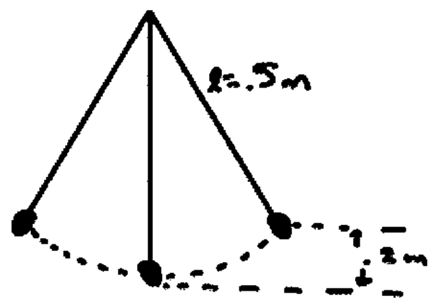
Simple Harmonic Motion Practice 1 Name: KEY

1. A pendulum has a length of 0.5 meters, and a mass of 0.5 kg. It is pulled to a height of 0.2 meters, and released, as shown.



   a. What is the potential energy of the mass at its highest point?
   b. What is the speed of the mass at its lowest point?
   c. What is the period and the frequency of the pendulum?
   d. How would the period change if a longer string were used? Explain.
   e. How would the period change if a greater mass was used? Explain.
   f. How would the period change, if the pendulum was on the moon, where gravity is smaller than on Earth? Explain.

a) $PE = mgh = (.5)(10)(.2) =$ ~~2.5 J~~ 1

b) at the bottom the mass has ~~2.5~~ 1 J of KE

$KE = \frac{1}{2}mv^2$

1 ~~2.5~~ $= \frac{1}{2}(.5)v^2$

~~v = 3.16 m/s~~ $\boxed{v = 2\ m/s}$

c) $T = 2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{.5}{10}} = 1.40\ s$

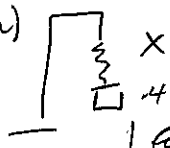
$f = \frac{1}{T} = \frac{1}{1.40} = .712\ 1/s$

d) T would increase → $T = 2\pi\sqrt{\frac{\ell}{g}}$ ← if ℓ is bigger, T is bigger.

e) T would not change → $T = 2\pi\sqrt{\frac{\ell}{g}}$ - no dependence on mass

f) T would increase → $T = 2\pi\sqrt{\frac{\ell}{g}}$ ← if g↓, T↑

2. A spring is hung from a ring stand, and a 0.4 kg mass is hung from it. The change in length of the spring is 0.6 meters. The mass is then lifted a distance of 0.2 meters, and dropped, causing the mass to oscillate.
    a. What is the spring constant of the spring?
    b. What will be the period and frequency of the mass-spring system?
    c. Write an equation for the height of the mass as a function of time.
    d. What will be the height of the mass 5 seconds after it is released?
    e. How would the period of the system change if a stronger spring (larger k) was used? Explain.
    f. How would the period change if the system were taken to the moon? Explain.

a) 

$F = kx$

$4 = k(.6)$

$k = 6.\bar{6}\ N/m$

b) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{.4}{6.\bar{6}}}$

$T = 1.54s$

$f = \frac{1}{T} = .650\ c/s$

c) $x = A\cos(2\pi f t)$

$x = .2\cos(2\pi(.650)t)$

$x = .2\cos(4.08t)$

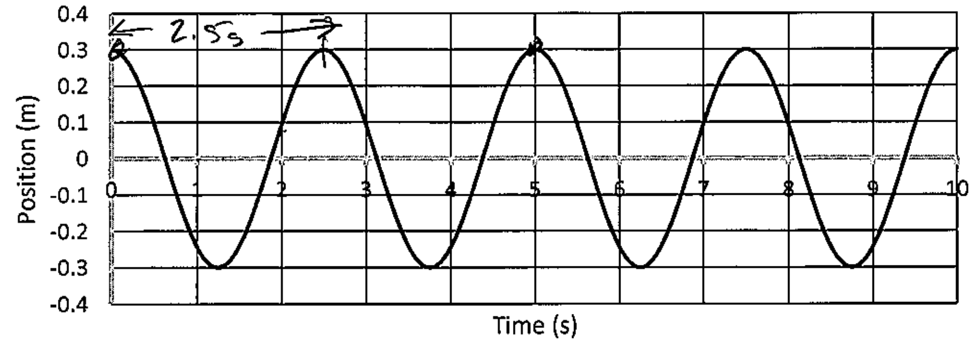
d) $x = .2\cos((4.08)(5))$

$x = .004m$

e) If $k\uparrow$, $T\downarrow$ → $T = 2\pi\sqrt{\frac{m}{k}}$ ← if $k\uparrow$, $T\downarrow$

f) No change. $T = 2\pi\sqrt{\frac{m}{k}}$ ← does not depend on "g"

3. A 0.5 kg mass is allowed to oscillate on a spring. A graph of its position as a function of time is given below:



a. What are the amplitude, period and frequency for the mass?
b. What is the spring constant?
c. How much energy is stored in the spring at the beginning of the problem?
d. What will be the maximum velocity of the mass?
e. Write an equation for the position of the mass as a function of time.
f. What will be the position of the mass after it has been oscillating for 12 seconds?

a) Amplitude = 0.3 m , T = 2.5 s , $f = \frac{1}{T} = \frac{1}{2.5} = .4$ c/s

b) $T = 2\pi\sqrt{\frac{m}{K}}$

$2.5 = 2\pi\sqrt{\frac{.5}{K}}$

$.398 = \sqrt{\frac{.5}{K}}$

$.158 = \frac{.5}{K}$

K = 3.16 N/m

c) $PE_s = \frac{1}{2}Kx^2$

$= \frac{1}{2}(3.16)(.3)^2$

PE_s = .142 J

d) $KE = \frac{1}{2}mv^2$

$.142 = \frac{1}{2}(.5)v^2$

V = .75 m/s

e) $x = A\cos(2\pi f t)$

$x = .3\cos(2\pi(.4)t)$

$x = .3\cos(2.513t)$

f) $x = .3\cos((2.513)(12))$

x = .092 m

Review Problems:

1. A 2 kg mass starts from rest, and is pulled by a force of 15 Newtons across a rough surface. It covers a distance of 1 meter in 0.67 seconds.
   a. What is the acceleration of the mass?
   b. Draw a free-body diagram showing all of the forces on the mass.
   c. Find the coefficient of friction between the mass and the surface.

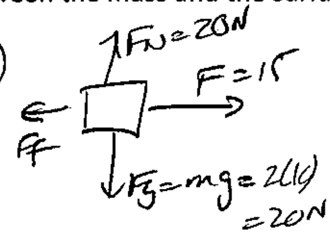
a) $\Delta x = 1m$, $t = .67s$, $v_i = 0$, $a = ?$

$\Delta x = v_0 t + \frac{1}{2} a t^2$

$1 = \frac{1}{2} a (.67)^2$

$a = 4.45 m/s^2$

b)



c) $F_{net} = ma$

$15 - F_f = 2(4.45)$

$F_f = 6.09N$

$F_f = \mu F_N$

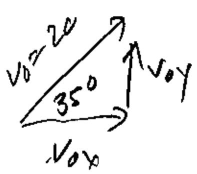
$6.09 = \mu (20)$

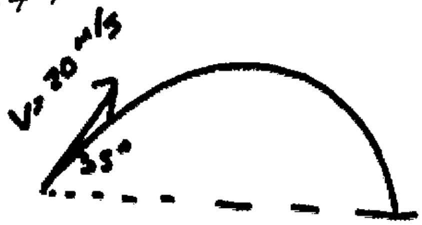
$\mu = .304$

2. A ball is thrown with a velocity of 20 m/s at an angle of 35⁰, as shown.

a) → $v_{0x} = 20\cos 35 = 16.38$

$v_{0y} = 20\sin 35 = 11.47$





   a. What are the vertical and horizontal components of the ball's velocity?
   b. How long will the ball be in the air?
   c. How far will it travel?

b)

x	y
$v_0 = 16.38$	$v_0 = 11.47$
$a = 0$	$v = 0$ (at top)
	$a = -10 m/s^2$
	$t = ?$

$v = v_0 + at$

$0 = 11.47 + (-10)t$

$t = 1.147$ (to top)

$t = 2(1.147) = 2.294s$

c) x

$\Delta x = v_0 t + \frac{1}{2} a t^2$

$= (16.38)(2.294)$

$\Delta x = 37.6m$