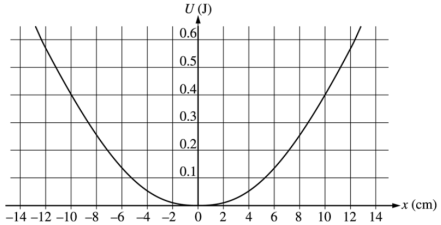
AP REVIEW #27 KEY (Harmonic Motion)



2002B1 (15 points) A 3.0 kg object subject to a restoring force F is undergoing simple harmonic motion with a small amplitude. The potential energy U of the object as a function of distance x from its equilibrium position is shown above. This particular object has a total energy E: of 0.4 J.

(a) What is the object's potential energy when its displacement is +4 cm from its equilibrium position?

(a) 1 point

For a reasonable value of U read from the graph — 1 point

$U \approx 0.05$ J

Alternate solution — *Alternate points*

Using the equation for the potential energy of simple harmonic motion:

$U = \frac{1}{2}kx^2$

Using the distance of 10 cm at which the maximum value of 0.4 J for U occurs allows determination of the force constant k.

$0.4\text{ J} = \frac{1}{2}k(0.1\text{ m})^2$

$k = 80\text{ N/m}$

For determining the value of U at 4 cm — *1 point*

$U = \frac{1}{2}(80\text{ N/m})(0.04\text{ m})^2$

$U = 0.06$ J

(b) What is the farthest the object moves along the x-axis in the positive direction? Explain your reasoning.

(b) 3 points

For indicating that the maximum possible position in the $+x$-direction is 10 cm — 1 point

For a complete explanation — 2 points

A full-credit explanation would indicate either of the following:

1) The particle stops at this point (or that the kinetic energy is zero here) because all of the energy is in the form of potential energy.
2) The maximum potential energy cannot be greater than the total energy.

Incomplete explanations (such as only saying that the potential energy is 0.4 J at 10 cm) received one point. However, only saying that the total energy is 0.4 J received no credit, since this is just a restatement of given information.

(c) Determine the object's kinetic energy when its displacement is -7 cm.

(c) 3 points

For a reasonable value of U read from the graph (between 0.18 J and 0.22 J) — 1 point

This point was also awarded for a correct calculation of U using the force constant determined in part (a).

For subtracting this value of U from the total energy to obtain the kinetic energy — 1 point

For a consistent final answer — 1 point

$K \approx 0.2$ J

(d) What is the object's speed at x = 0 ?

(d) 4 points

For any indication that the potential energy is zero at $x = 0$ — 1 point

For setting the kinetic energy equal to the total energy — 1 point

$K = \frac{1}{2}mv^2 = E_{total}$

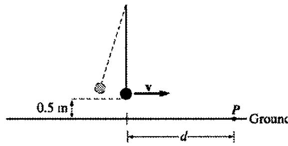
$v = \sqrt{2E_{total}/m}$

For correctly substituting the energy and mass into the above equation — 1 point

$v = \sqrt{2(0.4\text{ J})/(3.0\text{ kg})}$

For the correct answer — 1 point

$v = 0.5\text{ m/s}$



Note: Figure not drawn to scale.

(e) Suppose the object undergoes this motion because it is the bob of a simple pendulum as shown above. If the object breaks loose from the string at the instant the pendulum reaches its lowest point and hits the ground at point P shown, what is the horizontal distance d that it travels?

(e) 4 points

Using the kinematic equation for the vertical motion:

$y_f = y_i + v_{0y}t - \frac{1}{2}gt^2$

For correctly substituting $v_{0y} = 0$ — 1 point

Setting y_f also equal to zero and solving for t:

$t = \sqrt{2y_i/g} = \sqrt{2(0.5\text{ m})/(10\text{ m/s}^2)}$

For the correct value of t — 1 point

$t = 0.3$ s

Using the equation for the horizontal distance:

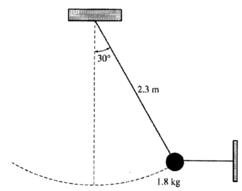
$d = v_x t$

For correctly substituting the value of t above and the value of v_x from part (d) — 1 point

$d = (0.5\text{ m/s})(0.3\text{ s})$

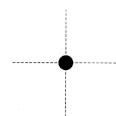
For the correct answer — 1 point

$d = 0.2$ m

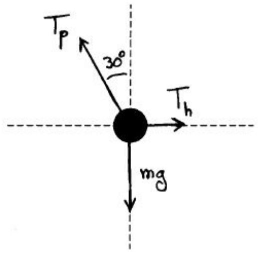


2005B2 (10 points)A simple pendulum consists of a bob of mass 1.8 kg attached to a string of length 2.3 m. The pendulum is held at an angle of 30° from the vertical by a light horizontal string attached to a wall, as shown above.

a. On the figure below, draw a free-body diagram showing and labeling the forces on the bob in the position shown above.



(a) 2 points



For each correctly drawn and labeled tension, with arrowhead in right direction	1 point each

One point was deducted for each of the following until score reached zero:
- No force of gravity
- Each extraneous force
- Any missing labels

Drawing all forces along correct lines with labels but no arrowheads received only one point.
Components of the tension in the pendulum string could be included in addition to or instead of the net tension, as long as they were clearly labeled as such.

b. Calculate the tension in the horizontal string.

(b) 4 points

For any indication that the net force is zero	1 point
For an attempt to determine the components of the tension in the pendulum string	1 point
For correctly determining these components	1 point

$T_h = T_p \sin 30°$

$mg = T_p \cos 30°$

$\frac{T_h}{mg} = \frac{\sin 30°}{\cos 30°} = \tan 30°$

$T_h = mg \tan 30°$

$T_h = (1.8 \text{ kg})(9.8 \text{ m/s}^2)\tan 30°$

For the correct answer with units — 1 point

$T_h = 10$ N

c. The horizontal string is now cut close to the bob, and the pendulum swings down. Calculate the speed of the bob at its lowest position.

(c) 4 points

For any indication of conservation of energy	1 point
For any indication of the need to use a change in height	1 point

$mgh_0 + \frac{1}{2}mv_0^2 = mgh_f + \frac{1}{2}mv_f^2$

For setting $v_0 = 0$ — 1 point

$\frac{1}{2}mv_f^2 = mg\,\Delta h$

$v_f = \sqrt{2g\,\Delta h}$

$\Delta h = L - L\cos 30°$

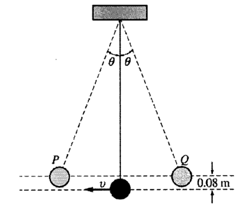
$v_f = \sqrt{2gL(1 - \cos 30°)}$

$v_f = \sqrt{2(9.8 \text{ m/s}^2)(2.3 \text{ m})(1 - \cos 30°)}$

For the correct answer, with units — 1 point

$v_f = 2.5$ m/s

2005Bb2—10 points) A simple pendulum consists of a bob of mass 0.085 kg attached to a string of length 1.5 m. The pendulum is raised to point *Q*, which is 0.08 m above its lowest position, and released so that it oscillates with small amplitude θ between the points *P* and *Q* as shown below.

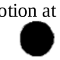


Note: Figure not drawn to scale.

a. On the figures below, draw free-body diagrams showing and labeling the forces acting on the bob in each of the situations described.

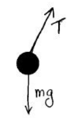
i. When it is at point P

ii. When it is in motion at its lowest position



(a)

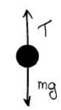
(i) 2 points



For each correctly drawn and labeled force	1 point each

One point was deducted if any extraneous forces were shown.

(ii) 2 points



b. Calculate the speed v of the bob at its lowest position.

(b) 2 points

For correctly applying conservation of energy — 1 point

$\frac{1}{2}mv^2 = mg\,\Delta h$

$v = \sqrt{2g\,\Delta h}$

$v = \sqrt{2(9.8 \text{ m/s}^2)(0.08 \text{ m})} = \sqrt{1.568}$ m/s

For the correct answer — 1 point

$v = 1.3$ m/s

c. Calculate the tension in the string when the bob is passing through its lowest position.

(c) 3 points

For using $\sum \mathbf{F} = m\mathbf{a}$	1 point
For a correct expression for the net force that is equated to a centripetal force	1 point

$T - mg = mv^2/r$

$T = m(g + v^2/r)$

$T = (0.085 \text{ kg})\left[9.8 \text{ m/s}^2 + (1.3 \text{ m/s})^2/(1.5 \text{ m})\right]$

For the correct answer with units — 1 point

$T = 0.93$ N (or equivalent answer depending on rounding or use of $g = 10 \text{ m/s}^2$)

d. Describe one modification that could be made to double the period of oscillation.

(d) 1 point

For a correct modification — 1 point

For example: Make the string four times longer.