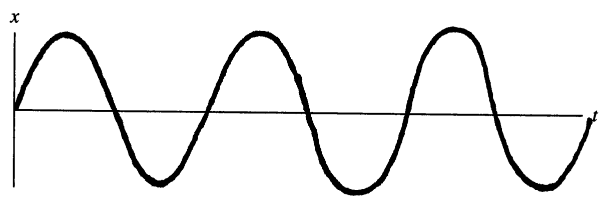
14 Oscillations

14.1 Equilibrium and Oscillation

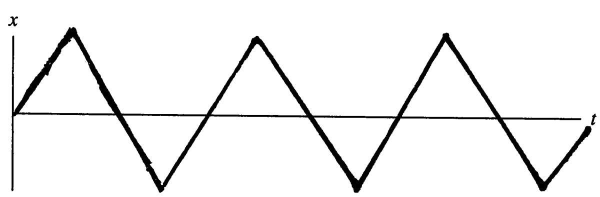
14.2 Linear Restoring Forces and Simple Harmonic Motion

1. On the axes below, sketch three cycles of the position-versus-time graph for:

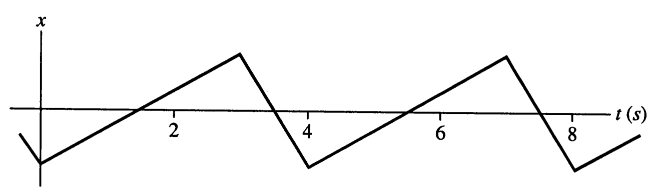
   a. A particle undergoing simple harmonic motion.

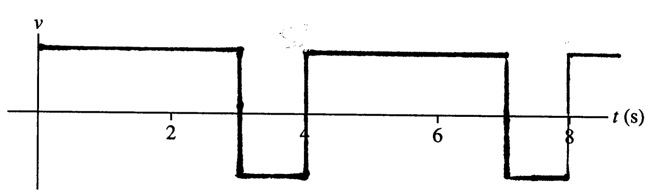


   b. A particle undergoing periodic motion that is not simple harmonic motion.



2. Consider the particle whose motion is represented by the x-versus-t graph below.





   a. Is this periodic motion? yes

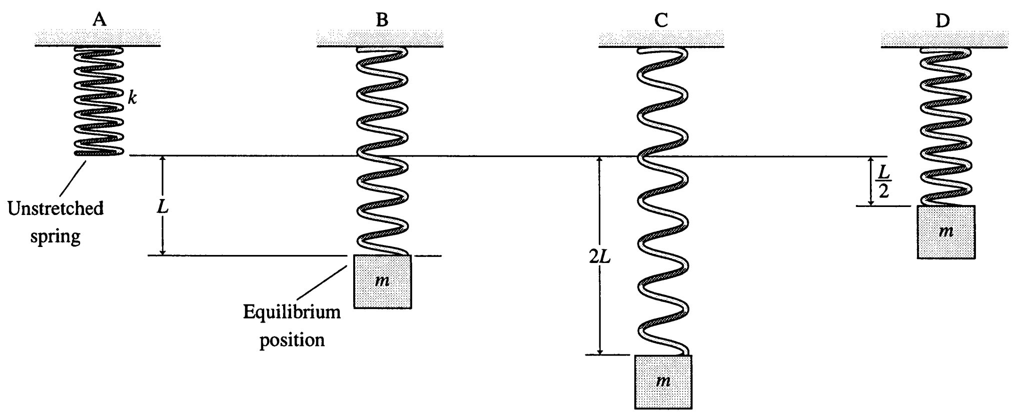
   b. Is this motion SHM? no

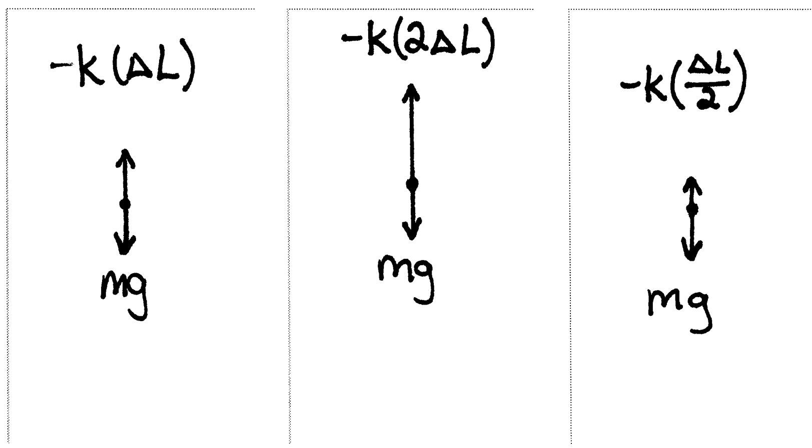
   c. What is the period? 4sec

   d. What is the frequency? $f = \frac{1}{T} = 0.25$ Hz

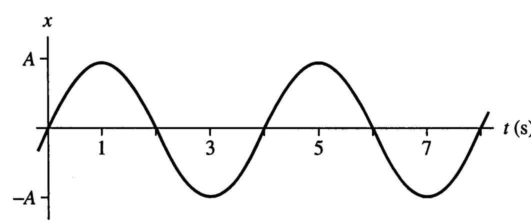
   e. You learned in Chapter 2 to relate velocity graphs to position graphs. Use that knowledge to draw the particle's velocity-versus-time graph on the axes provided.

3. Figure A shows an unstretched spring. Figure B shows a mass m hanging at rest from the spring. It has stretched the spring by L. Figures C and D show two instants in the oscillation of mass m about the equilibrium position. Draw free-body diagrams for the mass in the Figures B, C, and D.





4. The figure shows the position-versus-time graph of a particle in SHM.



a. At what times is the particle moving to the right at maximum speed?

0s, 4s, 8s

b. At what times is the particle moving to the left at maximum speed?

2s, 6s

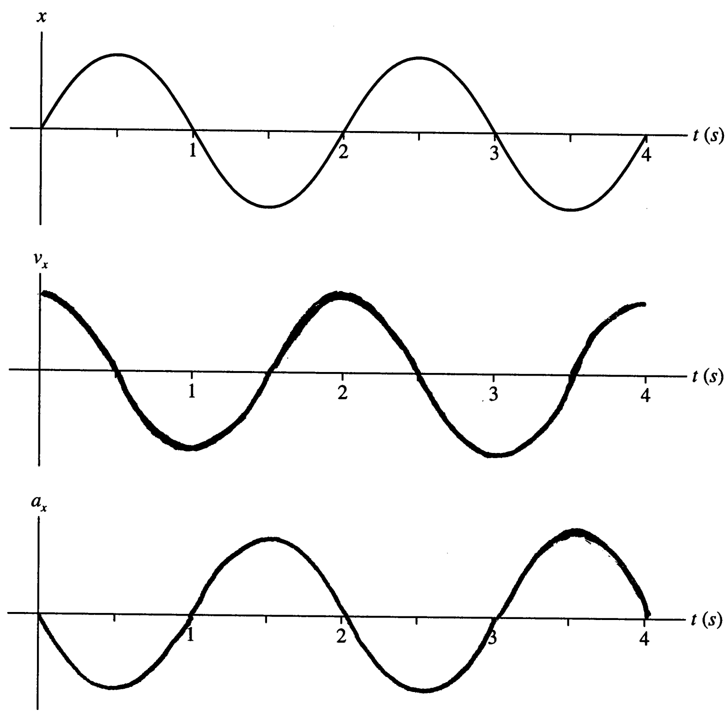
c. At what times is the particle instantaneously at rest?

1s, 3s, 5s, 7s

14.3 Describing Simple Harmonic Motion

5. The graph shown is the position-versus-time graph of an oscillating particle.
   a. Draw the corresponding velocity-versus-time graph.
   b. Draw the corresponding acceleration-versus-time graph.

   Hint: Remember that velocity is the slope of the position graph, and acceleration is the slope of the velocity graph.



   c. At what times is the position a maximum? 0.5s, 2.5s
      At those times, is the velocity a maximum, a minimum, or zero? zero
      At those times, is the acceleration a maximum, a minimum, or zero? minimum
   d. At what times is the position a minimum (most negative)? 1.5s, 3.5s
      At those times, is the velocity a maximum, a minimum, or zero? zero
      At those times, is the acceleration a maximum, a minimum, or zero? maximum
   e. At what times is the velocity a maximum? 0s, 2s, 4s
      At those times, where is the particle? at x = 0
   f. Can you find a simple relationship between the *sign* of the position and the *sign* of the acceleration at the same instant of time? If so, what is it?

      sign(a) = -sign(x)

6. Consider the function $x(t) = A\cos(2\pi t / T)$.

a. What are the units of the quantity $2\pi t / T$? radians

b. What is the value of x at $t = 0$? Explain.

x = A cos(0) = 1

c. What are the next *two* times at which x has the same value as it does at $t = 0$? Your answer will be in terms of the quantity T and, perhaps, constants such as π.

t = T, 2T (cos 2π = cos 4π = 1)

d. What is the first time after $t = 0$ when $x = 0$? Explain.

t = T/4, $\cos\frac{\pi}{2} = 0$

e. What is the value of x at $t = T/2$? Explain.

x = −A, cos π = −1

7. A mass on a spring oscillates with period T, amplitude A, maximum speed v_{max}, and maximum acceleration a_{max}.

a. If T doubles without changing A,

i. how does v_{max} change, if at all?

$v_{max} \rightarrow \frac{v_{max}}{2}$ The mass takes twice as long to travel the same distance.

ii. how does a_{max} change, if at all?

$a_{max} \rightarrow \frac{a_{max}}{4}$ The mass accelerates to half the speed in twice the time.

b. If A doubles without changing T,

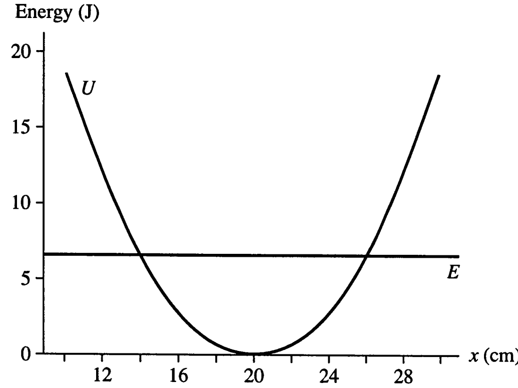
i. how does v_{max} change, if at all?

$v_{max} \rightarrow 2v_{max}$

ii. how does a_{max} change, if at all?

$a_{max} \rightarrow 2a_{max}$

14.4 Energy in Simple Harmonic Motion



8. The figure shows the potential-energy diagram and the total energy line of a particle oscillating on a spring.

   a. What is the spring's equilibrium length?

   20 cm

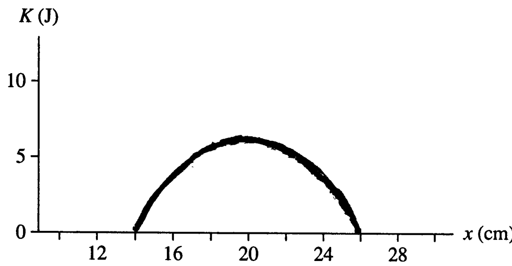
   b. Where are the turning points of the motion? Explain how you identify them.

   14 cm, 26 cm
   $U = E$

   c. What is the particle's maximum kinetic energy?

   ~ 6.5 J

   d. Draw a graph of the particle's kinetic energy as a function of position.



   e. What will be the turning points if the particle's total energy is doubled?

   11.6 cm, 28.4 cm $\Delta x \propto \sqrt{E}$ so $\sqrt{2}\,\Delta x$
   1.4(6) = 8.4 cm

9. Equation 14.23 in the textbook states that $\frac{1}{2}m(v_{max})^2 = \frac{1}{2}kA^2$. What does this mean? Write a couple of sentences explaining how to interpret this equation.

   The equation means that the maximum potential energy $\frac{1}{2}kA^2$ is the same as the maximum kinetic energy $\frac{1}{2}mv_{max}^2$ but it does not imply that these maxima occur at the same time. The energy shifts back and forth so that when the potential energy is a maximum, the kinetic energy is zero and vice-versa.

10. A mass oscillating on a spring with an amplitude of 5.0 cm has a period of 2.0 s.

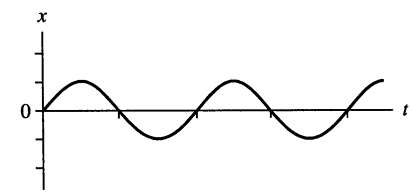
a. What will the period be if the amplitude is doubled to 10.0 cm without changing the mass? Explain.

2.0 s The period is independent of the amplitude for an ideal spring.

b. What will the period be if the mass is doubled without changing the 5.0 cm amplitude? Explain.

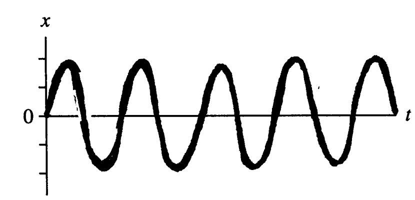
~2.8 s The period is proportional to the square root of the mass, so it increases by $\sqrt{2}$.

11. The top graph shows the position versus time for a mass oscillating on a spring. On the axes below, sketch the position-versus-time graph for this block for the following situations:

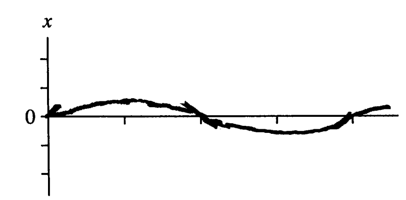


Note: The changes described in each part refer back to the original oscillation, not to the oscillation of the previous part of the question. Assume that all other parameters remain constant. Use the same horizontal and vertical scales as the original oscillation graph.

a. The amplitude and the frequency are both doubled.



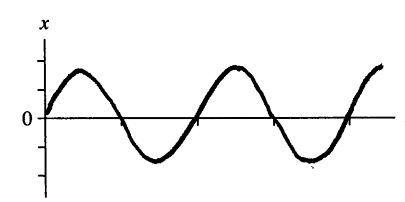
b. The amplitude is halved and the mass is quadrupled.



c. The energy is doubled.

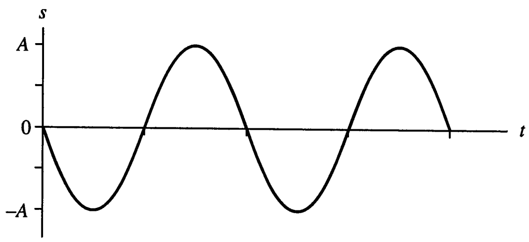
Amplitude increases by $\sqrt{2}$.

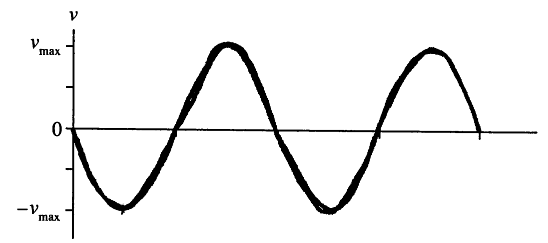
$E = \frac{1}{2} k A^2$



14.5 Pendulum Motion

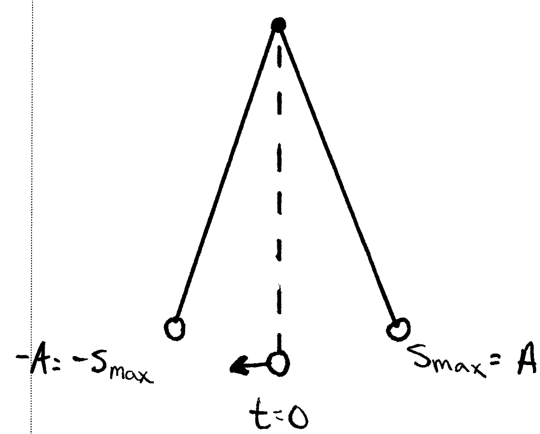
12. The graph shows the displacement s versus time for an oscillating pendulum.





a. Draw the pendulum's velocity-versus-time graph.

b. In the space at the right, draw a *picture* of the pendulum that shows (and labels!)
- The extremes of its motion.
- Its position at $t = 0$ s.
- Its direction of motion (using an arrow) at $t = 0$ s.



13. A pendulum on planet X, where the value of g is unknown, oscillates with a period of 2 seconds. What is the period of this pendulum if:

a. Its mass is increased by a factor of 4?
Note: You do not know the values of m, L, or g, so do not assume any specific values.

2s. The period is independent of the mass.

b. Its length is increased by a factor of 4?

4s. The period is proportional to the square-root of the length.

c. Its oscillation amplitude is increased by a factor of 4?

~2s. The period is approximately independent of the amplitude.

14.6 Damped Oscillations

14. For a damped oscillation, is the time constant τ greater than or less than the time in which the oscillation amplitude decays to half of its initial value? Explain.

$\tau > t_{1/2}$ τ = time to decay by 1/e ~.37

15. The amplitude of a damped oscillation decays to one-half of its initial value in 4.0 s. How much *additional* time will it take until the amplitude is one-quarter of its initial value? Explain.

4.0s $1/4 = 1/2 \times 1/2$ so the additional time is $t_{1/2}$.

16. If the time constant τ of an oscillator is decreased,

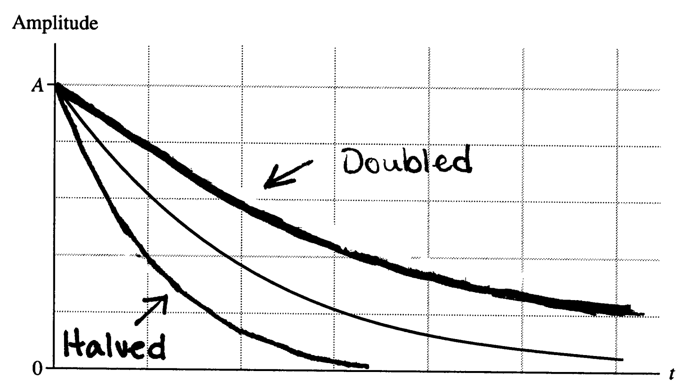
a. Is the drag force increased or decreased?

Increased

b. Do the oscillations damp out more quickly or less quickly?

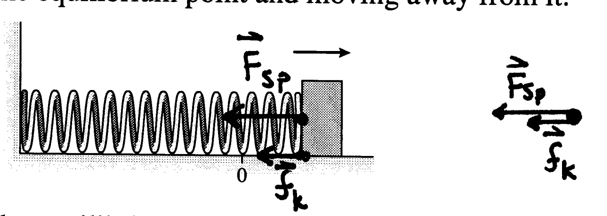
more quickly

17. The figure below shows the decreasing amplitude of a damped oscillator. (The oscillations are occurring rapidly and are not shown; this shows only their amplitude.) On the same axes, draw the amplitude if (a) the time constant is doubled and (b) the time constant is halved. Label your two curves "doubled" and "halved."

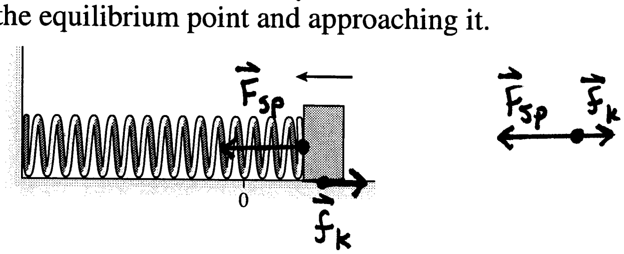


18. A block on a spring oscillates horizontally on a table *with friction*. Draw and label force vectors on the block to show all *horizontal* forces on the block.

a. The mass is to the right of the equilibrium point and moving away from it.



b . The mass is to the right of the equilibrium point and approaching it.



14.7 Driven Oscillations and Resonance

19. A car drives along a bumpy road on which the bumps are equally spaced. At a speed of 20 mph, the frequency of hitting bumps is equal to the natural frequency of the car bouncing on its springs.

   a. Draw a graph of the car's vertical bouncing amplitude as a function of its speed if the car has new shock absorbers (large damping coefficient).

   b. Draw a graph of the car's vertical bouncing amplitude as a function of its speed if the car has worn out shock absorbers (small damping coefficient).

   Draw both graphs on the same axes, and label them as to which is which.