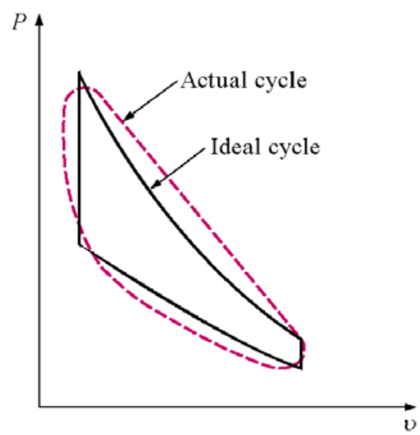
Chapter 8: Gas Power Cycles

Our study of gas power cycles will involve the study of those heat engines in which the working fluid remains in the gaseous state throughout the cycle. We often study the ideal cycle in which internal irreversibilities and complexities (the actual intake of air and fuel, the actual combustion process, and the exhaust of products of combustion among others) are removed.



We will be concerned with how the major parameters of the cycle affect the performance of heat engines. The performance is often measured in terms of the cycle efficiency.

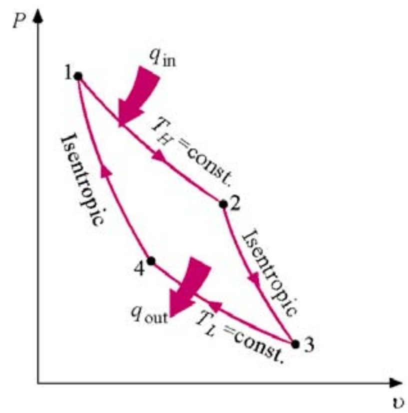
$$\eta_{th} = \frac{W_{net}}{Q_{in}}$$

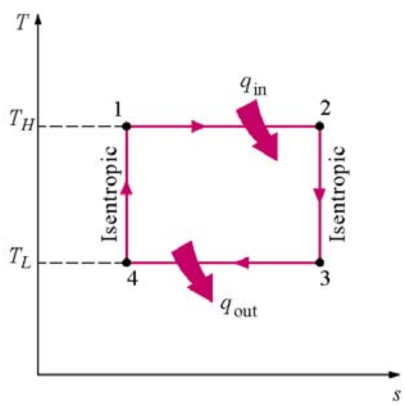
Carnot Cycle

The Carnot cycle was introduced in Chapter 5 as the most efficient heat engine that can operate between two fixed temperatures T_H and T_L. The Carnot cycle is described by the following four processes.

Carnot Cycle

Process	Description
1-2	Isothermal heat addition
2-3	Isentropic expansion
3-4	Isothermal heat rejection
4-1	Isentropic compression





Note the processes on both the *P-v* and *T-s* diagrams. The areas under the process curves on the *P-v* diagram represent the work done for closed systems. The net cycle work done is the area enclosed by the cycle on the *P-v* diagram. The areas under the process curves on the *T-s* diagram represent the heat transfer for the processes. The net heat added to the cycle is the area that is enclosed by the cycle on the *T-s* diagram. For a cycle we know $W_{net} = Q_{net}$; therefore, the areas enclosed on the *P-v* and *T-s* diagrams are equal.

$$\eta_{th,\,Carnot} = 1 - \frac{T_L}{T_H}$$

We often use the Carnot efficiency as a means to think about ways to improve the cycle efficiency of other cycles. One of the observations about the efficiency of both ideal and actual cycles comes from the Carnot efficiency: Thermal efficiency increases with an increase in the average temperature at which heat is supplied to the system or with a decrease in the average temperature at which heat is rejected from the system.

Air-Standard Assumptions

In our study of gas power cycles, we assume that the working fluid is air, and the air undergoes a thermodynamic cycle even though the working fluid in the actual power system does not undergo a cycle.

To simplify the analysis, we approximate the cycles with the following assumptions:

- The air continuously circulates in a closed loop and always behaves as an ideal gas.

- All the processes that make up the cycle are internally reversible.

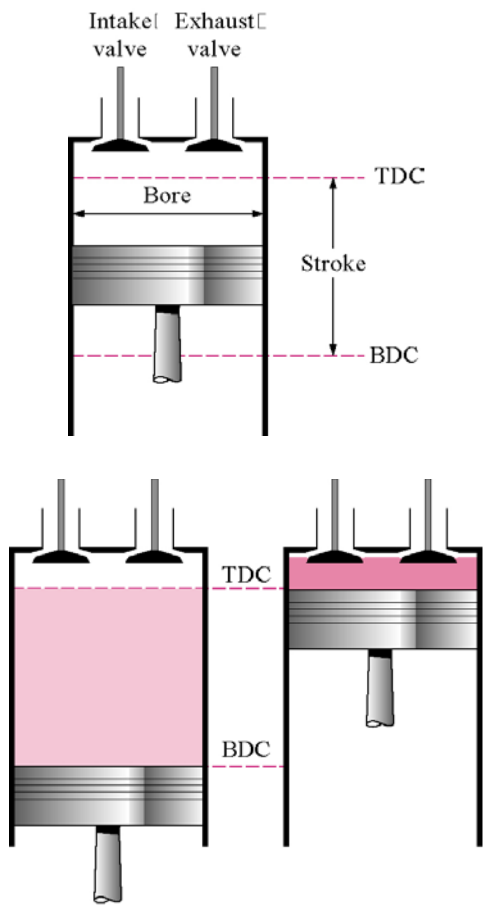
- The combustion process is replaced by a heat-addition process from an external source.

- A heat rejection process that restores the working fluid to its initial state replaces the exhaust process.

- The cold-air-standard assumptions apply when the working fluid is air and has constant specific heat evaluated at room temperature (25^{o}C or 77^{o}F).

Terminology for Reciprocating Devices

The following is some terminology we need to understand for reciprocating engines—typically piston-cylinder devices. Let's look at the following figures for the definitions of top dead center (TDC), bottom dead center (BDC), stroke, bore, intake valve, exhaust valve, clearance volume, displacement volume, compression ratio, and mean effective pressure.

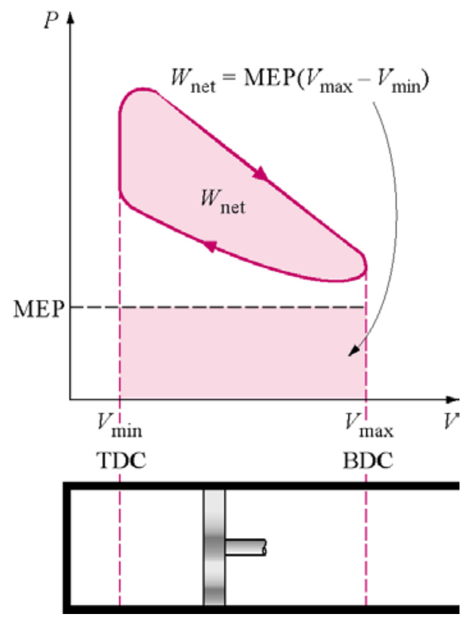


(*a*) Displacement volume (*b*) Clearance volume

The compression ratio *r* of an engine is the ratio of the maximum volume to the minimum volume formed in the cylinder.

$$r = \frac{V\max}{V\min} = \frac{V_{BDC}}{V_{TDC}}$$

The mean effective pressure **(MEP*)*** is a fictitious pressure that, if it operated on the piston during the entire power stroke, would produce the same amount of net work as that produced during the actual cycle.



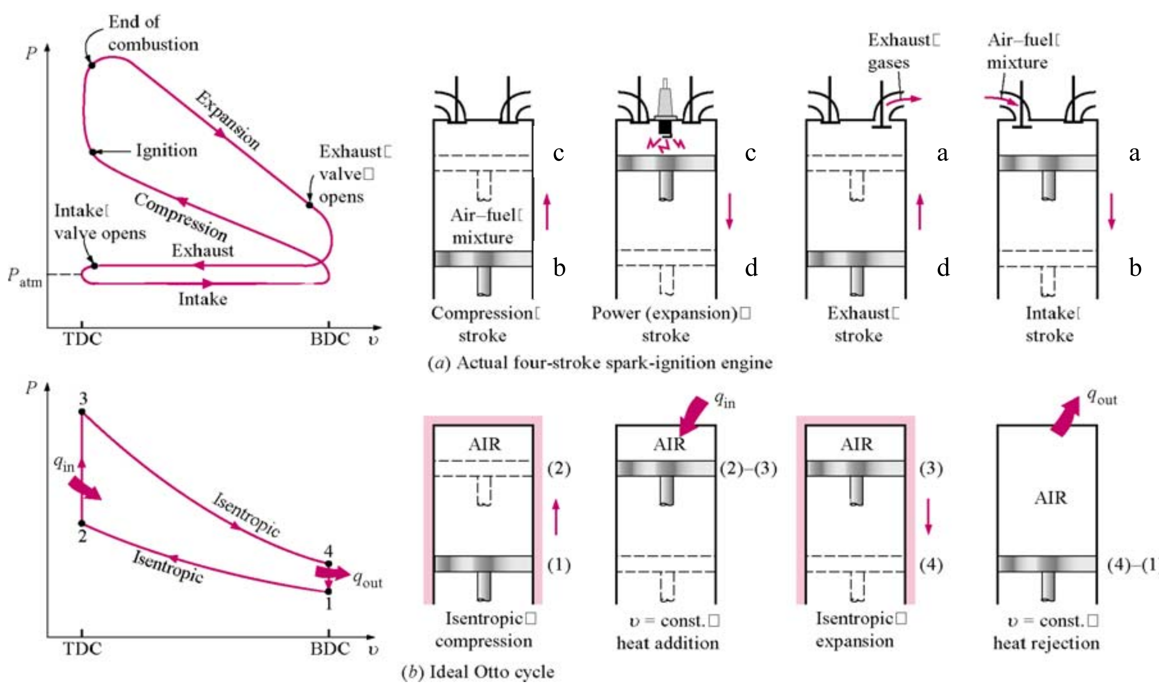
$$MEP = \frac{W_{net}}{V_{\max} - V_{\min}} = \frac{w_{net}}{v_{\max} - v_{\min}}$$

Otto Cycle: The Ideal Cycle for Spark-Ignition Engines

Consider the automotive spark-ignition power cycle.

Processes
Intake stroke
Compression stroke
Power (expansion) stroke
Exhaust stroke

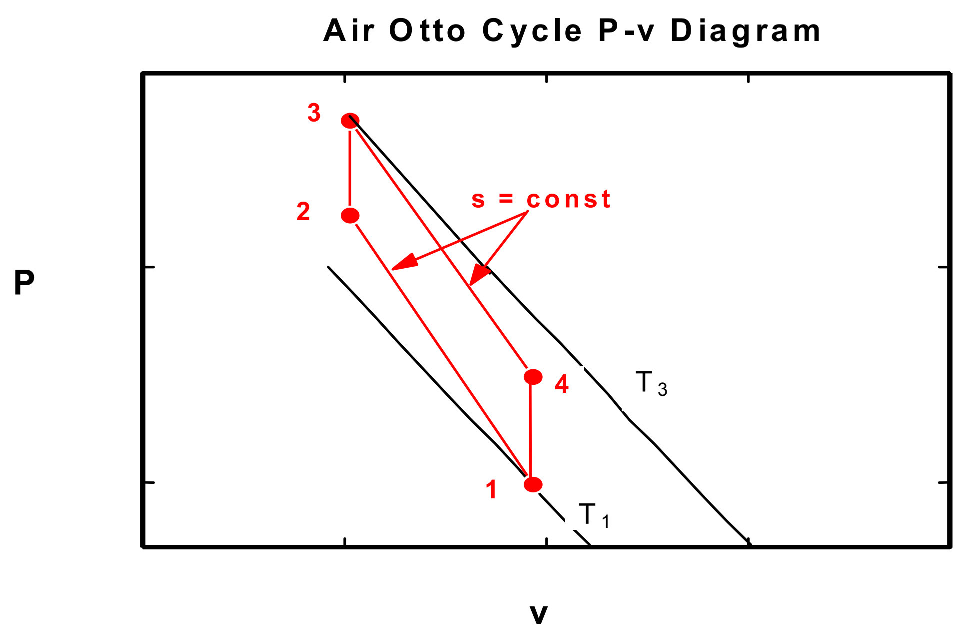
Often the ignition and combustion process begins before the completion of the compression stroke. The number of crank angle degrees before the piston reaches TDC on the number one piston at which the spark occurs is called the engine timing. What are the compression ratio and timing of your engine in your car, truck, or motorcycle?

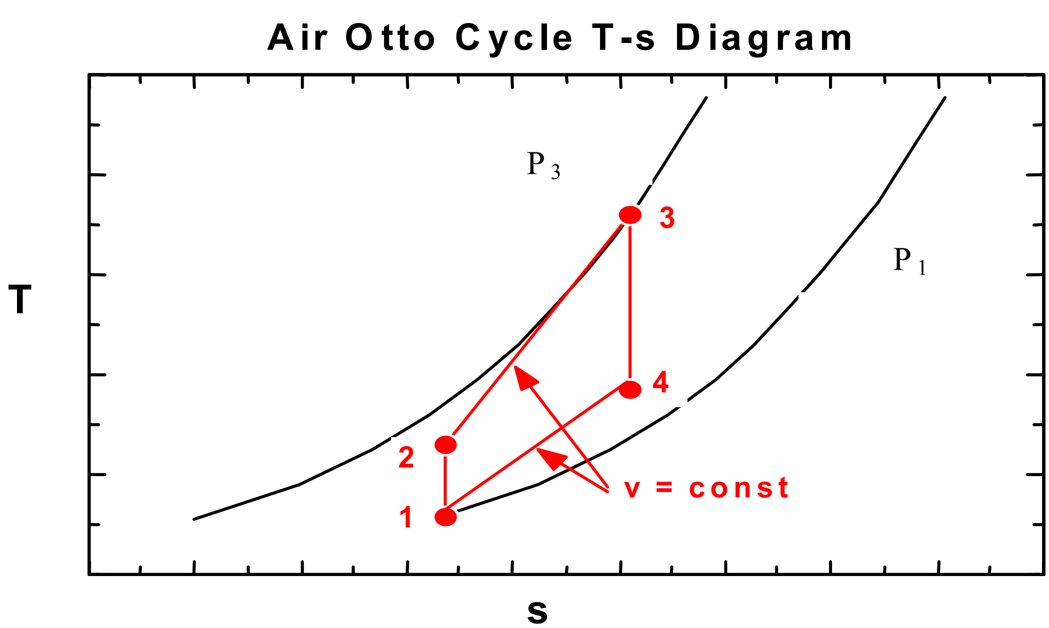


The air-standard Otto cycle is the ideal cycle that approximates the spark-ignition combustion engine.

Process	Description
1-2	Isentropic compression
2-3	Constant volume heat addition
3-4	Isentropic expansion
4-1	Constant volume heat rejection

The *P-v* and *T-s* diagrams are





Thermal Efficiency of the Otto cycle:

$$\eta_{th} = \frac{W_{net}}{Q_{in}} = \frac{Q_{net}}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

Now to find Q_{in} and Q_{out}.

Apply first law closed system to process 2-3, V = constant.

$$Q_{net,\,23} - W_{net,\,23} = \Delta U_{23}$$

$$W_{net,\,23} = W_{other,\,23} + W_{b,\,23} = 0 + \int_2^3 P\,dV = 0$$

Thus, for constant specific heats,

$$Q_{net,\,23} = \Delta U_{23}$$

$$Q_{net,\,23} = Q_{in} = mC_v(T_3 - T_2)$$

Apply first law closed system to process 4-1, V = constant.

$$Q_{net,\,41} - W_{net,\,41} = \Delta U_{41}$$

$$W_{net,\,41} = W_{other,\,41} + W_{b,\,41} = 0 + \int_4^1 P\,dV = 0$$

Thus, for constant specific heats,

$$Q_{net,\,41} = \Delta U_{41}$$

$$Q_{net,\,41} = -Q_{out} = mC_v(T_1 - T_4)$$

$$Q_{out} = -mC_v(T_1 - T_4) = mC_v(T_4 - T_1)$$

The thermal efficiency becomes

$$\eta_{th,\,Otto} = 1 - \frac{Q_{out}}{Q_{in}}$$

$$= 1 - \frac{mC_v(T_4 - T_1)}{mC_v(T_3 - T_2)}$$

$$\eta_{th,\,Otto} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

$$= 1 - \frac{T_1(T_4 / T_1 - 1)}{T_2(T_3 / T_2 - 1)}$$

Recall processes 1-2 and 3-4 are isentropic, so

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1} \quad and \quad \frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{k-1}$$

Since $V_3 = V_2$ and $V_4 = V_1$, we see that

$$\frac{T_2}{T_1} = \frac{T_3}{T_4}$$

$$or$$

$$\frac{T_4}{T_1} = \frac{T_3}{T_2}$$

The Otto cycle efficiency becomes

$$\eta_{th,\,Otto} = 1 - \frac{T_1}{T_2}$$

Is this the same as the Carnot cycle efficiency?

Since process 1-2 is isentropic,

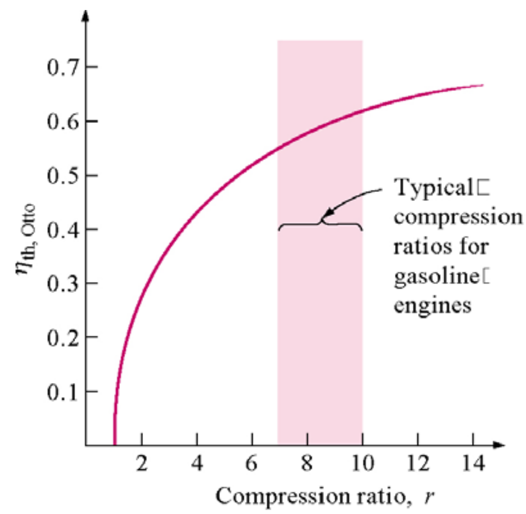
$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1}$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{k-1} = \left(\frac{1}{r}\right)^{k-1}$$

where the compression ratio is $r = V_1/V_2$ and

$$\eta_{th,\,Otto} = 1 - \frac{1}{r^{k-1}}$$

We see that increasing the compression ratio increases the thermal efficiency. However, there is a limit on r depending upon the fuel. Fuels under high temperature resulting from high compression ratios will prematurely ignite, causing knock.



Example 8-1

An Otto cycle having a compression ratio of 9:1 uses air as the working fluid. Initially P_1 = 95 kPa, T_1 = 17°C, and V_1 = 3.8 liters. During the heat addition process, 7.5 kJ of heat are added. Determine all *T*'s, *P*'s, η_{th}, the back work ratio, and the mean effective pressure.

Process Diagrams: Review the *P-v* and *T-s* diagrams given above for the Otto cycle.

Assume constant specific heats with C_v = 0.718 kJ/kg ·K, *k* = 1.4. (Use the 300 K data from Table A-2)

Process 1-2 is isentropic; therefore, recalling that $r = V_1/V_2 = 9$,

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{k-1} = T_1(r)^{k-1}$$

$$= (17+273)K(9)^{1.4-1}$$

$$= 698.4K$$

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^k = P_1(r)^k$$

$$= 95kPa(9)^{1.4}$$

$$= 2059\,kPa$$

The first law closed system for process 2-3 was shown to reduce to (your homework solutions must be complete; that is, develop your equations from the application of the first law for each process as we did in obtaining the Otto cycle efficiency equation)

$$Q_{in} = mC_v(T_3 - T_2)$$

Let $q_{\text{in}} = Q_{\text{in}} / m$ and $m = V_1 / v_1$

$$v_1 = \frac{RT_1}{P_1}$$

$$= \frac{0.287\dfrac{kJ}{kg \cdot K}(290\,K)}{95\,kPa}\frac{m^3 kPa}{kJ}$$

$$= 0.875\frac{m^3}{kg}$$

$$q_{in} = \frac{Q_{in}}{m} = Q_{in}\frac{v_1}{V_1}$$

$$= 7.5kJ\frac{0.875\frac{m^3}{kg}}{3.8\cdot 10^{-3}m^3}$$

$$= 1727\frac{kJ}{kg}$$

Then,

$$T_3 = T_2 + \frac{q_{in}}{C_v}$$

$$= 698.4K + \frac{1727\frac{kJ}{kg}}{0.718\frac{kJ}{kg\cdot K}}$$

$$= 3103.7\,K$$

Using the combined gas law

$$P_3 = P_2\frac{T_3}{T_2} = 9.15\,MPa$$

Process 3-4 is isentropic; therefore,

$$T_4 = T_3\left(\frac{V_3}{V_4}\right)^{k-1} = T_3\left(\frac{1}{r}\right)^{k-1}$$

$$= (3103.7)K\left(\frac{1}{9}\right)^{1.4-1}$$

$$= 1288.8\,K$$

$$P_4 = P_3\left(\frac{V_3}{V_4}\right)^{k} = P_3\left(\frac{1}{r}\right)^{k}$$

$$= 9.15\,MPa\left(\frac{1}{9}\right)^{1.4}$$

$$= 422\,kPa$$

Process 4-1 is constant volume. So the first law for the closed system gives, on a mass basis,

$$Q_{out} = mC_v(T_4 - T_1)$$

$$q_{out} = \frac{Q_{out}}{m} = C_v(T_4 - T_1)$$

$$= 0.718\frac{kJ}{kg\cdot K}(1288.8 - 290)K$$

$$= 717.1\frac{kJ}{kg}$$

For the cycle, $\Delta u = 0$, and the first law gives

$$w_{net} = q_{net} = q_{in} - q_{out}$$
$$= (1727 - 717.4)\frac{kJ}{kg}$$
$$= 1009.6\frac{kJ}{kg}$$

The thermal efficiency is

$$\eta_{th,\,Otto} = \frac{w_{net}}{q_{in}} = \frac{1009.6\dfrac{kJ}{kg}}{1727\dfrac{kJ}{kg}}$$
$$= 0.585 \;\; or \;\;\; 58.5\%$$

The mean effective pressure is

$$MEP = \frac{W_{net}}{V_{\max} - V_{\min}} = \frac{w_{net}}{v_{\max} - v_{\min}}$$
$$= \frac{w_{net}}{v_1 - v_2} = \frac{w_{net}}{v_1(1 - v_2 / v_1)} = \frac{w_{net}}{v_1(1 - 1/r)}$$
$$= \frac{1009.6\dfrac{kJ}{kg}}{0.875\dfrac{m^3}{kg}(1 - \dfrac{1}{9})}\frac{m^3 kPa}{kJ} = 1298 \; kPa$$

The back work ratio is (can you show that this is true?)

$$BWR = \frac{w_{comp}}{w_{\exp}} = \frac{\Delta u_{12}}{-\Delta u_{34}}$$

$$= \frac{C_v(T_2 - T_1)}{C_v(T_3 - T_4)} = \frac{(T_2 - T_1)}{(T_3 - T_4)}$$

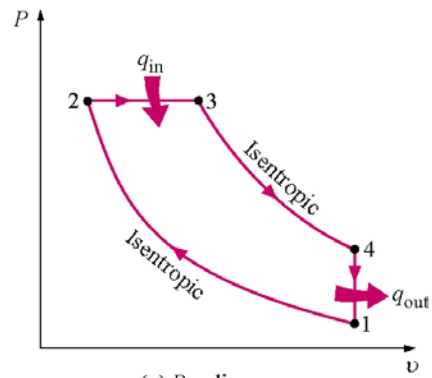
$$= 0.225 \;\; or \;\; 22.5\%$$

Air-Standard Diesel Cycle

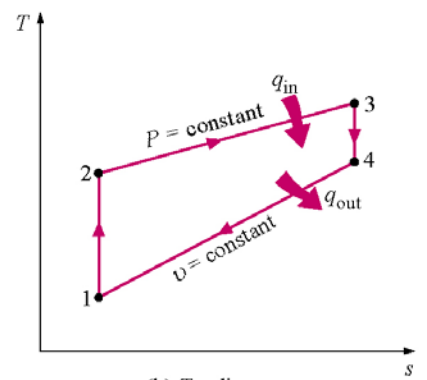
The air-standard Diesel cycle is the ideal cycle that approximates the Diesel combustion engine

Process	Description
1-2	Isentropic compression
2-3	Constant pressure heat addition
3-4	Isentropic expansion
4-1	Constant volume heat rejection

The *P-v* and *T-s* diagrams are



(a) P-υdiagram



(b) T-s diagram

Thermal efficiency of the Diesel cycle

$$\eta_{th,\,Diesel} = \frac{W_{net}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

Now to find Q_{in} and Q_{out}.

Apply the first law closed system to process 2-3, P = constant.

$$E_{in} - E_{out} = \Delta E$$

$$Q_{net,\,23} - W_{net,\,23} = \Delta U_{23}$$

$$W_{net,\,23} = W_{other,\,23} + W_{b,\,23} = 0 + \int_2^3 PdV$$

$$= P_2(V_3 - V_2)$$

Thus, for constant specific heats

$$Q_{net,\,23} = \Delta U_{23} + P_2(V_3 - V_2)$$

$$Q_{net,\,23} = Q_{in} = mC_v(T_3 - T_2) + mR(T_3 - T_2)$$

$$Q_{in} = mC_p(T_3 - T_2)$$

Apply the first law closed system to process 4-1, V = constant (just like the Otto cycle)

$$E_{in} - E_{out} = \Delta E$$

$$Q_{net,\,41} - W_{net,\,41} = \Delta U_{41}$$

$$W_{net,\,41} = W_{other,\,41} + W_{b,\,41} = 0 + \int_4^1 PdV = 0$$

Thus, for constant specific heats

$$Q_{net,\,41} = \Delta U_{41}$$

$$Q_{net,\,41} = -Q_{out} = mC_v(T_1 - T_4)$$

$$Q_{out} = -mC_v(T_1 - T_4) = mC_v(T_4 - T_1)$$

The thermal efficiency becomes

$$\eta_{th,\, Diesel} = 1 - \frac{Q_{out}}{Q_{in}}$$

$$= 1 - \frac{mC_v(T_4 - T_1)}{mC_p(T_3 - T_2)}$$

$$\eta_{th,\, Diesel} = 1 - \frac{C_v(T_4 - T_1)}{C_p(T_3 - T_2)}$$

$$= 1 - \frac{1}{k}\frac{T_1(T_4 / T_1 - 1)}{T_2(T_3 / T_2 - 1)}$$

What is T_3/T_2 ?

$$\frac{P_3 V_3}{T_3} = \frac{P_2 V_2}{T_2} \quad \text{where } P_3 = P_2$$

$$\frac{T_3}{T_2} = \frac{V_3}{V_2} = r_c$$

where r_c is called the cutoff ratio, defined as V_3 / V_2, and is a measure of the duration of the heat addition at constant pressure. Since the fuel is injected directly into the cylinder, the cutoff ratio can be related to the number of degrees that the crank rotated during the fuel injection into the cylinder.

What is T_4/T_1 ?

$$\frac{P_4 V_4}{T_4} = \frac{P_1 V_1}{T_1} \quad \text{where } V_4 = V_1$$

$$\frac{T_4}{T_1} = \frac{P_4}{P_1}$$

Recall processes 1-2 and 3-4 are isentropic, so

$$P_1 V_1^k = P_2 V_2^k \quad \text{and} \quad P_4 V_4^k = P_3 V_3^k$$

Since $V_4 = V_1$ and $P_3 = P_2$, we divide the second equation by the first equation and obtain

$$\frac{P_4}{P_1} = \left(\frac{V_3}{V_2}\right)^k = r_c^k$$

Therefore,

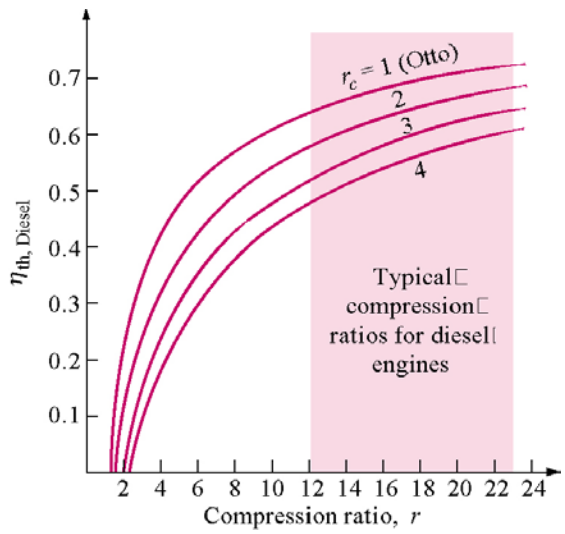
$$\eta_{th,\,Diesel} = 1 - \frac{1}{k}\frac{T_1(T_4 / T_1 - 1)}{T_2(T_3 / T_2 - 1)}$$

$$= 1 - \frac{1}{k}\frac{T_1}{T_2}\frac{r_c^k - 1}{(r_c - 1)}$$

$$= 1 - \frac{1}{r^{k-1}}\frac{r_c^k - 1}{k(r_c - 1)}$$

What happens as r_c goes to 1? Sketch the *P*-*v* diagram for the Diesel cycle and show r_c approaching1 in the limit.

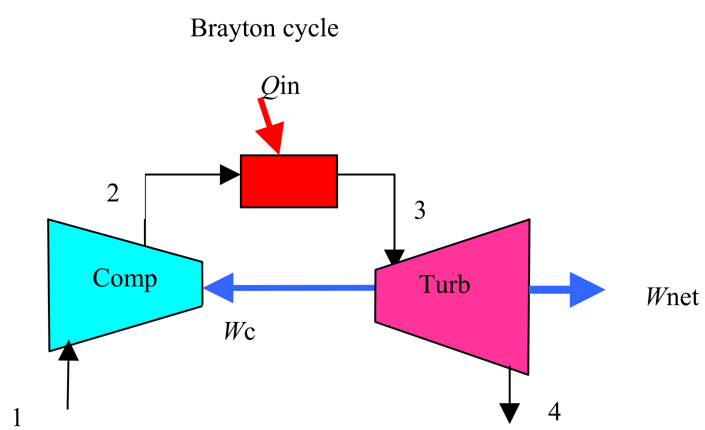
When $r_c > 1$ for a fixed *r*, $\eta_{th,\, Diesel} < \eta_{th,\, Otto}$. But, since $r_{Diesel} > r_{Otto}$, $\eta_{th,\, Diesel} > \eta_{th,\, Otto}$.



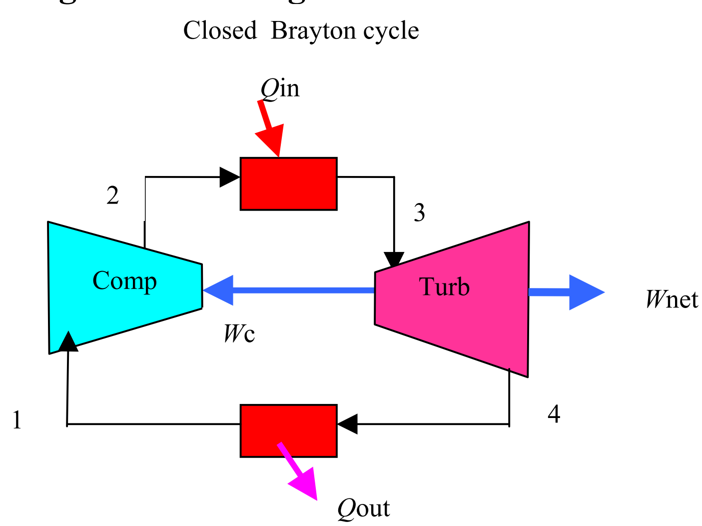
Brayton Cycle

The Brayton cycle is the air-standard ideal cycle approximation for the gas-turbine engine. This cycle differs from the Otto and Diesel cycles in that the processes making the cycle occur in open systems or control volumes. Therefore, an open system, steady-flow analysis is used to determine the heat transfer and work for the cycle.

We assume the working fluid is air and the specific heats are constant and will consider the cold-air-standard cycle.

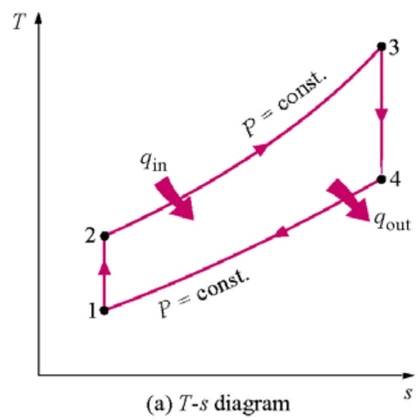


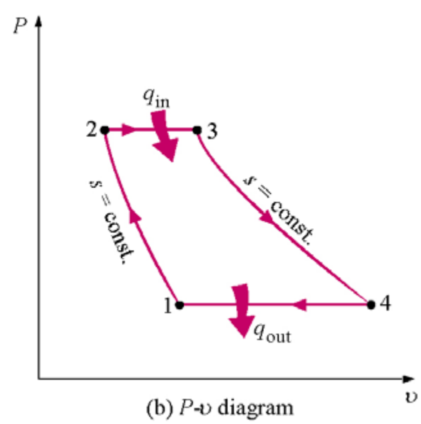
The closed cycle gas-turbine engine



Process	Description
1-2	Isentropic compression (in a compressor)
2-3	Constant pressure heat addition
3-4	Isentropic expansion (in a turbine)
4-1	Constant pressure heat rejection

The *T-s* and *P-v* diagrams are





Thermal efficiency of the Brayton cycle

$$\eta_{th,\ Brayton} = \frac{W_{net}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

Now to find Q_{in} and Q_{out}.

Apply the conservation of energy to process 2-3 for P = constant (no work), steady-flow, and neglect changes in kinetic and potential energies.

$$\dot{E}_{in} = \dot{E}_{out}$$
$$\dot{m}_2 h_2 + \dot{Q}_{in} = \dot{m}_3 h_3$$

The conservation of mass gives

$$\dot{m}_{in} = \dot{m}_{out}$$
$$\dot{m}_2 = \dot{m}_3 = \dot{m}$$

For constant specific heats, the heat added per unit mass flow is

$$\dot{Q}_{in} = \dot{m}(h_3 - h_2)$$
$$\dot{Q}_{in} = \dot{m}C_p(T_3 - T_2)$$
$$q_{in} = \frac{\dot{Q}_{in}}{\dot{m}} = C_p(T_3 - T_2)$$

The conservation of energy for process 4-1 yields for constant specific heats (let's take a minute for you to get the following result)

$$\dot{Q}_{out} = \dot{m}(h_4 - h_1)$$
$$\dot{Q}_{out} = \dot{m}C_p(T_4 - T_1)$$
$$q_{out} = \frac{\dot{Q}_{out}}{\dot{m}} = C_p(T_4 - T_1)$$

The thermal efficiency becomes

$$\eta_{th,\,Brayton} = 1 - \frac{\dot{Q}_{out}}{\dot{Q}_{in}} = 1 - \frac{q_{out}}{q_{in}}$$

$$= 1 - \frac{C_p(T_4 - T_1)}{C_p(T_3 - T_2)}$$

$$\eta_{th,\,Brayton} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

$$= 1 - \frac{T_1(T_4 / T_1 - 1)}{T_2(T_3 / T_2 - 1)}$$

Recall processes 1-2 and 3-4 are isentropic, so

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} \quad and \quad \frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{(k-1)/k}$$

Since $P_3 = P_2$ and $P_4 = P_1$, we see that

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \quad \text{or} \quad \frac{T_4}{T_1} = \frac{T_3}{T_2}$$

The Brayton cycle efficiency becomes

$$\eta_{th,\,Brayton} = 1 - \frac{T_1}{T_2}$$

Is this the same as the Carnot cycle efficiency?

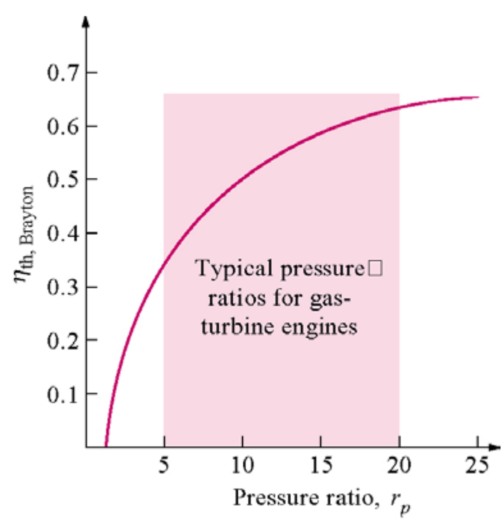
Since process 1-2 is isentropic,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = r_p^{(k-1)/k}$$

$$\frac{T_1}{T_2} = \frac{1}{r_p^{(k-1)/k}}$$

where the pressure ratio is $r_p = P_2/P_1$ and

$$\eta_{th,\,Brayton} = 1 - \frac{1}{r_p^{(k-1)/k}}$$



Extra Assignment

Evaluate the Brayton cycle efficiency by determining the net work directly from the turbine work and the compressor work. Compare your result with the above expression. Note that this approach does not require the closed cycle assumption.

Example 8-2

The ideal air-standard Brayton cycle operates with air entering the compressor at 95 kPa, 22°C. The pressure ratio r_p is 6:1 and the air leaves the heat addition process at 1100 K. Determine the compressor work and the turbine work per unit mass flow, the cycle efficiency, the back work ratio, and compare the compressor exit temperature to the turbine exit temperature. Assume constant properties.

Apply the conservation of energy for steady-flow and neglect changes in kinetic and potential energies to process 1-2 for the compressor. Note that the compressor is isentropic.

$$\dot{E}_{in} = \dot{E}_{out}$$
$$\dot{m}_1 h_1 + \dot{W}_{comp} = \dot{m}_2 h_2$$

The conservation of mass gives

$$\dot{m}_{in} = \dot{m}_{out}$$
$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

For constant specific heats, the compressor work per unit mass flow is

$$\dot{W}_{comp} = \dot{m}(h_2 - h_1)$$
$$\dot{W}_{comp} = \dot{m}C_p(T_2 - T_1)$$
$$w_{comp} = \frac{\dot{W}_{comp}}{\dot{m}} = C_p(T_2 - T_1)$$

Since the compressor is isentropic

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = r_p^{(k-1)/k}$$
$$T_2 = T_1 r_p^{(k-1)/k}$$
$$= (22 + 273)K(6)^{(1.4-1)/1.4}$$
$$= 492.5\,K$$

$$w_{comp} = C_p(T_2 - T_1)$$
$$= 1.005\frac{kJ}{kg \cdot K}(492.5 - 295)K$$
$$= 198.15\frac{kJ}{kg}$$

The conservation of energy for the turbine, process 3-4, yields for constant specific heats (let's take a minute for you to get the following result)

$$\dot{W}_{turb} = \dot{m}(h_3 - h_4)$$
$$\dot{W}_{turb} = \dot{m}C_p(T_3 - T_4)$$
$$w_{turb} = \frac{\dot{W}_{turb}}{\dot{m}} = C_p(T_3 - T_4)$$

Since process 3-4 is isentropic

$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{(k-1)/k}$$

Since $P_3 = P_2$ and $P_4 = P_1$, we see that

$$\frac{T_4}{T_3} = \left(\frac{1}{r_p}\right)^{(k-1)/k}$$

$$T_4 = T_3\left(\frac{1}{r_p}\right)^{(k-1)/k}$$

$$= 1100K\left(\frac{1}{6}\right)^{(1.4-1)/1.4}$$

$$= 659.1\,K$$

$$w_{turb} = C_p(T_3 - T_4) = 1.005\frac{kJ}{kg \cdot K}(1100 - 659.1)K$$

$$= 442.5\frac{kJ}{kg}$$

We have already shown the heat supplied to the cycle per unit mass flow in process 2-3 is

$$\dot{m}_2 = \dot{m}_3 = \dot{m}$$

$$\dot{m}_2 h_2 + \dot{Q}_{in} = \dot{m}_3 h_3$$

$$q_{in} = \frac{\dot{Q}_{in}}{\dot{m}} = h_3 - h_2$$

$$= C_p (T_3 - T_2) = 1.005 \frac{kJ}{kg \cdot K}(1100 - 492.5)K$$

$$= 609.6 \frac{kJ}{kg}$$

The net work done by the cycle is

$$w_{net} = w_{turb} - w_{comp}$$

$$= (442.5 - 198.15)\frac{kJ}{kg}$$

$$= 244.3 \frac{kJ}{kg}$$

The cycle efficiency becomes

$$\eta_{th,\ Brayton} = \frac{w_{net}}{q_{in}}$$

$$= \frac{244.3 \frac{kJ}{kg}}{609.6 \frac{kJ}{kg}} = 0.40 \quad or \quad 40\%$$

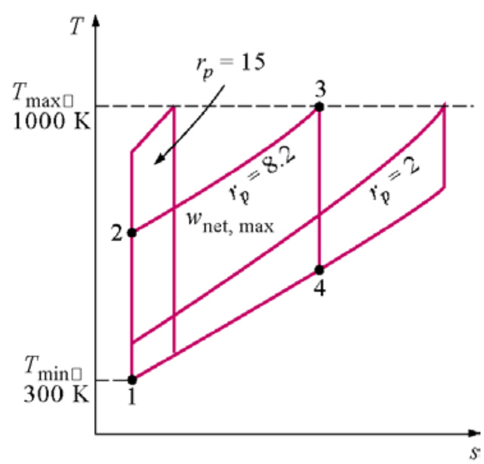
The back work ratio is defined as

$$BWR = \frac{w_{in}}{w_{out}} = \frac{w_{comp}}{w_{turb}}$$

$$= \frac{198.15\frac{kJ}{kg}}{442.5\frac{kJ}{kg}} = 0.448$$

Note that T_4 = 659.1 K > T_2 = 492.5 K, or the turbine outlet temperature is greater than the compressor exit temperature. Can this result be used to improve the cycle efficiency?

What happens to η_{th}, w_{in}/w_{out}, and w_{net} as the pressure ratio r_p is increased?



Let's take a closer look at the effect of the pressure ratio on the net work done.

$$
\begin{aligned}
w_{net} &= w_{turb} - w_{comp} \\
&= C_p(T_3 - T_4) - C_p(T_2 - T_1) \\
&= C_p T_3(1 - T_4 / T_3) - C_p T_1(T_2 / T_1 - 1) \\
&= C_p T_3\left(1 - \frac{1}{r_p^{(k-1)/k}}\right) - C_p T_1(r_p^{(k-1)/k} - 1)
\end{aligned}
$$

Note that the net work is **zero** when

$$
r_p = 1 \quad and \quad r_p = \left(\frac{T_3}{T_1}\right)^{(k-1)/k}
$$

For fixed T_3 and T_1, the pressure ratio that makes the work a maximum is obtained from:

$$
\frac{dw_{net}}{dr_p} = 0
$$

This is easier to do if we let $X = r_p^{(k-1)/k}$

$$
w_{net} = C_p T_3\left(1 - \frac{1}{X}\right) - C_p T_1(X - 1)
$$

$$
\frac{dw_{net}}{dX} = C_p T_3[0 - (-1)X^{-2}] - C_p T_1[1 - 0] = 0
$$

Solving for X

$$X^2 = \frac{T_3}{T_1} = \left(r_p\right)^{2(k-1)/k}$$

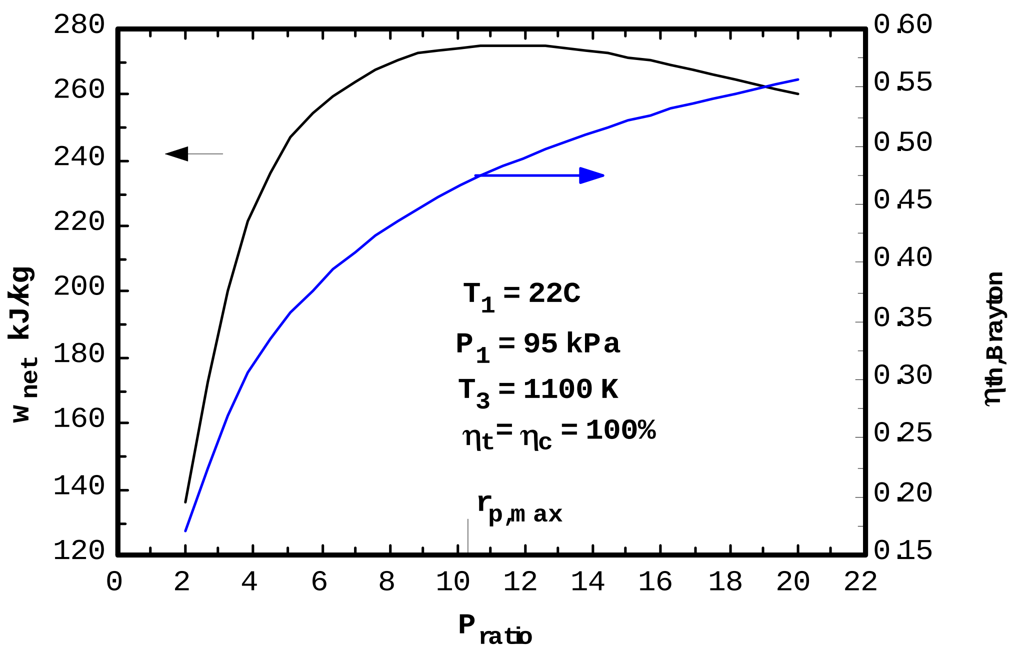
Then, the r_p that makes the work a maximum for the constant property case and fixed T_3 and T_1 is

$$r_{p,\text{ max work}} = \left(\frac{T_3}{T_1}\right)^{k/[2(k-1)]}$$

For the ideal Brayton cycle, show that the following results are true.

- When $r_p = r_{p,\text{ max work}}$, $T_4 = T_2$
- When $r_p < r_{p,\text{ max work}}$, $T_4 > T_2$
- When $r_p > r_{p,\text{ max work}}$, $T_4 < T_2$

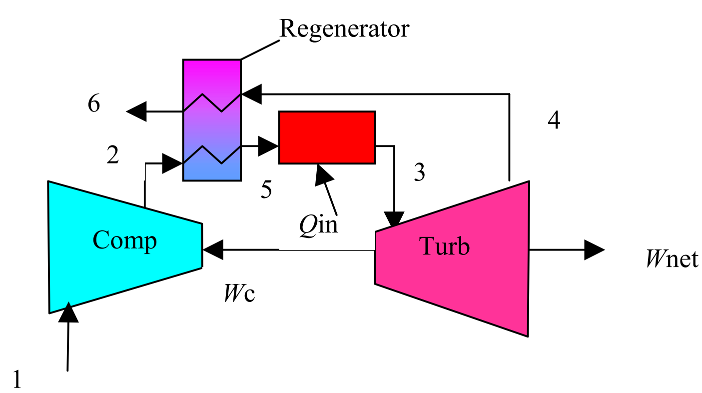
The following is a plot of net work per unit mass and the efficiency for the above example as a function of the pressure ratio.

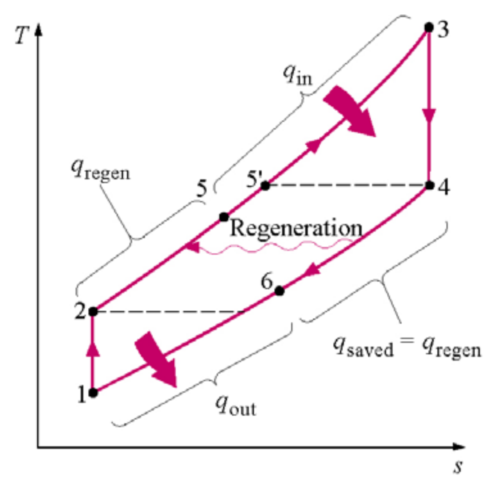


Regenerative Brayton Cycle

For the Brayton cycle, the turbine exhaust temperature is greater than the compressor exit temperature. Therefore, a heat exchanger can be placed between the hot gases leaving the turbine and the cooler gases leaving the compressor. This heat exchanger is called a regenerator or recuperator. The sketch of the regenerative Brayton cycle is shown below.

Regenerative Brayton cycle





We define the regenerator effectiveness ε_{regen} as the ratio of the heat transferred to the compressor gases in the regenerator to the maximum possible heat transfer to the compressor gases.

$$q_{regen,\,act} = h_5 - h_2$$

$$q_{regen,\,\max} = h_{5'} - h_2 = h_4 - h_2$$

$$\varepsilon_{regen} = \frac{q_{regen,\,act}}{q_{regen,\,\max}} = \frac{h_5 - h_2}{h_4 - h_2}$$

For ideal gases using the cold-air-standard assumption with constant specific heats, the regenerator effectiveness becomes

$$\varepsilon_{regen} \cong \frac{T_5 - T_2}{T_4 - T_2}$$

Using the closed cycle analysis and treating the heat addition and heat rejection as steady-flow processes, the regenerative cycle thermal efficiency is

$$\eta_{th,\,regen} = 1 - \frac{q_{out}}{q_{in}}$$

$$= 1 - \frac{h_6 - h_1}{h_3 - h_5}$$

Notice that the heat transfer occurring within the regenerator is **not** included in the efficiency calculation because this energy is not a heat transfer across the cycle boundary.

Assuming an ideal regenerator $\varepsilon_{regen} = 1$ and constant specific heats, the thermal efficiency becomes (take the time to show this on your own)

$$\eta_{th,\,regen} = 1 - \frac{T_1}{T_3}\left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

$$= 1 - \frac{T_1}{T_3}(r_p)^{(k-1)/k}$$

When does the efficiency of the air-standard Brayton cycle equal the efficiency of the air-standard regenerative Brayton cycle? If we set $\eta_{\text{th, Brayton}} = \eta_{\text{th, regen}}$ then

$$\eta_{th,\,Brayton} = \eta_{th,\,regen}$$

$$1 - \frac{1}{(r_p)^{(k-1)/k}} = 1 - \frac{T_1}{T_3}(r_p)^{(k-1)/k}$$

$$r_p = \left(\frac{T_3}{T_1}\right)^{k/[2(k-1)]}$$

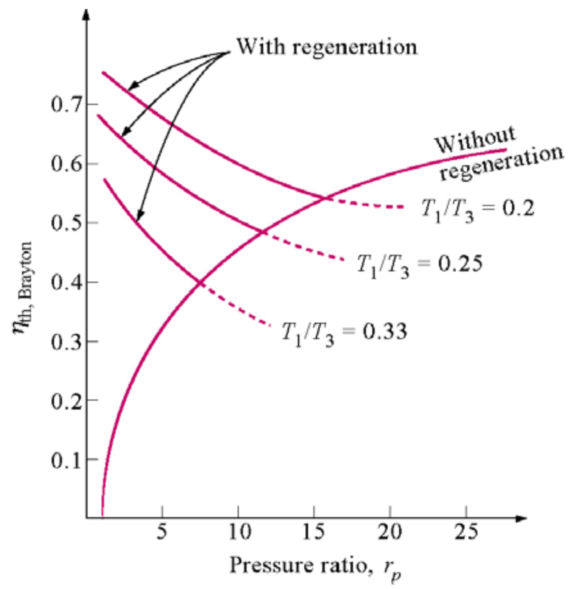
Recall that this is the pressure ratio that maximizes the net work for the simple Brayton cycle and makes $T_4 = T_2$. What happens if the regenerative Brayton cycle operates at a pressure ratio larger than this value?

For fixed T_3 and T_1, pressure ratios greater than this value cause T_4 to be less than T_2, and the regenerator is not effective.

What happens to the net work when a regenerator is added?

What happens to the heat supplied when a regenerator is added?

The following shows a plot of the regenerative Brayton cycle efficiency as a function of the pressure ratio and minimum to maximum temperature ratio, T_1/T_3.



Example 8-3: Regenerative Brayton Cycle

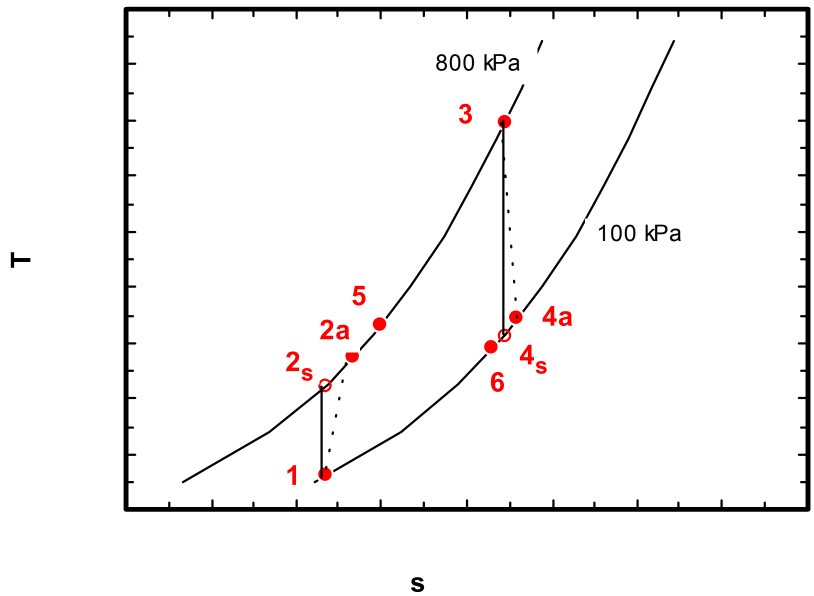
Air enters the compressor of a regenerative gas-turbine engine at 100 kPa and 300 K and is compressed to 800 kPa. The regenerator has an effectiveness of 65 percent, and the air enters the turbine at 1200 K. For a compressor efficiency of 75 percent and a turbine efficiency of 86 percent, determine
(a) The heat transfer in the regenerator.
(b) The back work ratio.
(c) The cycle thermal efficiency.

Compare the results for the above cycle with the ones listed below that have the same common data as required.
(a) The actual cycle with no regeneration, $\varepsilon = 0$.
(b) The actual cycle with ideal regeneration, $\varepsilon = 1.0$.
(b) The ideal cycle with regeneration, $\varepsilon = 0.65$.
(d) The ideal cycle with no regeneration, $\varepsilon = 0$.
(e) The ideal cycle with ideal regeneration, $\varepsilon = 1.0$.

We assume air is an ideal gas with constant specific heats, that is, we use the cold-air-standard assumption.

The cycle schematic is the same as above and the *T-s* diagram showing the effects of compressor and turbine efficiencies is below.

T-s Diagram for Gas Turbine with Regeneration



Summary of Results

Cycle type	Actual	Actual	Actual	Ideal	Ideal	Ideal
ε_{regen}	0.00	0.65	1.00	0.00	0.65	1.00
η_{comp}	0.75	0.75	0.75	1.00	1.00	1.00
η_{turb}	0.86	0.86	0.86	1.00	1.00	1.00
q_{in} kJ/kg	578.3	504.4	464.6	659.9	582.2	540.2
w_{comp} kJ/kg	326.2	326.2	326.2	244.6	244.6	244.6
w_{turb} kJ/kg	464.6	464.6	464.6	540.2	540.2	540.2
w_{comp}/w_{turb}	0.70	0.70	0.70	0.453	0.453	0.453
η_{th}	24.0%	27.5%	29.8%	44.8%	50.8%	54.7%

Compressor analysis

The isentropic temperature at compressor exit is

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

$$T_{2s} = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

$$= 300K\left(\frac{800kPa}{100kPa}\right)^{(1.4-1)/1.4} = 543.4K$$

To find the actual temperature at compressor exit, T_{2a}, we apply the compressor efficiency

$$\eta_{comp} = \frac{w_{isen,comp}}{w_{act,comp}} = \frac{h_{2s} - h_1}{h_{2a} - h_1} \cong \frac{T_{2s} - T_1}{T_{2a} - T_1}$$

$$T_{2a} = T_1 + \frac{1}{\eta_{comp}}(T_{2s} - T_1)$$

$$= 300K + \frac{1}{0.75}(543.4 - 300)K$$

$$= 624.6K$$

Since the compressor is adiabatic and has steady-flow

$$w_{comp} = h_{2a} - h_1 = C_p(T_{2a} - T_1)$$

$$= 1.005\frac{kJ}{kg \cdot K}(624.6 - 300)K = 326.2\frac{kJ}{kg}$$

Turbine analysis

The conservation of energy for the turbine, process 3-4, yields for constant specific heats (let's take a minute for you to get the following result)

$$\dot{W}_{turb} = \dot{m}(h_3 - h_{4a})$$
$$\dot{W}_{turb} = \dot{m}C_p(T_3 - T_{4a})$$
$$w_{turb} = \frac{\dot{W}_{turb}}{\dot{m}} = C_p(T_3 - T_{4a})$$

Since $P_3 = P_2$ and $P_4 = P_1$, we can find the isentropic temperature at the turbine exit.

$$\frac{T_{4s}}{T_3} = \left(\frac{P_4}{P_3}\right)^{(k-1)/k}$$

$$T_{4s} = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k}$$
$$= 1200K(\frac{100kPa}{800kPa})^{(1.4-1)/1.4} = 662.5K$$

To find the actual temperature at turbine exit, T_{4a}, we apply the turbine efficiency.

$$\eta_{turb} = \frac{w_{act,turb}}{w_{isen,turb}} = \frac{h_3 - h_{4a}}{h_3 - h_{4s}} \cong \frac{T_3 - T_{4a}}{T_3 - T_{4s}}$$

$$\begin{aligned} T_{4a} &= T_3 - \eta_{turb}(T_3 - T_{4s}) \\ &= 1200K - 0.86(1200 - 662.5)K \\ &= 737.7K > T_{2a} \end{aligned}$$

The turbine work becomes

$$\begin{aligned} w_{turb} &= h_3 - h_{4a} = C_p(T_3 - T_{4a}) \\ &= 1.005\frac{kJ}{kg \cdot K}(1200 - 737.7)K \\ &= 464.6\frac{kJ}{kg} \end{aligned}$$

The back work ratio is defined as

$$\begin{aligned} BWR &= \frac{w_{in}}{w_{out}} = \frac{w_{comp}}{w_{turb}} \\ &= \frac{326.2\frac{kJ}{kg}}{464.6\frac{kJ}{kg}} = 0.70 \end{aligned}$$

Regenerator analysis

To find T_5, we apply the regenerator effectiveness.

$$\varepsilon_{regen} \cong \frac{T_5 - T_{2a}}{T_{4a} - T_{2a}}$$

$$\begin{aligned} T_5 &= T_{2a} + \varepsilon_{regen}(T_{4a} - T_{2a}) \\ &= 624.6K + 0.65(737.7 - 624.6)K \\ &= 698.1K \end{aligned}$$

To find the heat transferred from the turbine exhaust gas to the compressor exit gas, apply the steady-flow conservation of energy to the compressor gas side of the regenerator.

$$\begin{aligned} \dot{m}_{2a}h_{2a} + \dot{Q}_{regen} &= \dot{m}_5 h_5 \\ \dot{m}_{2a} &= \dot{m}_5 = \dot{m} \\ q_{regen} &= \frac{\dot{Q}_{regen}}{\dot{m}} = h_5 - h_{2a} \\ &= C_p(T_5 - T_{2a}) \\ &= 1.005\frac{kJ}{kg \cdot K}(698.1 - 624.6)K \\ &= 73.9\frac{kJ}{kg} \end{aligned}$$

Using q_{regen}, we can determine the turbine exhaust gas temperature at the regenerator exit.

$$\dot{m}_{4a} h_{4a} = \dot{Q}_{regen} + \dot{m}_6 h_6$$
$$\dot{m}_{4a} = \dot{m}_6 = \dot{m}$$
$$q_{regen} = \frac{\dot{Q}_{regen}}{\dot{m}} = h_{4a} - h_6 = C_p (T_{4a} - T_6)$$

$$T_6 = T_{4a} - \frac{q_{regen}}{C_p} = 737.7K - \frac{73.9\frac{kJ}{kg}}{1.005\frac{kJ}{kg \cdot K}}$$
$$= 664.2K$$

Heat supplied to cycle

Apply the steady-flow conservation of energy to the heat exchanger for process 5-3. We obtain a result similar to that for the simple Brayton cycle.

$$q_{in} = h_3 - h_5 = C_p (T_3 - T_5)$$
$$= 1.005 \frac{kJ}{kg \cdot K} (1200 - 698.1) K$$
$$= 504.4 \frac{kJ}{kg}$$

Cycle thermal efficiency

The net work done by the cycle is

$$w_{net} = w_{turb} - w_{comp}$$
$$= (464.6 - 326.2) \frac{kJ}{kg} = 138.4 \frac{kJ}{kg}$$

The cycle efficiency becomes

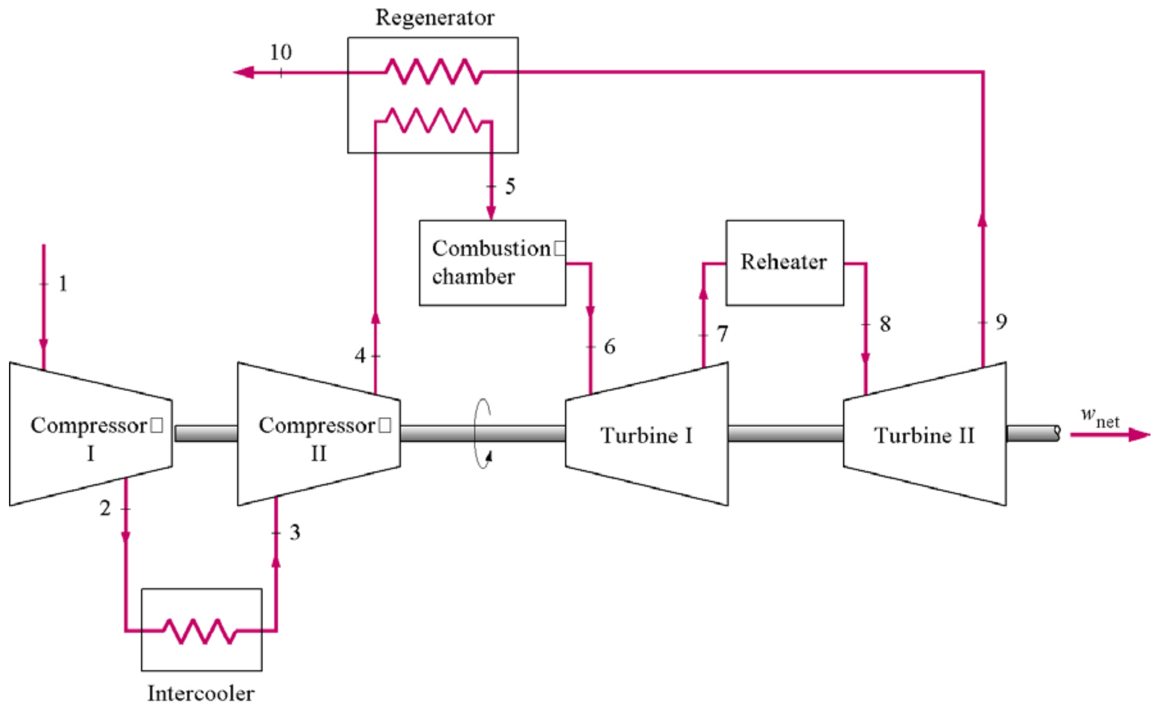
$$\eta_{th,\,Brayton} = \frac{w_{net}}{q_{in}}$$

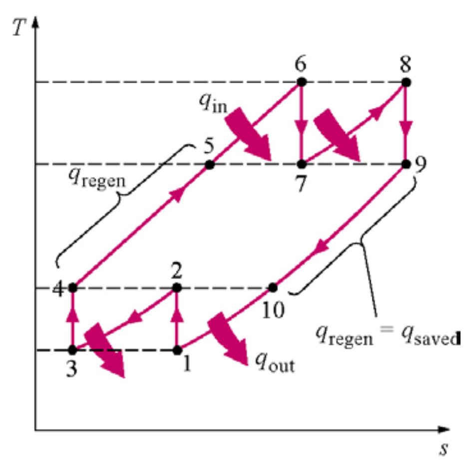
$$= \frac{138.4\,\dfrac{kJ}{kg}}{504.4\,\dfrac{kJ}{kg}} = 0.274 \quad or \quad 27.4\%$$

You are encouraged to complete the calculations for the other values found in the summary table.

Other Ways to Improve Brayton Cycle Performance

Intercooling and reheating are two important ways to improve the performance of the Brayton cycle with regeneration.





Intercooling

When using multistage compression, cooling the working fluid between the stages will reduce the amount of compressor work required. The compressor work is reduced because cooling the working fluid reduces the average specific volume of the fluid and thus reduces the amount of work on the fluid to achieve the given pressure rise.

To determine the intermediate pressure at which intercooling should take place to minimize the compressor work, we follow the approach shown in Chapter 6.

For the adiabatic, steady-flow compression process, the work input to the compressor per unit mass is

$$w_{comp} = \int_1^4 v\,dP = \int_1^2 v\,dP + \cancelto{0}{\int_2^3 v\,dP} + \int_3^4 v\,dP$$

For the isentropic compression process

$$
\begin{aligned}
w_{comp} &= \frac{k}{k\text{-}1}(P_2 v_2 - P_1 v_1) + \frac{k}{k\text{-}1}(P_4 v_4 - P_3 v_3) \\
&= \frac{k}{k\text{-}1} R(T_2 - T_1) + \frac{kR}{k\text{-}1}(T_4 - T_3) \\
&= \frac{k}{k\text{-}1} R\left[T_1(T_2 / T_1 - 1) + T_3(T_4 / T_3 - 1)\right] \\
&= \frac{k}{k\text{-}1} R\left[T_1\left(\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1\right) + T_3\left(\left(\frac{P_4}{P_3}\right)^{(k-1)/k} - 1\right)\right]
\end{aligned}
$$

Notice that the fraction $kR/(k\text{-}1) = C_p$.

$$
w_{comp} = C_p\left[T_1\left(\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1\right) + T_3\left(\left(\frac{P_4}{P_3}\right)^{(k-1)/k} - 1\right)\right]
$$

Can you obtain this relation another way? Hint: apply the first law to processes 1-4.

For two-stage compression, let's assume that intercooling takes place at constant pressure and the gases can be cooled to the inlet temperature for the compressor, such that $P_3 = P_2$ and $T_3 = T_1$.

The total work supplied to the compressor becomes

$$
\begin{aligned}
w_{comp} &= C_p T_1 \left[\left(\left(\frac{P_2}{P_1} \right)^{(k-1)/k} - 1 \right) + \left(\left(\frac{P_4}{P_2} \right)^{(k-1)/k} - 1 \right) \right] \\
&= C_p T_1 \left[\left(\frac{P_2}{P_1} \right)^{(k-1)/k} + \left(\frac{P_4}{P_2} \right)^{(k-1)/k} - 2 \right]
\end{aligned}
$$

To find the unknown pressure P_2 that gives the minimum work input for fixed compressor inlet conditions T_1, P_1, and exit pressure P_4, we set

$$
\frac{dw_{comp}(P_2)}{dP_2} = 0
$$

This yields

$$
P_2 = \sqrt{P_1 P_4}
$$

or, the pressure ratios across the two compressors are equal.

$$
\frac{P_2}{P_1} = \frac{P_4}{P_2} = \frac{P_4}{P_3}
$$

Intercooling is almost always used with regeneration. During intercooling the compressor exit temperature is reduced; therefore, more heat must be supplied in the heat addition process. Regeneration can make up part of the required heat transfer.

To supply only compressed air, using intercooling requires less work input. The next time you go to a home supply store where air compressors are sold, check the larger air compressors to see if intercooling is used. For the larger air compressors, the compressors are made of two piston-cylinder chambers. The intercooling heat exchanger may be only a pipe with a few attached fins that connects the large piston-cylinder chamber with the smaller piston-cylinder chamber.

Extra Assignment

Obtain the expression for the compressor total work by applying conservation of energy directly to the low- and high-pressure compressors.

Reheating

When using multistage expansion through two or more turbines, reheating between stages will increase the net work done (it also increases the required heat input). The regenerative Brayton cycle with reheating is shown above.

The optimum intermediate pressure for reheating is the one that maximizes the turbine work. Following the development given above for intercooling and assuming reheating to the high-pressure turbine inlet temperature in a constant pressure steady-flow process, we can show the optimum reheat pressure to be

$$P_7 = \sqrt{P_6 P_9}$$

or the pressure ratios across the two turbines are equal.

$$\frac{P_6}{P_7} = \frac{P_7}{P_9} = \frac{P_8}{P_9}$$