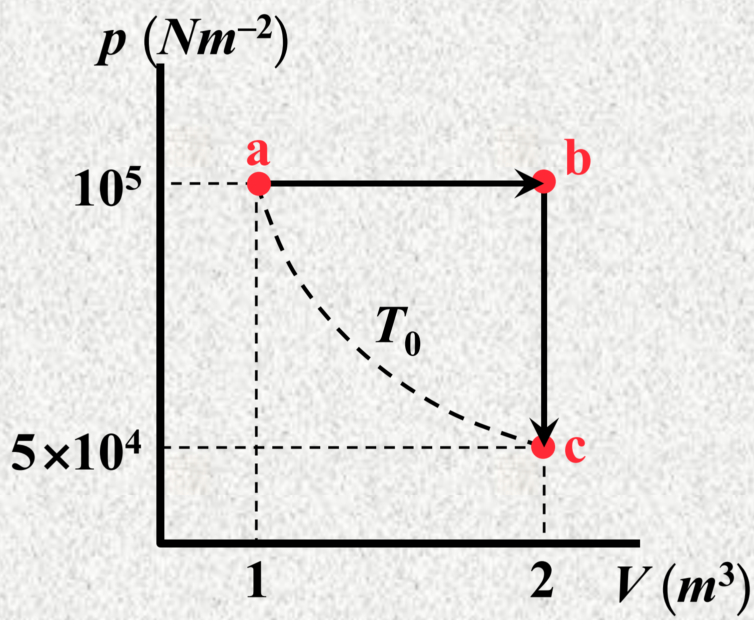
$$p_c V_c = p_a V_a \Rightarrow V_c = \frac{p_a}{p_c} V_a = 2\ m^3$$

first law of thermodynamics: $\Delta E_{int} = Q - W\ (= nC_V \Delta T)$

ideal gas law: $pV = nRT$

Clicker question 4

Consider the *p-V* diagram below in which the system evolves from a → b → c. What is the net change in internal energy, ΔE_{int}?



a) $0\ J$

b) $5.0\times10^4\ J$

c) about $7.0\times10^4\ J$

d) $10^5\ J$

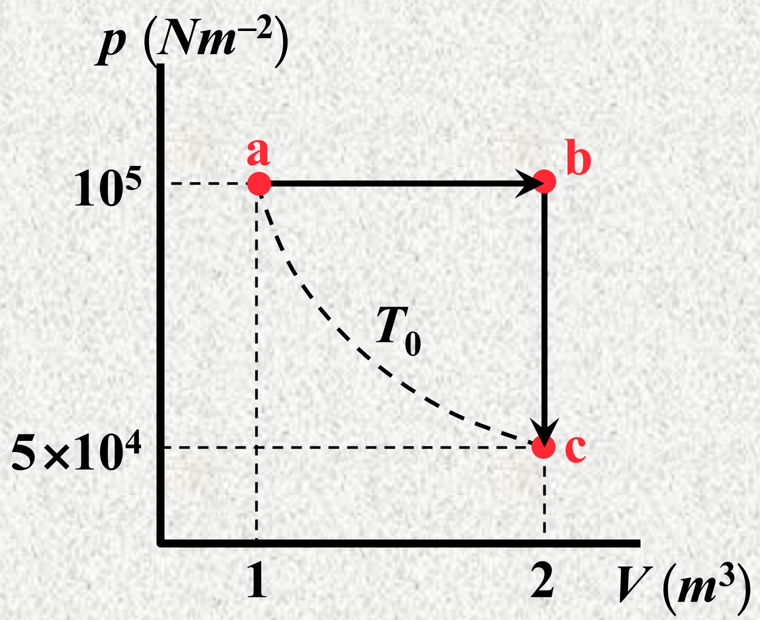
e) Not enough information to tell

first law of thermodynamics: $\Delta E_{int} = Q - W\ (= nC_V\Delta T)$

ideal gas law: $pV = nRT$

Clicker question 4

Consider the p-V diagram below in which the system evolves from a → b → c. What is the net change in internal energy, ΔE_{int}?



a) $0\ J$

b) $5.0\times10^{4}\ J$

c) about $7.0\times10^{4}\ J$

d) $10^{5}\ J$

e) Not enough information to tell

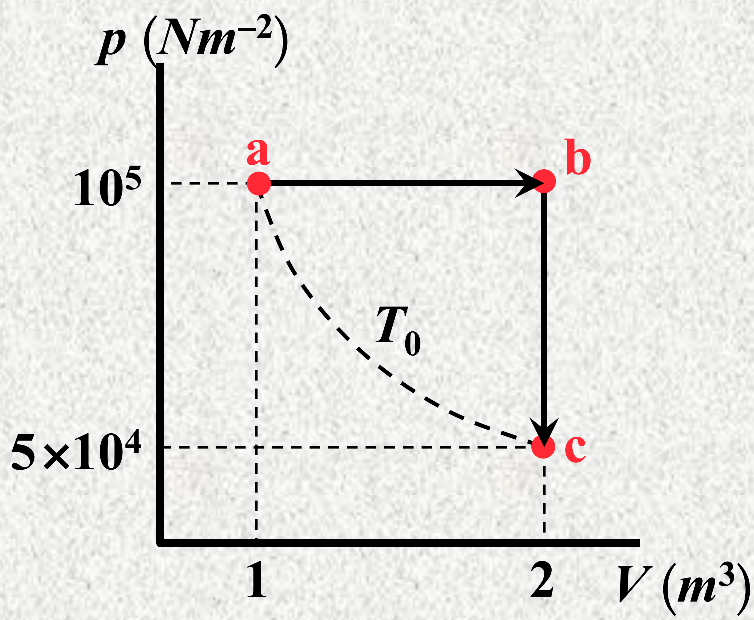
$$\Delta E_{int} = nC_V\Delta T$$

first law of thermodynamics: $\Delta E_{int} = Q - W \ (= nC_V\Delta T)$

ideal gas law: $pV = nRT$

Clicker question 5

Consider the *p*-*V* diagram below in which the system evolves from **a → b → c**. What is the net work done *by* the system *on* its environment, W?



a) **0 *J***

b) 5.0×10^4 *J*

c) about 7.0×10^4 *J*

d) 10^5 *J*

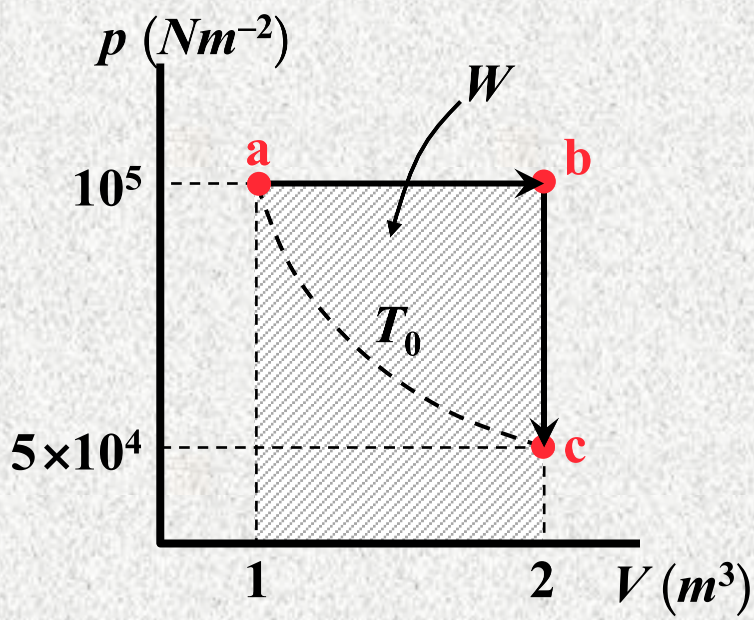
e) Not enough information to tell

first law of thermodynamics: $\Delta E_{int} = Q - W \;(= nC_V\Delta T)$

ideal gas law: $pV = nRT$

Clicker question 5

Consider the p-V diagram below in which the system evolves from **a → b → c**. What is the net work done _by_ the system _on_ its environment, W?



a) $0\ J$

b) $5.0\times10^4\ J$

c) about $7.0\times10^4\ J$

d) $10^5\ J$

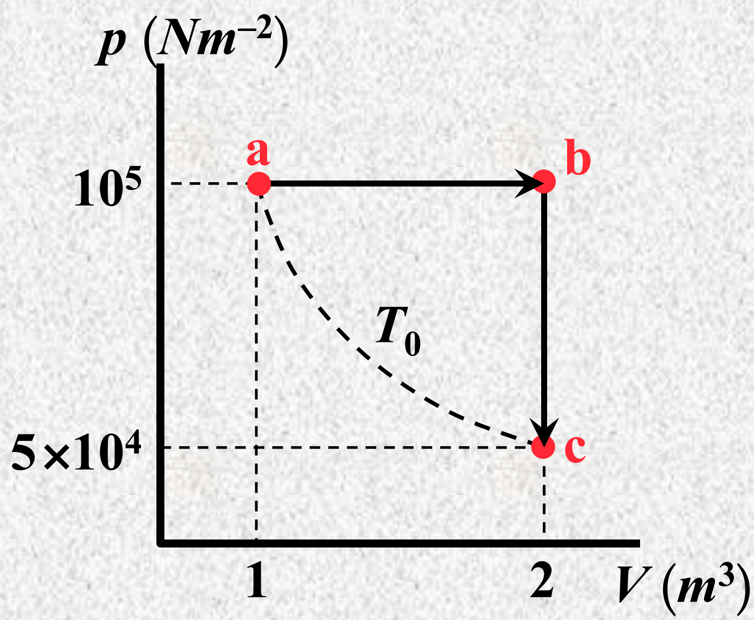
e) Not enough information to tell

first law of thermodynamics: $\Delta E_{int} = Q - W \ (= nC_V\Delta T)$

ideal gas law: $pV = nRT$

Clicker question 6

Consider the *p-V* diagram below in which the system evolves from **a → b → c**. What is the net heat transferred into the system, *Q*?



a) $-5.0\times10^4\ J$

b) $5.0\times10^4\ J$

c) $-10^5\ J$

d) $10^5\ J$

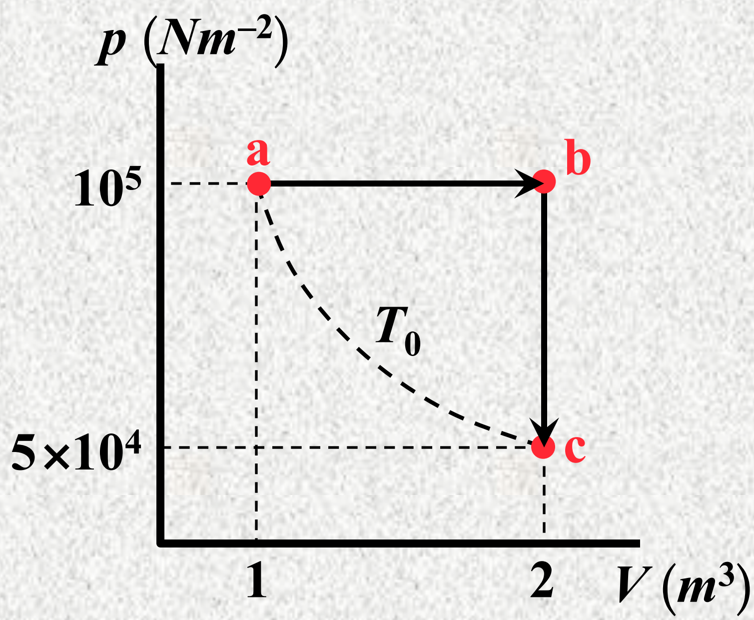
e) Not enough information to tell

first law of thermodynamics: $\Delta E_{int} = Q - W\ (= nC_V\Delta T)$

ideal gas law: $pV = nRT$

Clicker question 6

Consider the *p-V* diagram below in which the system evolves from **a** → **b** → **c**. What is the net heat transferred into the system, Q?



a) $-5.0\times10^4\ J$

b) $5.0\times10^4\ J$

c) $-10^5\ J$

d) $10^5\ J$

e) Not enough information to tell

$$Q = \Delta E_{int} + W = 0 + 10^5\ J$$

first law of thermodynamics: $\Delta E_{int} = Q - W\ (= nC_V\Delta T)$

ideal gas law: $pV = nRT$

Internal Energy (revisited)

$$E_{\text{int}} = nC_V T = \frac{f}{2}nRT = \frac{f}{2}NkT \qquad C_p = C_V + R$$

n = number of moles; 1 mole = 6.0221×10^{22} particles (N_A)
N = number of particles
R = gas constant = 8.3147 J mol^{-1} K^{-1}
k = Boltzmann's constant = 1.3807×10^{-23} J K^{-1}

type of gas	degrees of freedom (f)	specific heat at constant volume (C_V)	internal energy (E_{int})	specific heat at constant pressure (C_p)	γ (C_p/C_V)
monatomic	3	$\frac{3}{2}R$	$\frac{3}{2}nRT$	$\frac{5}{2}R$	$\frac{5}{3}$
diatomic	5	$\frac{5}{2}R$	$\frac{5}{2}nRT$	$\frac{7}{2}R$	$\frac{7}{5}$
polyatomic (≥3)	~6	$3R$	$3nRT$	$4R$	$\frac{4}{3}$

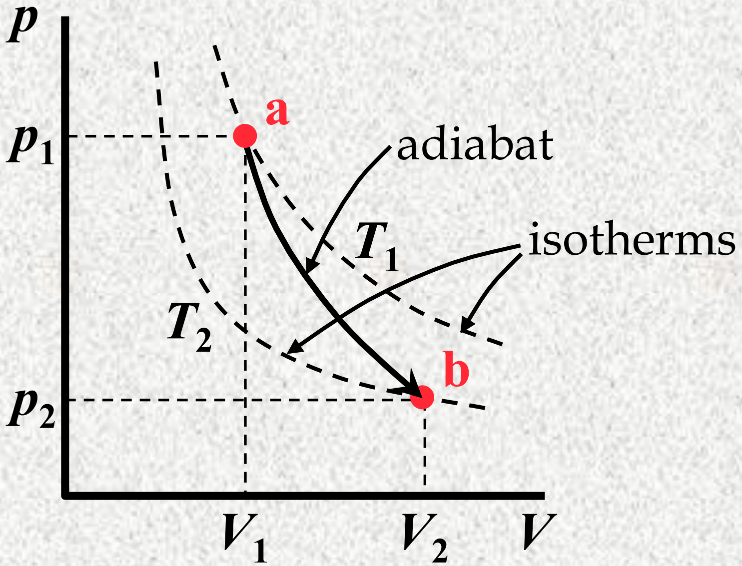
Adiabatic processes

reversible

$\mathbf{a} = (p_1, V_1, T_1)$

$\mathbf{b} = (p_2, V_2, T_2)$

$pV^{\gamma} = \text{constant}$

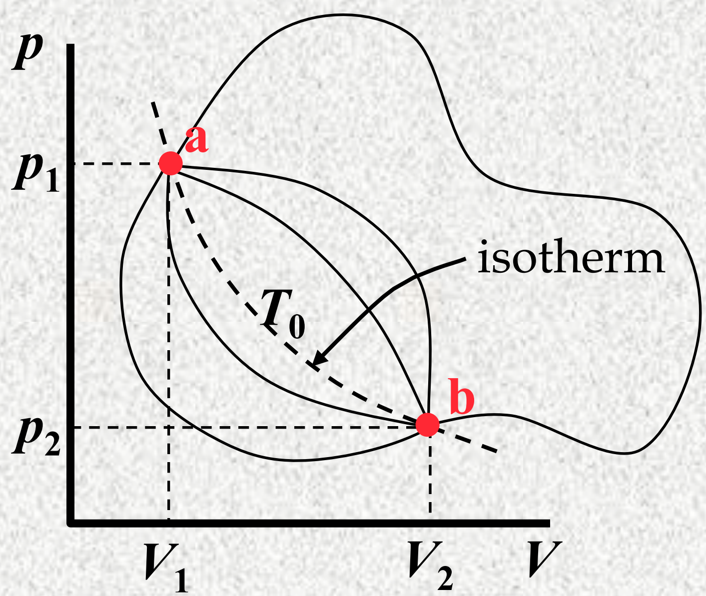


irreversible

$\mathbf{a} = (p_1, V_1, T_0)$

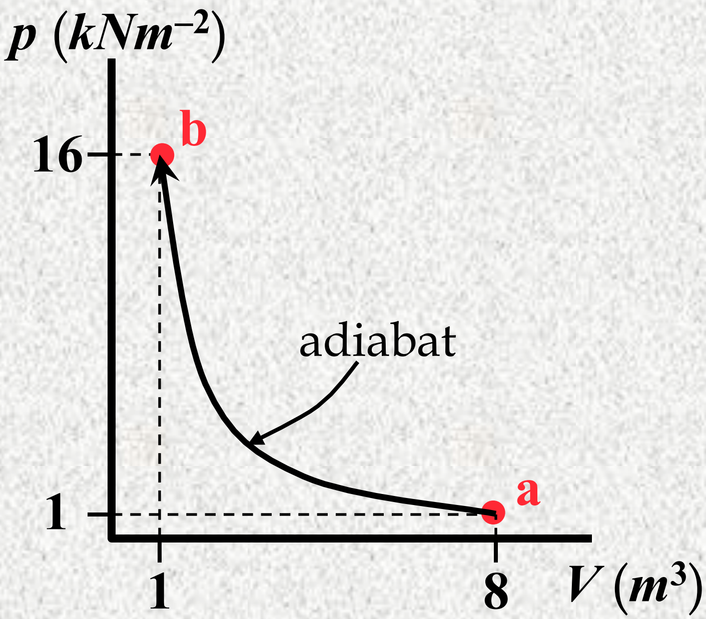
$\mathbf{b} = (p_2, V_2, T_0)$

$p_1 V_1 = p_2 V_2$



Clicker question 7

Consider the *p-V* diagram below in which the system evolves reversibly along the adiabat from state **a** to state **b**. This gas is…



$$pV^{\gamma} = \text{constant}$$

a) monatomic (γ = 5/3)

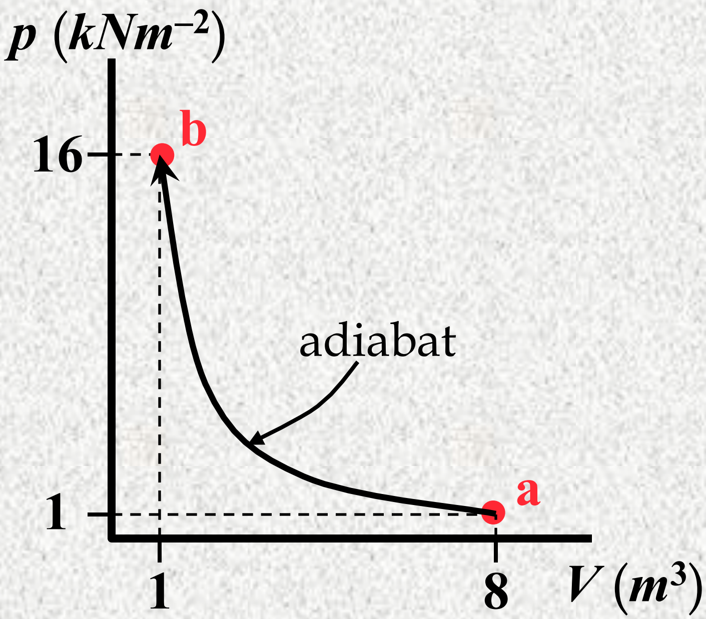
b) diatomic (γ = 7/5)

c) polyatomic (γ = 4/3)

d) not enough information to tell

Clicker question 7

Consider the *p-V* diagram below in which the system evolves reversibly along the adiabat from state **a** to state **b**. This gas is…



$$pV^{\gamma} = \text{constant}$$

a) monatomic (γ = 5/3)

b) diatomic (γ = 7/5)

c) polyatomic (γ = 4/3)

d) not enough information to tell

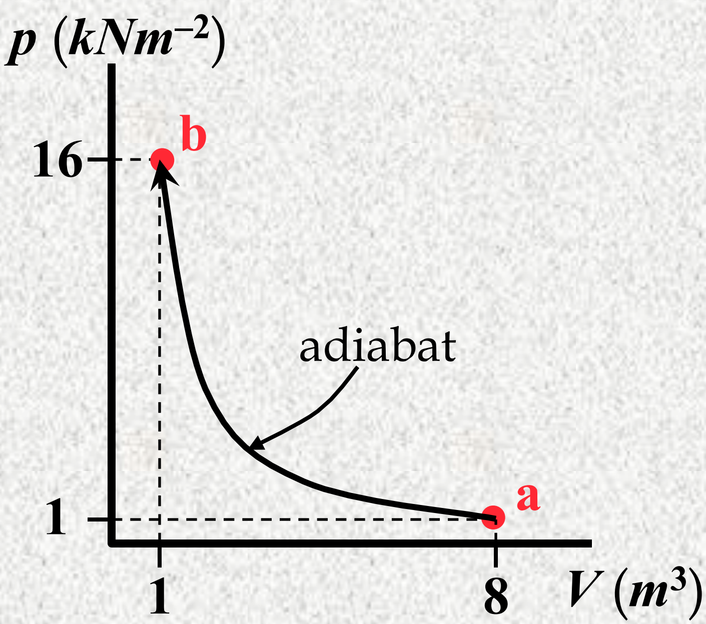
$$p_a V_a^{\gamma} = 1(8)^{\gamma} = 2^{3\gamma} =$$

$$p_b V_b^{\gamma} = 16(1)^{\gamma} = 2^4$$

$\Rightarrow \gamma = 4/3 \Rightarrow$ **polyatomic**

Clicker question 8

Consider the *p-V* diagram below in which the system evolves reversibly along the adiabat from state **a** to state **b**. How much heat is transferred to the system?



$$pV^{\gamma} = \text{constant}$$

a) 0 J

b) 8 kJ

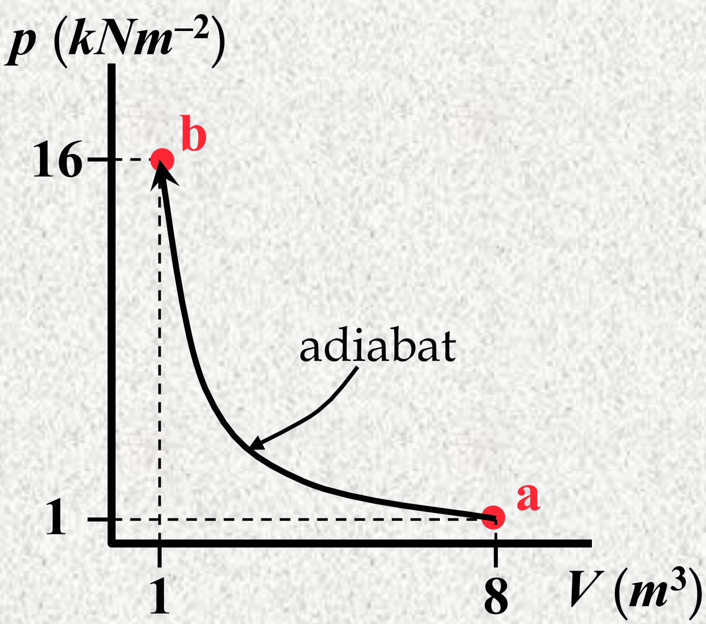
c) 16 kJ

d) 128 kJ

e) not enough information to tell

Clicker question 8

Consider the *p-V* diagram below in which the system evolves reversibly along the adiabat from state **a** to state **b**. How much heat, Q, is transferred to the system?



$pV^{\gamma} = \text{constant}$

a) 0 J

b) 8 kJ

c) 16 kJ

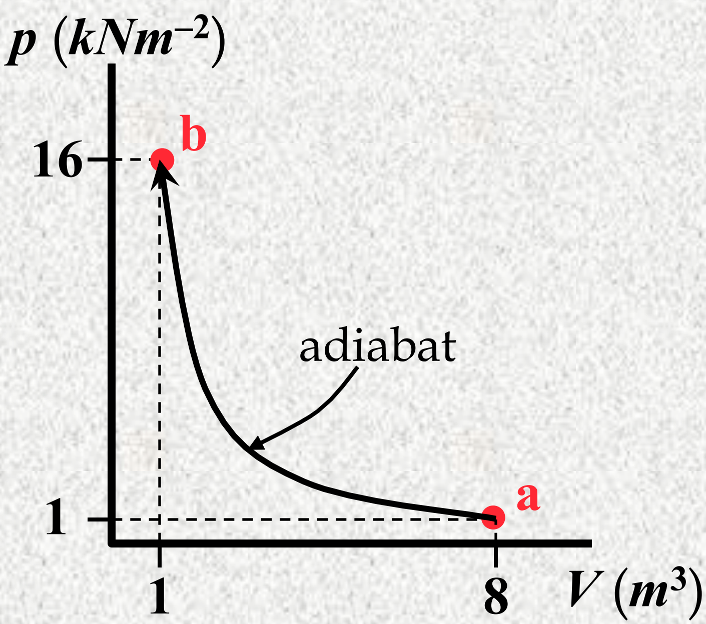
d) 128 kJ

e) not enough information to tell

By definition, $Q = 0$ for all adiabatic processes.

Clicker question 9

Consider the *p-V* diagram below in which the system evolves reversibly along the adiabat from state **a** to state **b**. How much work, W, does the system do on its environment?



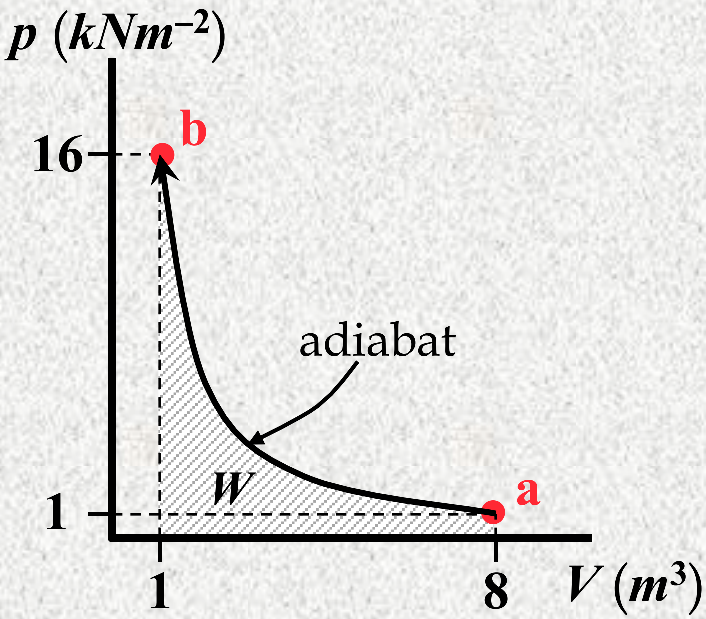
a) 20 kJ
b) –20 kJ
c) 24 kJ
d) –24 kJ
e) 32 kJ
f) –32 kJ
g) not enough information to tell

$$E_{int} = nC_V T = n3RT$$ (for a polyatomic gas)

first law: $\Delta E_{int} = Q - W$ ideal gas law: $pV = nRT$

Clicker question 9

Consider the *p-V* diagram below in which the system evolves reversibly along the adiabat from state **a** to state **b**. How much work, *W*, does the system do on its environment?



a) 20 kJ b) –20 kJ

c) 24 kJ d) –24 kJ ✔

e) 32 kJ f) –32 kJ

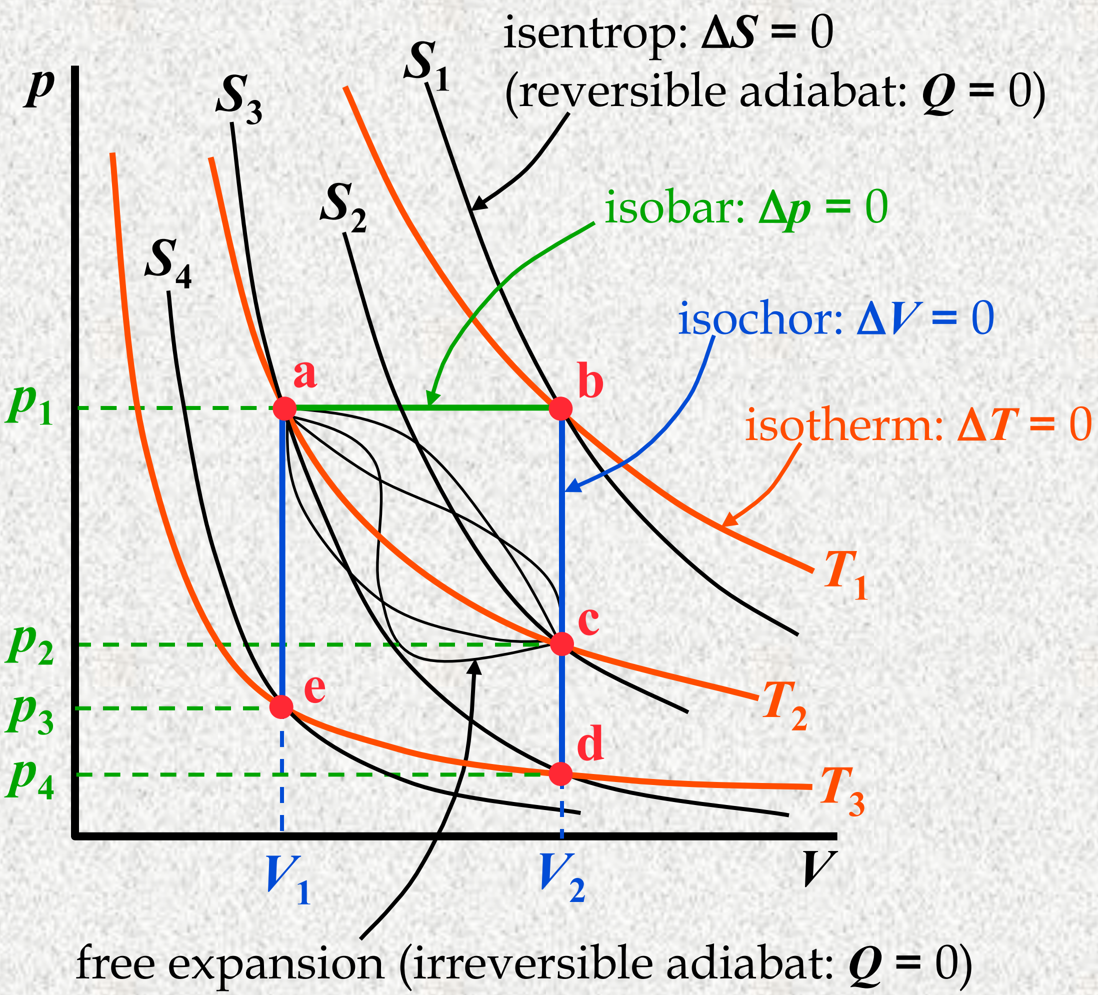
g) not enough information to tell

$$\Delta E_{int} = 0 - W = 3\Delta(nRT) = 3\Delta(pV)$$
$$= 3(16 - 8) = 24 \quad \Rightarrow \quad W = -24\ \text{kJ}$$

$$E_{int} = nC_V T = n3RT$$ (for a polyatomic gas)

first law: $\Delta E_{int} = Q - W$ ideal gas law: $pV = nRT$

Summary of Processes



$\mathbf{a} = (p_1, V_1, T_2, S_3)$

$\mathbf{b} = (p_1, V_2, T_1, S_1)$

$\mathbf{c} = (p_2, V_2, T_2, S_2)$

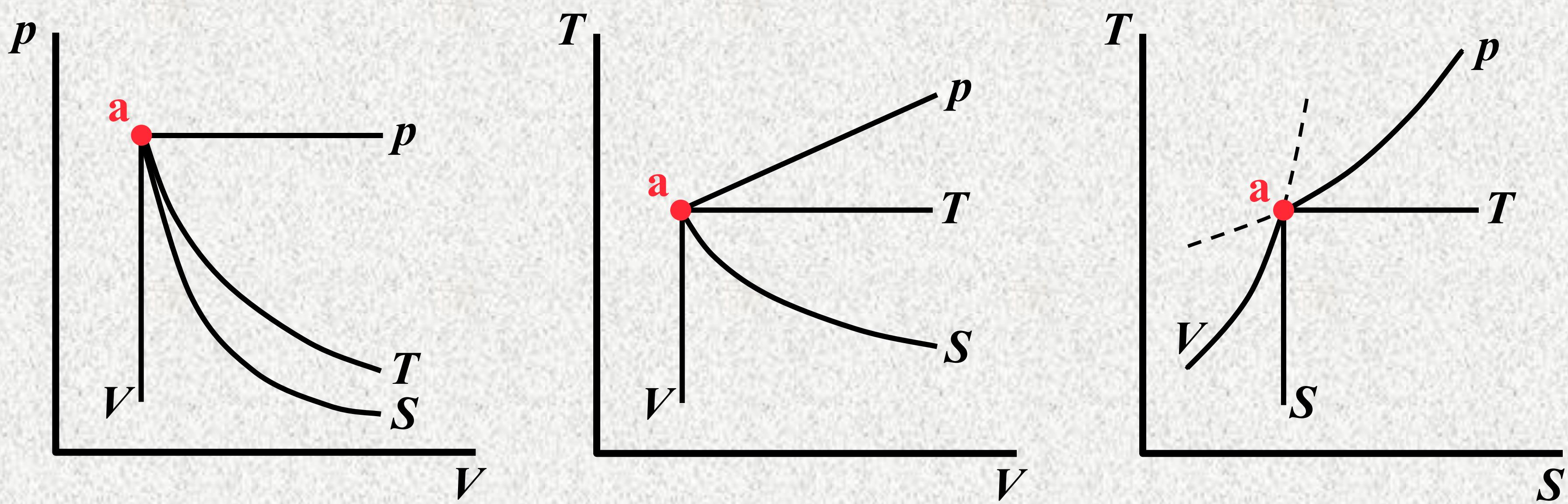
$\mathbf{d} = (p_4, V_2, T_3, S_3)$

$\mathbf{e} = (p_3, V_1, T_3, S_4)$

Summary of Processes

process	W	Q	$\Delta E_{int} = nC_V\Delta T$	ΔS
isobar	$p(V_2 - V_1)$	$\frac{pC_p}{R}(V_2 - V_1)$	$\frac{pC_V}{R}(V_2 - V_1)$	$nC_p \ln\left(\frac{V_2}{V_1}\right)$
isochor	0	$\frac{VC_V}{R}(p_2 - p_1)$	$\frac{VC_V}{R}(p_2 - p_1)$	$nC_V \ln\left(\frac{p_2}{p_1}\right)$
isotherm	$nRT \ln\left(\frac{V_2}{V_1}\right)$	$nRT \ln\left(\frac{V_2}{V_1}\right)$	0	$nR \ln\left(\frac{V_2}{V_1}\right)$
isentrop	$\frac{p_1V_1 - p_2V_2}{\gamma - 1}$	0	$\frac{p_2V_2 - p_1V_1}{\gamma - 1}$	0
free expansion	0	0	0	$nR \ln\left(\frac{V_2}{V_1}\right)$

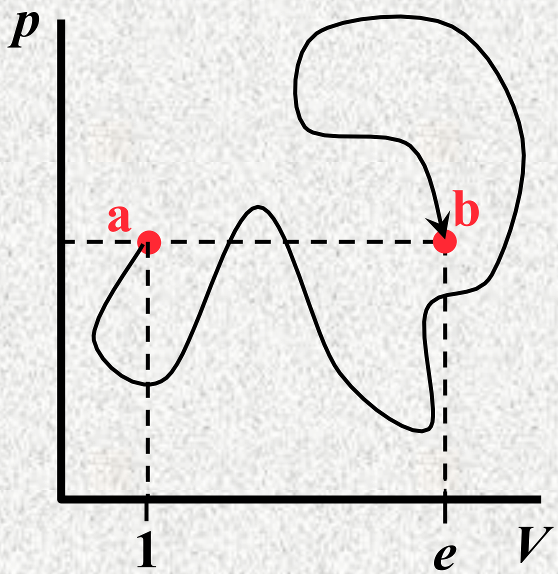
All processes on *p-V*, *T-V*, and *T-S* diagrams



An isobar (*p*), isotherm (*T*), isentrop (*S*), and isochor (*V*) emanating from the same initial state (**a**) as manifest on a *p-V*, *T-V*, and a *T-S* diagram.

Clicker question 10

Consider the p-V diagram below in which n = 1 mole of gas evolves reversibly from state **a** to state **b** along the path shown. What is the net change in entropy? (Note, e = 2.71828 = Euler's number, and thus ln(e) = 1.)



a) C_p

b) $C_p e$

c) C_V

d) $C_V e$

e) R

f) $R e$

g) no where near enough information!!

some formulae, in case they help…

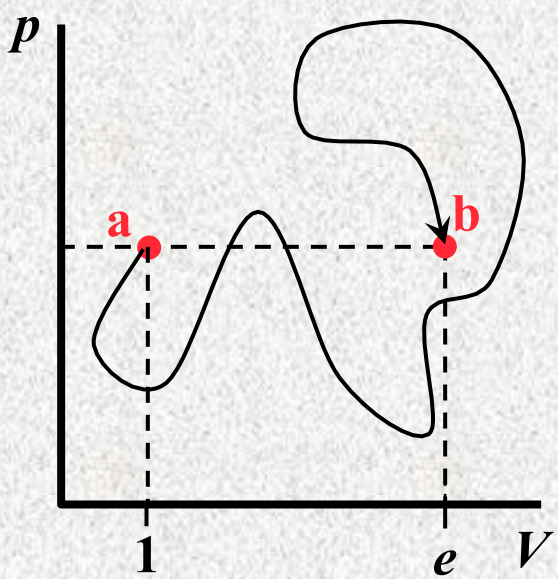
$$\Delta S = nC_p \ln\left(\frac{V_2}{V_1}\right) \quad \text{(isobar)}$$

$$\Delta S = nC_V \ln\left(\frac{p_2}{p_1}\right) \quad \text{(isochor)}$$

$$\Delta S = nR \ln\left(\frac{V_2}{V_1}\right) \quad \text{(isotherm)}$$

Clicker question 10

Consider the p-V diagram below in which n = 1 mole of gas evolves revers -ibly from state **a** to state **b** along the path shown. What is the net change in entropy? (Note, e = 2.71828 = Euler's number, and thus ln(e) = 1.)



a) C_p

b) $C_p e$

c) C_V

d) $C_V e$

e) R

f) Re

g) Yes there is!!

Since S is a state variable, it doesn't matter which path from **a** to **b** you choose. Thus, choose the isobar.

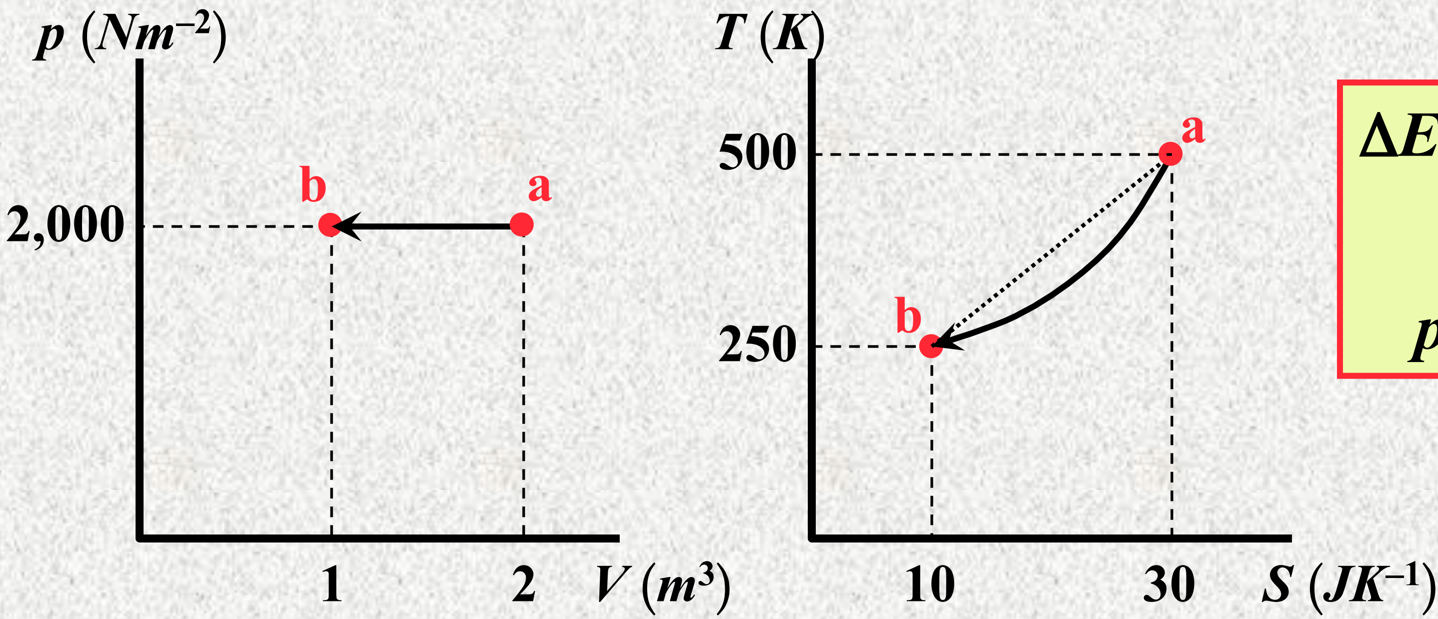
$$\Delta S = nC_p \ln\left(\frac{V_2}{V_1}\right) \quad \text{(isobar)}$$

$$\Delta S = nC_V \ln\left(\frac{p_2}{p_1}\right) \quad \text{(isochor)}$$

$$\Delta S = nR \ln\left(\frac{V_2}{V_1}\right) \quad \text{(isotherm)}$$

Clicker question 11

The evolution of a system from state **a** to state **b** is shown on both the p-V and T-S diagrams below. What is the change in internal energy?

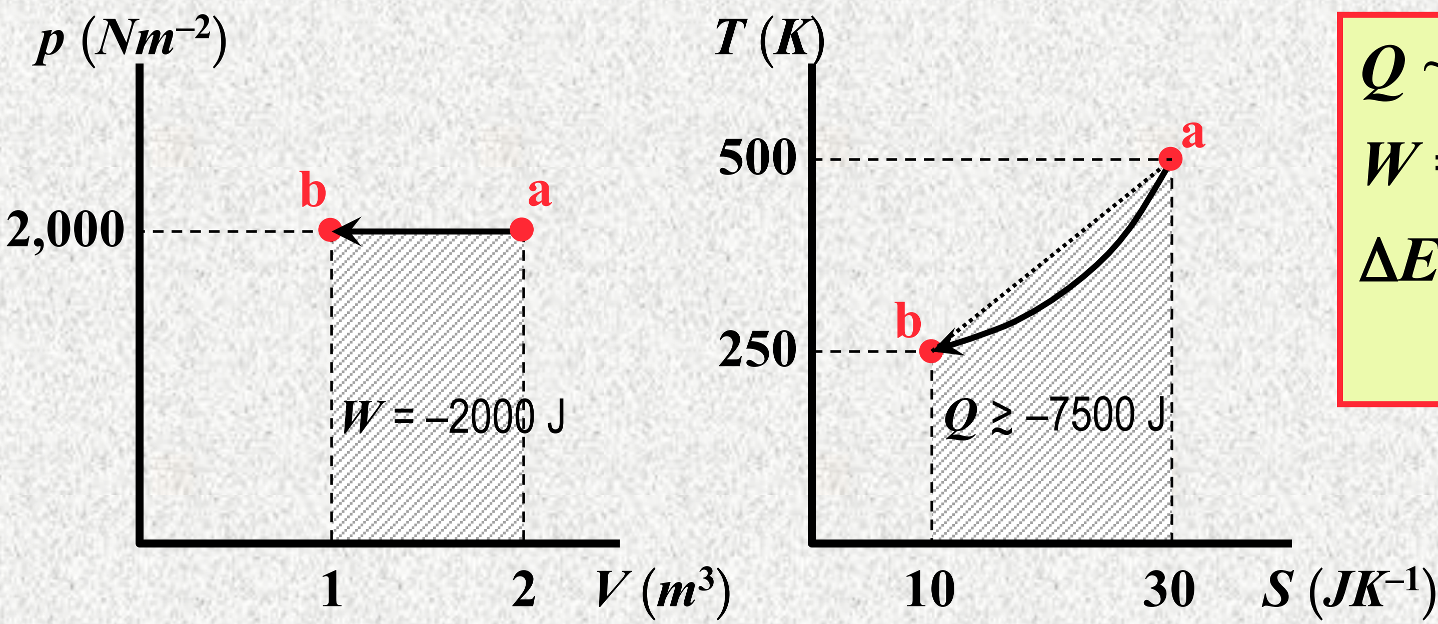


$$\Delta E_{int} = Q - W$$
$$= nC_V \Delta T$$
$$pV = nRT$$

a) 2000 J
b) –2000 J
c) 5000 J
d) –5000 J
e) 7000 J
f) –7000 J

Clicker question 11

The evolution of a system from state **a** to state **b** is shown on both the p-V and T-S diagrams below. What is the change in internal energy?



$Q \sim -7000$ J

$W = -2000$ J

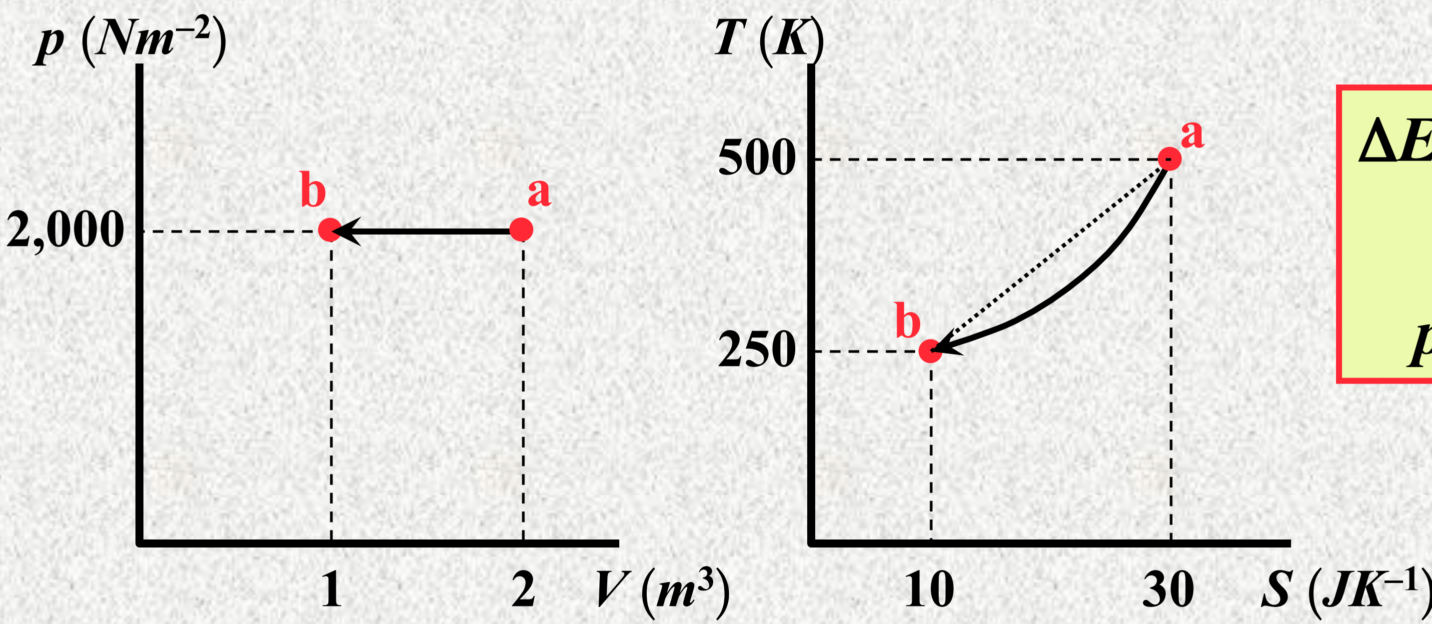
$\Delta E_{int} = Q - W$

~ -5000 J

a) 2000 J
b) –2000 J
c) 5000 J
d) –5000 J ✓
e) 7000 J
f) –7000 J

Clicker question 12

The evolution of a system from state **a** to state **b** is shown on both the p-V and T-S diagrams below. About how many moles of gas are in the system? (Take $R = 8$.)



$$\Delta E_{int} = Q - W$$
$$= nC_V \Delta T$$
$$pV = nRT$$

a) 0.5

b) 1

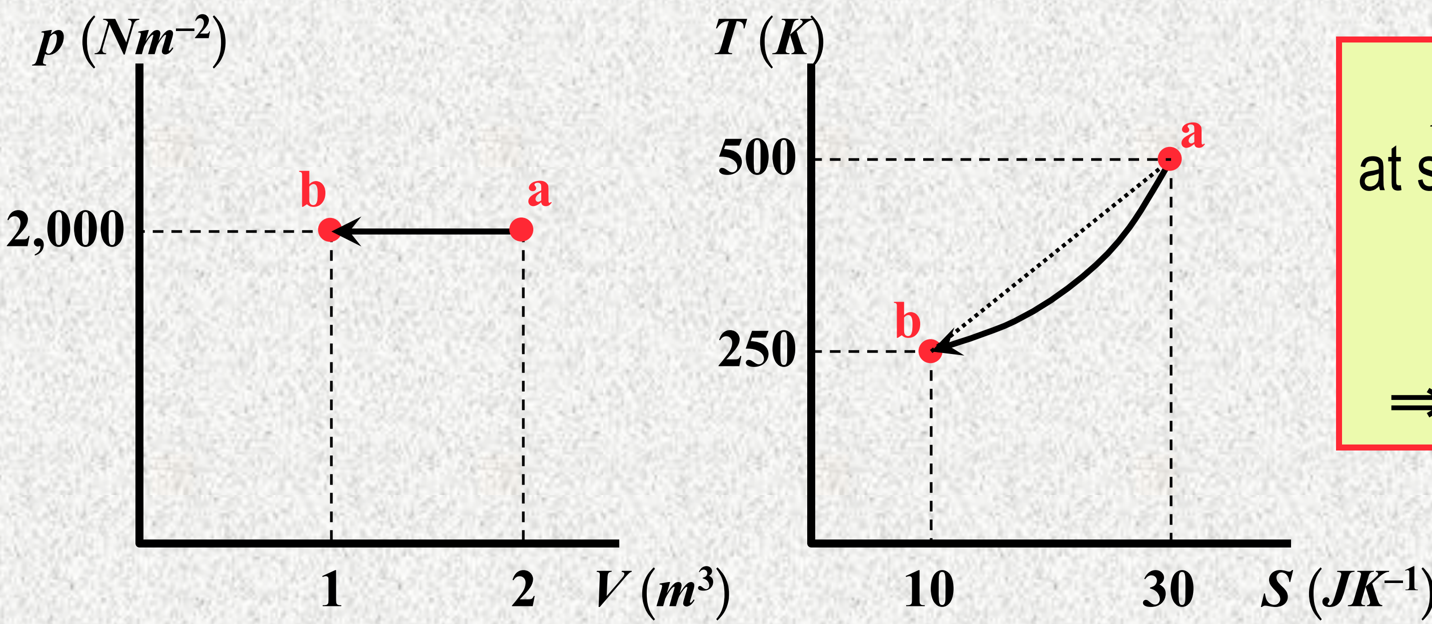
c) 1.5

d) 2

e) not enough information to tell

Clicker question 12

The evolution of a system from state **a** to state **b** is shown on both the p-V and T-S diagrams below. About how many moles of gas are in the system? (Take $R = 8$.)



$$pV = nRT$$

at state **b**:

$$pV = 2000$$

$$RT \sim 2000$$

$$\Rightarrow n \sim 1$$

a) 0.5

b) 1 ✔

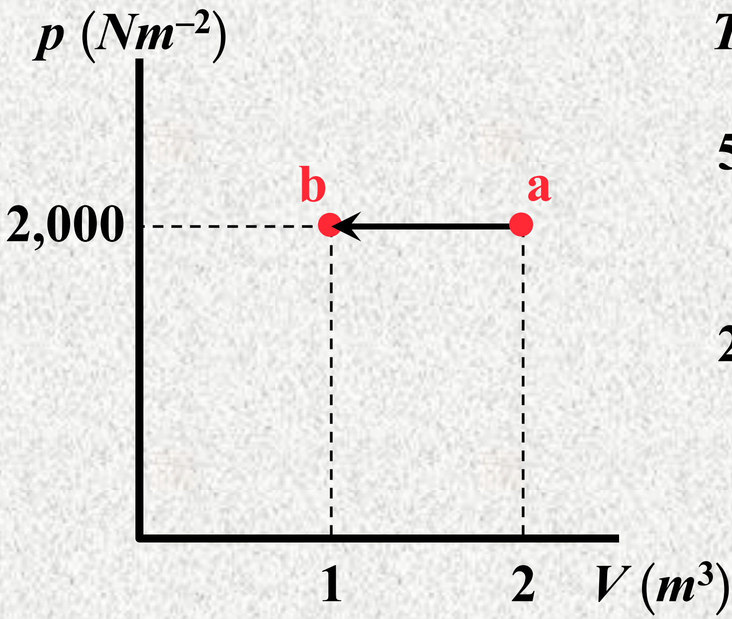
c) 1.5

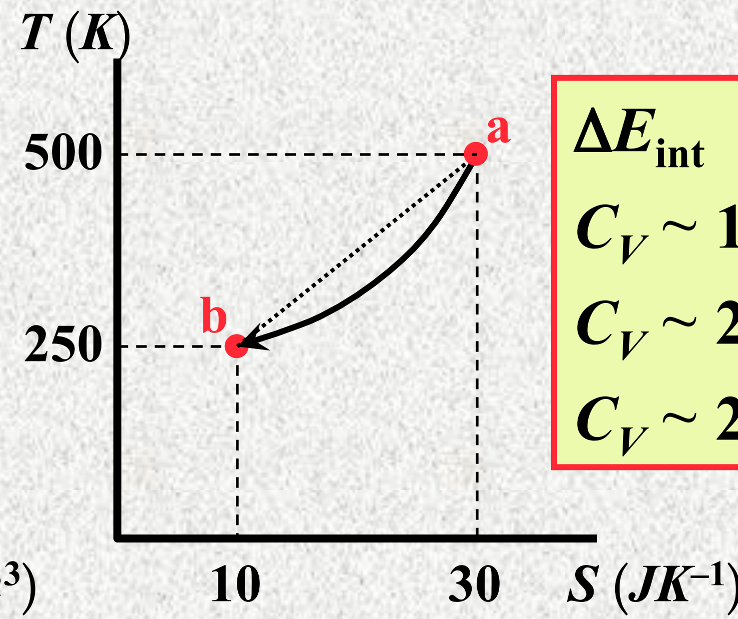
d) 2

e) not enough information to tell

Clicker question 13

The evolution of a system from state **a** to state **b** is shown on both the p-V and T-S diagrams below. With n = 1 and ΔE_{int} = –5000 J, this gas is:





$\Delta E_{int} = nC_V\Delta T$

$C_V \sim 12$ (monatomic)

$C_V \sim 21$ (diatomic)

$C_V \sim 25$ (polyatomic)

a) monatomic

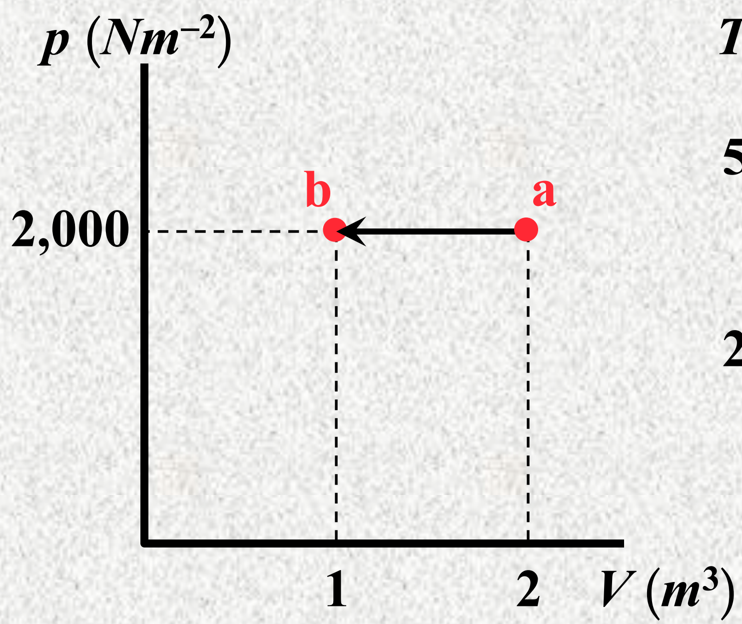
b) diatomic

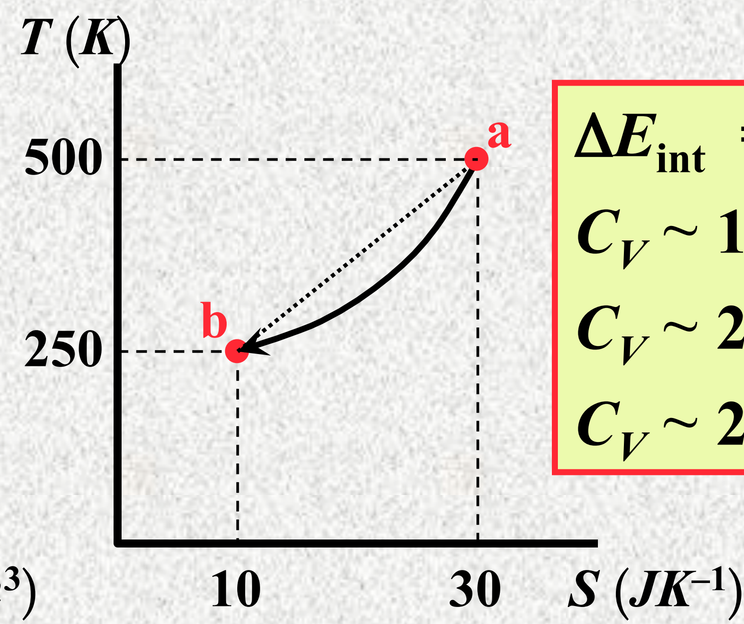
c) polyatomic

d) not enough information to tell

Clicker question 13

The evolution of a system from state **a** to state **b** is shown on both the p-V and T-S diagrams below. With $n = 1$ and $\Delta E_{int} = -5000$ J, this gas is:





$\Delta E_{int} = nC_V\Delta T$

$C_V \sim 12$ (monatomic)

$C_V \sim 21$ (diatomic)

$C_V \sim 25$ (polyatomic)

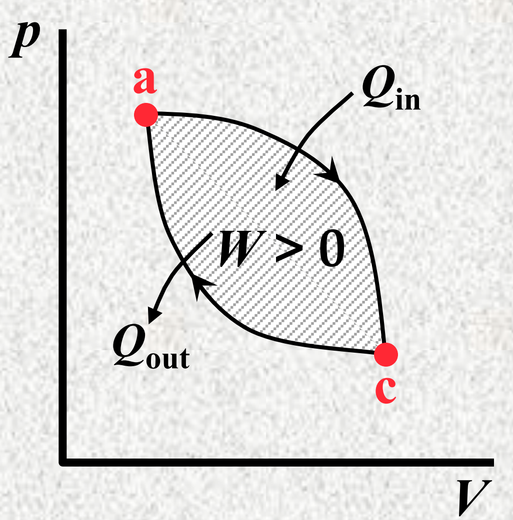
a) monatomic

b) diatomic ✔

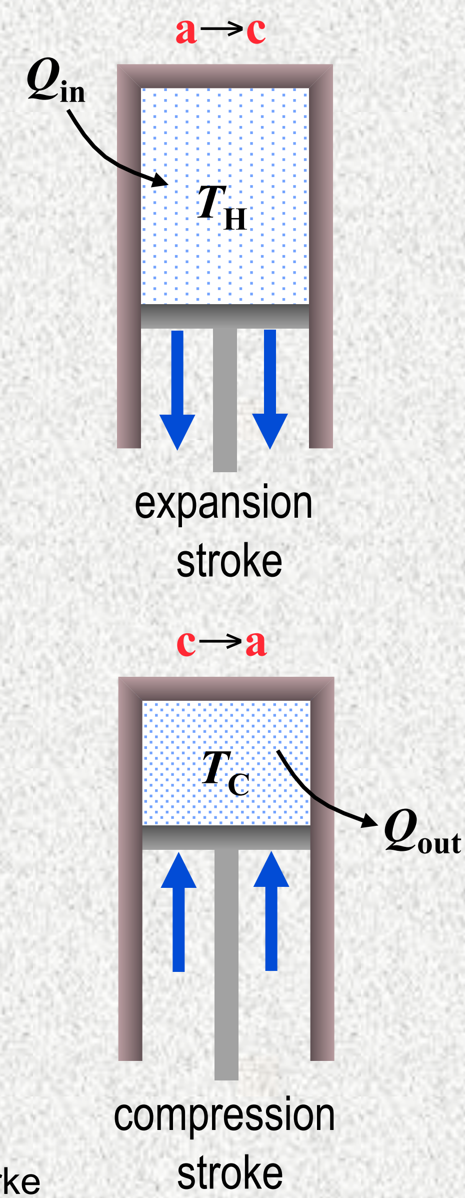
c) polyatomic

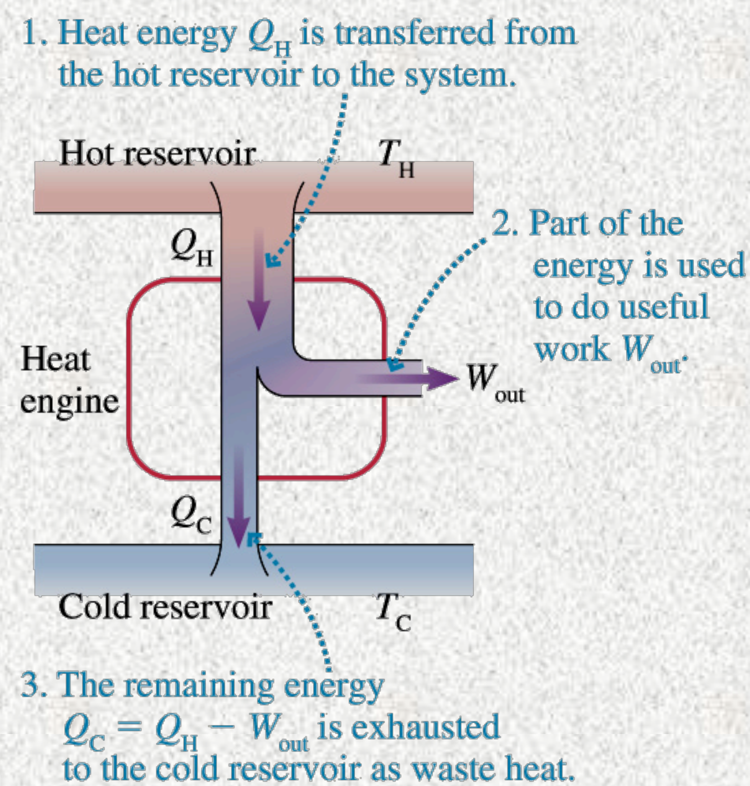
d) not enough information to tell

Three pictorial representations of an engine



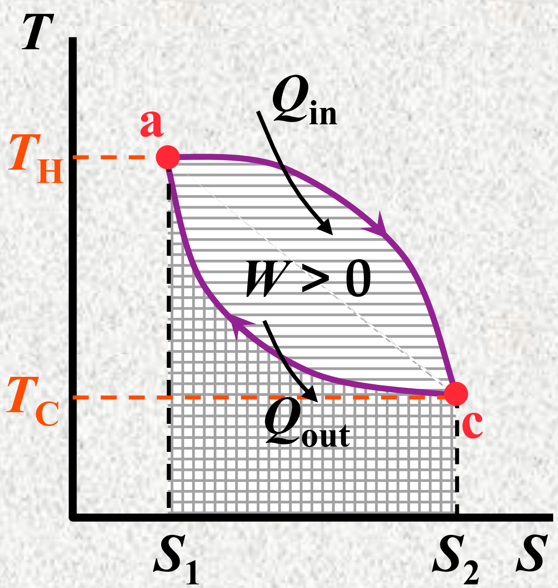
In a complete thermodynamical cycle, gas expands at high pressure and compresses at low pressure allowing work, W, to be extracted in each cycle.



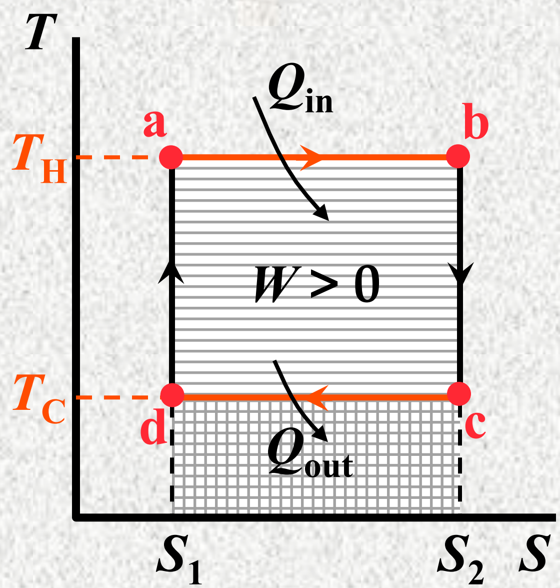


$$Q_H = Q_{in}$$
$$Q_C = Q_{out}$$
$$W_{out} = W$$

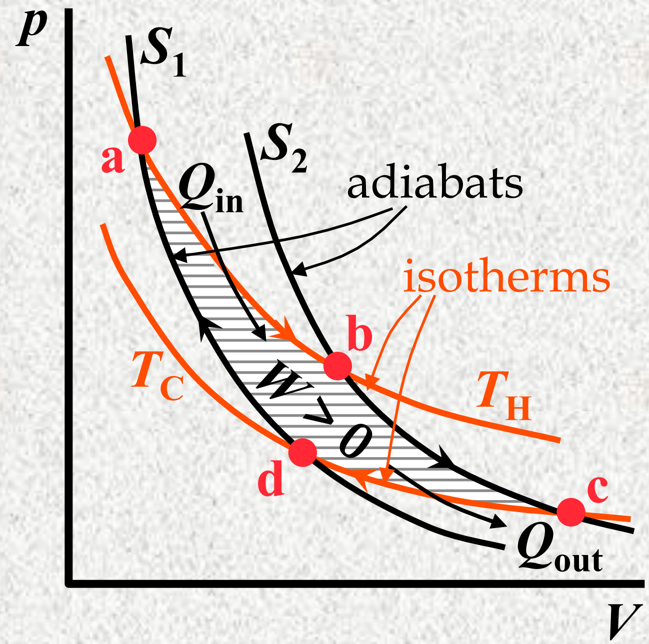
Maximum efficiency and the Carnot cycle



non-optimal thermo-dynamical cycle for an engine



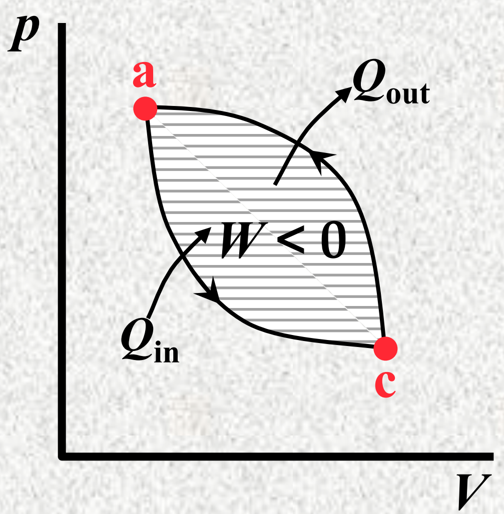
optimal thermodyn-amical cycle for an engine (Carnot cycle)



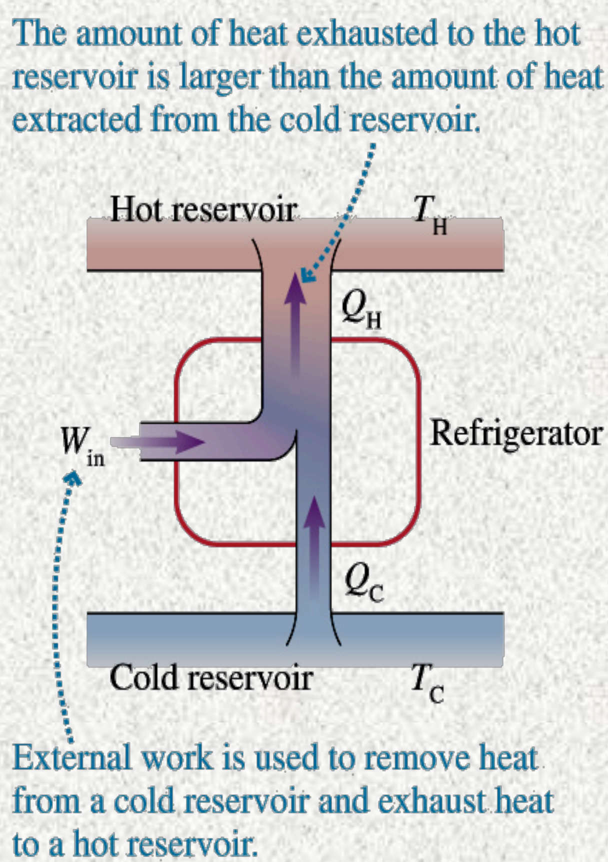
the Carnot engine cycle on a ***p*-*V*** diagram

Note that for an engine, the thermodynamical cycle is always *clockwise.*

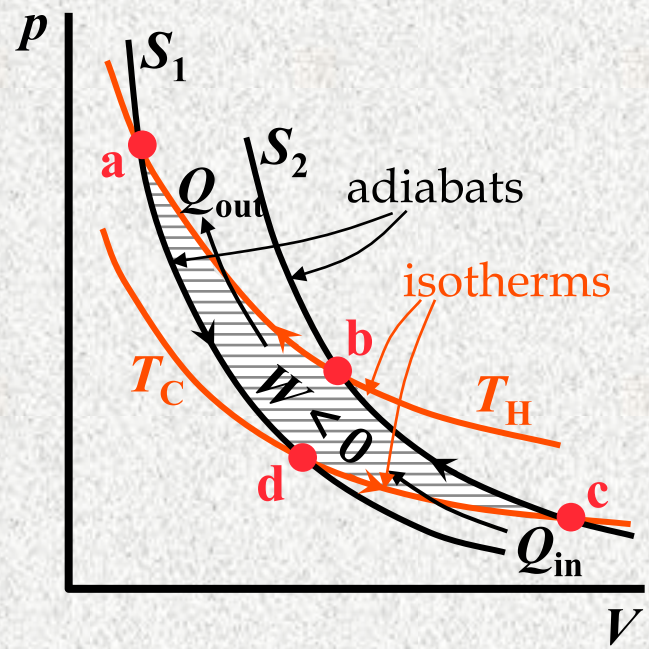
Refrigerators (heat pumps)



For a refrigerator, the cycle is always *counterclockwise*. Expansion happens at low pressure, compression at high pressure and this takes work. Heat is drawn in at T_C and expelled at T_H.



$$Q_C = Q_{in}$$
$$Q_H = Q_{out}$$
$$W_{in} = -W$$



As for an engine, the most optimal thermodynamical cycle for a refrigerator is the Carnot cycle traversed in the counterclockwise direction (opposite to the engine).