16

SUPERPOSITION AND STANDING WAVES

Q16.1. Reason: Where there is a change in medium—in particular a change in the wave speed—then reflection can occur.
Assess: Light travels at different speeds in water and air, and so some is reflected at a water-air interface.

Q16.2. Reason: When a wave encounters a discontinuity in the medium (sudden change in medium properties—in this case depth) there is partial reflection and partial transmission. The incident and reflected waves will have the same speed because the medium is the same. The reflected and transmitted wave will have a different speed, and this speed depends on the critical property (here the depth of the medium). If we consider the limiting case where the depth of the water goes to zero, the speed of the waves goes to zero. From this we conclude that as the depth of the water decreases, the speed of the water wave decreases.
Assess: The amplitude of the reflected and transmitted wave will be smaller than for the incident wave because the energy of the incident wave is divided between them. The phase of the reflected wave might change, depending on the relative magnitude of the critical factor on either side of the discontinuity.

Q16.3. Reason: See Figure 16.10 in the text.
(a) In Chapter 15 we saw that the speed of a wave on a stretched string is $v = \sqrt{T_s/\mu}$. Since the left side of the string has a lower speed, the linear density must be greater there.
(b) You would start a pulse from the left side, in the part with the greater linear density, in order to have the reflection not inverted.
Assess: This would be a fairly easy experiment to set up at home with two different strings tied together.

Q16.4. Reason: Knowing that the frequency of a vibrating string is determined by $f_n = (n/(2L))\sqrt{T/\mu}$, we can conclude that the frequency is directly proportional to the square root of the tension in the string. As a result, if the tension in the string increases, the rate at which it vibrates (the frequency) will also increase.
Assess: The next time you are around a guitar or other string instrument give this a try.

Q16.5. Reason: The frequency of vibration of your vocal cords is related to their linear mass density by $f_m = (m/(2L))\sqrt{T/\mu}$. Due to the inflammation, the vocal cords are more massive and hence have a greater linear mass density. Since the inflamed vocal cords have a greater linear mass density, the frequency of vibration of the cords and hence the frequency of the sound generated will decrease.
Assess: You are no doubt aware that the frequency of your voice lowers with certain illnesses.

Q16.6. Reason: When the frequency is doubled the wavelength is halved. This halving of the wavelength will increase the number of antinodes to six.
Assess: Calculating the number of antinodes in this situation is easier than calculating the number of nodes because there are nodes on each end of the string, so the number is not doubled: It goes from four to seven.

Q16.7. Reason: (a) When standing waves are set up in a tube that is open at both ends, the length of the tube is an integral number of half wavelengths $L = m\lambda/2$. Looking at the figure we see four half wavelengths, hence $m = 4$, and the air column is vibrating in the fourth harmonic. **(b)** Since this is sound, we have a longitudinal wave and the air molecules are vibrating horizontally, parallel to the tube.
Assess: The wave diagram superimposed on top of the open-ended tube sketch is a representation of the pressure at different points in the gas. It is not an air molecule displacement sketch.

Q16.8. Reason: According to Conceptual Example 16.7, a flute may be modeled as an open-open tube. Since the piccolo is considered to be a small flute, it may also be modeled by an open-open tube. When standing wave resonance is set up in an open-open tube, the length of the tube is some integral number of half wavelengths.

$$L = m\lambda/2 \quad \text{or} \quad \lambda = 2(L/m)$$

Inserting this expression for the wavelength into the relationship for speed, frequency and wavelength obtain

$$v = f\lambda = f(2L/m) \quad \text{or} \quad f = mv/(2L)$$

This last expression for the frequency in terms of the length of the tube allows us to conclude that the frequency in an open-open tube is inversely proportional to the length of the tube. Hence a shorter instrument (smaller L), such as the piccolo, will produce higher-frequency sounds. In fact, for the same fingering, a piccolo sounds one octave higher than a flute.
Assess: Having the ability to inspect a function such as the one previously discussed for the frequency in order to determine what will occur is a skill that physics will help you develop.

Q16.9. Reason: The advantage of having low-frequency organ pipes closed at one end is that they will sound an octave lower without being twice as long as an open-open pipe. Because a pipe closed on one end contains only 1/4 of a wavelength in the lowest mode, the wavelength is twice as long as for an open-open pipe of the same length. If the wavelength is twice as long then the frequency is half, and that corresponds to a musical interval of an octave.
Assess: Pipes closed on one end also have a different sound (timbre) than open-open pipes sounding the same note.

Q16.10. Reason: While the fundamental frequencies of normal voices are below 3000 Hz, there are higher harmonics mixed in that give the voice its characteristic sound and convey information. These harmonics are lost when the telephone cuts off frequencies above 3000 Hz, and it is a bit harder to understand what is said.
Assess: The range of human hearing is usually quoted as 20 Hz–20 kHz; a large portion of that range is above 3000 Hz. This principle holds for music as well as spoken word. CDs are designed to reproduce frequencies clearly up to 20 kHz (and even a tiny bit higher), even though the musical notes themselves (the fundamental frequencies) are not nearly that high. Those higher harmonics give the sound definition and sharpness; the music sounds muddled if the high frequencies are cut out.

Q16.11. Reason: Treating the instrument as an open-closed tube, the frequencies are $f_m = m(v/(4L))$. Inspecting this relationship, we expect the pitch of the instrument to be slightly higher since the speed of sound is greater in helium.
Assess: Having the ability to inspect a function such as the one previously discussed for the frequency in order to determine what will occur is a skill that physics will help you develop.

Q16.12. Reason: The sound you hear is the vibration of the glass; it is set into motion by the disturbance of the flowing, sloshing liquid. The disturbances are of many frequencies, but the natural frequency of the glass resonates and is amplified by the glass while other frequencies are quickly damped out.
The reason the pitch rises as the glass fills is that the natural frequency of the glass changes; as the glass fills the resonance frequency rises.
Assess: This is also the basis of the kitchen orchestra; get a set of glasses and fill them with varying amounts of water, then tap them lightly on the side with a spoon. You can tune them by adding or removing small amounts of water. Make a whole octave scale and play your favorite tunes.

Q16.13. Reason: The introduction of helium into the mouth allows harmonics of higher frequencies to be excited more than in the normal voice. The fundamental frequency of the voice is the same but the quality has changed. Our perception of the quality is a function of which harmonics are present.
Assess: The fundamental frequency of a complex tone from the voice is determined by the vibration of the vocal cords and depends on the tension and linear mass density of the cords, not the gas in the mouth.

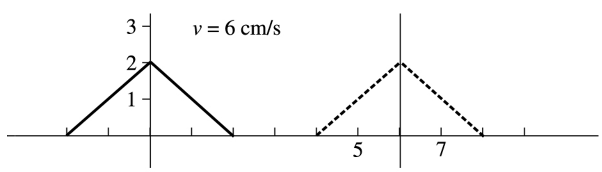
Q16.14. Reason: It is the harmonics and formants that help us understand what is being said in the first place (that is, the harmonics of the vocal cords, modified by the formants of the vocal tract, make an "ee" sound different from an "oo" sound at the same pitch). The harmonics are multiples of the fundamental, and when the fundamental is high (1000 Hz or more), the harmonics are out of the range of hearing for many people. The formants emphasize different harmonics and allow us to distinguish different vocalizations, but if the harmonics cannot be heard then it would be difficult to tell the difference between an "ee" sound and an "oo" sound.
Assess: The upper limit on the frequencies people can hear varies from person to person. Men and people who have endured lengthy periods of very loud sounds cannot hear frequencies as high as women and people who've protected their hearing, on average.

Q16.15. Reason: The settings on the synthesizer change the amount and proportion of higher harmonics that are mixed in (added to) the fundamental frequency.
Assess: Adding a lot of 2^{nd} and 4^{th} harmonics makes it sound more like a flute.

Q16.16. Reason: The quality of the sound of your voice depends on the resonant frequencies of the cavities (throat, mouth, and nose) in your vocal tract. When one of these cavities is stuffed up, the quality of your voice will be affected.
Assess: In this case the fundamental frequency has not changed but the ability to create and enhance certain harmonics has. The harmonics play an important role in the quality of the sound of your voice.

Q16.17. Reason: The pitch may be the same, but the vocal tract is considerably different. The vocal tract determines the formants that emphasize different harmonics and allow us to distinguish different vocalizations.
Assess: Due to the formants, it is easy to distinguish the voice of a child from the voice of an adult.

Q16.18. Reason: The sketch below shows the displacement at $t = 0$ s and again at $t = 1$ s. The displacement at $x = 7$ cm is read from the sketch to be 1.0 mm.

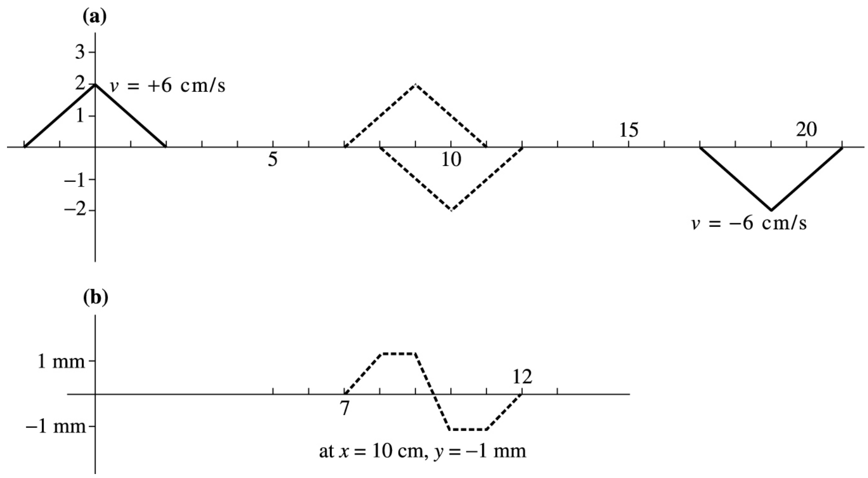


At $t = 6$ s, $x = 7$ cm, $y = 1$ mm

The correct choice is D.
Assess: The ability to inspect a figure and determine what will occur is a skill that physics will help you develop.

Q16.19. Reason: The earliest (and only) time that y will equal 2 mm at the point $x = 3$ cm is when the top of the triangular peak on the left moves right to the point $x = 3$ cm. That wave pulse is moving at 6 cm/s, so it will take 0.5 s to move 3 cm.
The correct choice is A.
Assess: Because the pulse moving from right to left is a negative pulse, it will make $y = -2$ mm at $t = 3$ s, but not a positive $+2$ mm.

Q16.20. Reason: Sketch **(a)** below shows each pulse at $t = 0\text{ s}$ and again at $t = 1.5\text{ s}$. Sketch **(b)** shows the resultant interference as the two pulses pass through each other. Note that at $x = 10\text{ cm}$ the resultant amplitude is -1.0 mm.



The correct choice is B.
Assess: The ability to inspect a figure and determine what will occur is a skill that physics will help you develop.

Q16.21. Reason: The maximum displacement will be the sum of the contributions from the two traveling waves at each point and at each time; however, you are not guaranteed to be watching a point where a crest will meet a crest. It is true that you might be watching an antinode where the maximum displacement would be $2A$, but it is also possible that you might be watching a node that doesn't move at all—or any place in between.
The correct choice is D.
Assess: If you watch the whole string, you will see points whose maximum displacement is $2A$ and other points whose maximum displacement is 0 and other points in between.

Q16.22. Reason: Since the lowest frequency that creates resonance is 20 Hz, we know that the fundamental frequency is $f_1 = 20\text{ Hz}$. Note that the next harmonic is three times as much. This information tells us that the student is using an open-closed tube as reasoned next.
Successive frequencies for an open–open tube are determined by

$$f_m = mf_1, \quad \text{where} \quad m = 1, 2, 3, \ldots$$

Successive frequencies for an open-closed tube are determined by

$$f_m = mf_1, \quad \text{where} \quad m = 1, 3, 5, \ldots$$

Since the next frequency is three times the fundamental frequency, the student is using an open-closed tube. As a result the next frequency will be $f_5 = 5f_1 = 100\text{ Hz}$.
The correct choice is B.
Assess: Open-closed tubes have only the odd numbered harmonics.

Q16.23. Reason: We know that the speed, frequency, and wavelength of a traveling wave are related by $f = v/\lambda$. As the air and pipe warm from 20°C to 25°C, there is an insignificant expansion of the pipe, hence the resonant wavelengths remain the same. However, the change in temperature of the air is significant compared to the temperature of the air. Since the speed of sound increases with temperature, the frequency will increase. The correct response is A.
Assess: Your ability to look at an expression that describes a situation and predict what will happen under certain circumstances will be enhanced by your study of physics.

Q16.24. Reason: The speed of sound is 350 m/s. Model the dog's ear canal as an open-closed tube for which the resonant frequencies are

$$f_m = m\left(\frac{v}{4L}\right) \qquad m = 1, 3, 5, \ldots$$

For $\mathrm{m} = 1, 2,$ the frequencies are 1700 Hz and 5100 Hz. The correct response is B.

Assess: Of course dogs vary in the length of their ear canal.

Q16.25. Reason: For the lowest standing wave mode on a string, the wavelength is twice the length of the string. If this is the case the speed of the disturbance is determined by

$$v = f_1\lambda_1 = (20\ \text{Hz})(2.0\ \text{m}) = 40\ \text{m/s}$$

The correct choice is D.

Assess: For the lowest standing wave mode on a string, one-half a wavelength fits into the string.

Q16.26. Reason: $\lambda_{\text{new}} = (4/5)\lambda_{\text{old}}$. But the properties of the string (T_s and μ) haven't changed, so v hasn't changed. That is, $v_{\text{new}} = v_{\text{old}}$.

Let's solve the fundamental relationship for the frequency $f = v/\lambda$.

$$\frac{f_{\text{new}}}{f_{\text{old}}} = \frac{v_{\text{new}}/\lambda_{\text{new}}}{v_{\text{old}}/\lambda_{\text{old}}} = \frac{v_{\text{old}}/\left(\frac{4}{5}\lambda_{\text{old}}\right)}{v_{\text{old}}/\lambda_{\text{old}}} = \frac{5}{4}$$

$$f_{\text{new}} = \frac{5}{4}v_{\text{old}} = \frac{5}{4}(440\ \text{Hz}) = 550\ \text{Hz}$$

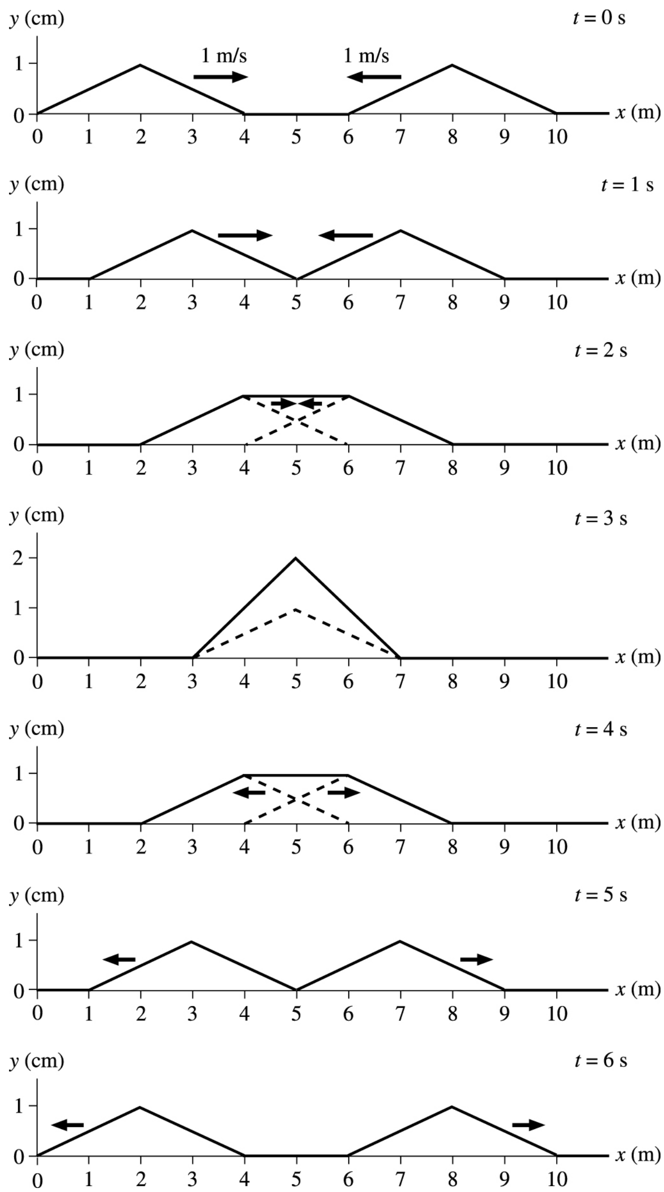
The correct choice is D.

Assess: We (especially the musicians among us) already knew (or suspected) that a shorter string would produce a higher frequency, so we tentatively eliminated choices A and B. The answer we came up with jibes well with our intuition.

Problems

P16.1. Prepare: The principle of superposition comes into play whenever the waves overlap. The waves are approaching each other at a speed of 1 m/s, that is, each part of each wave is moving 1 m every second.

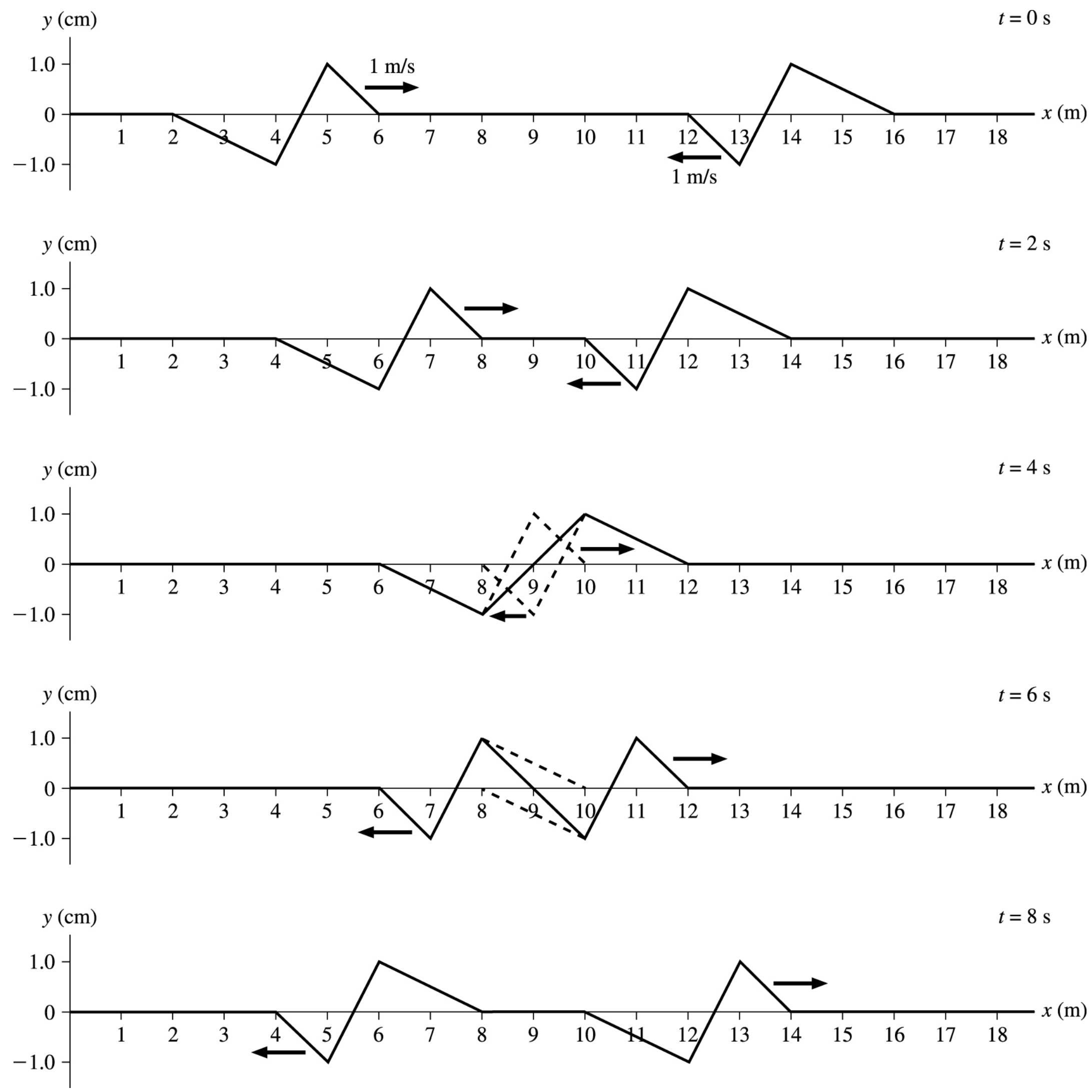
Solve: The graph at $t = 1\ \text{s}$ differs from the graph at $t = 0\ \text{s}$ in that the left wave has moved to the right by 1 m and the right wave has moved to the left by 1 m. This is because the distance covered by the wave pulse in 1 s is 1 m. The snapshot graphs at $t = 2\ \text{s}, 3\ \text{s},$ and 4 s are a superposition of the left- and the right-moving waves. The overlapping parts of the two waves are shown by the dotted lines.



Assess: This is an excellent problem because it allows you to see the progress of each wave and the superposition (addition) of the waves. As time progresses, you know exactly what has happened to each wave and to the superposition of these waves.

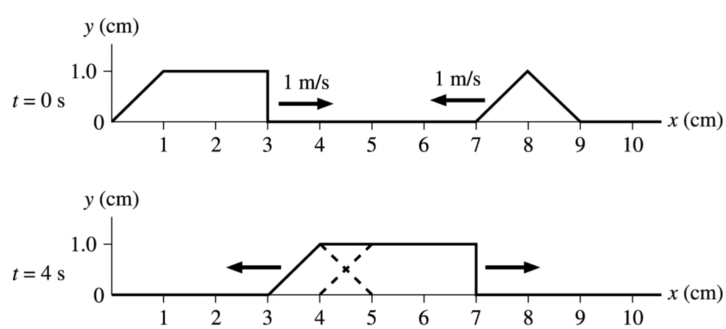
P16.2. **Prepare:** The principle of superposition comes into play whenever the waves overlap. The waves are approaching each other at a speed of 1 m/s, that is, each part of each wave is moving 1 m every second.

Solve: The graph at $t = 2\ \text{s}$ differs from the graph at $t = 0\ \text{s}$ in that the left wave has moved to the right by 2 m and the right wave has moved to the left by 2 m. This is because the distance covered by the wave pulse in 2 s is 2 m. The snapshot graphs at $t = 4\ \text{s}$ and $t = 6\ \text{s}$ are a superposition of the left- and the right-moving waves. The overlapping parts of the two are shown by the dotted lines.



Assess: This is an excellent problem because it allows you to see the progress of each wave and the superposition (addition) of the waves. As time progresses, you know exactly what has happened to each wave and to the superposition of these waves.

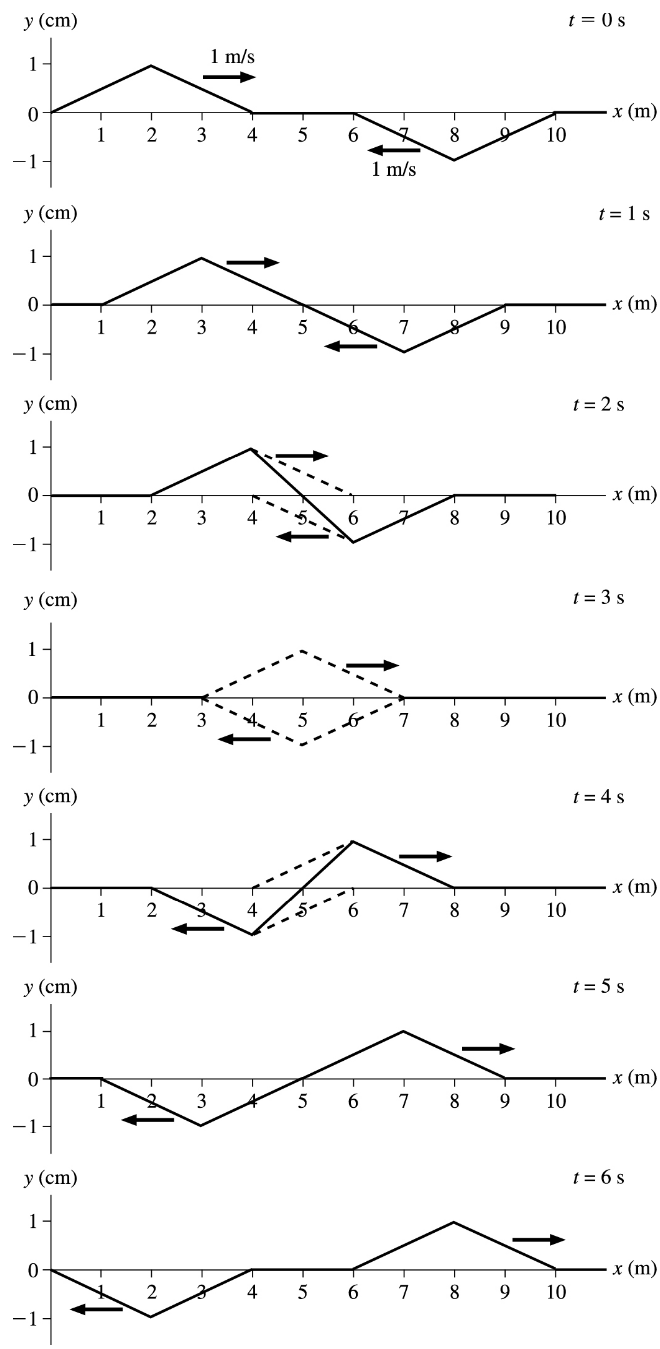
P16.3. Prepare: The principle of superposition comes into play whenever the waves overlap.
Solve: As graphically illustrated in the following figure, the snapshot graph of Figure P16.3 was taken at $t = 4$ s.



Assess: This is an excellent problem because it allows you to see the progress of each wave and the superposition (addition) of the waves. As time progresses, you know exactly what has happened to each wave and to the superposition of these waves.

P16.4. Prepare: The principle of superposition comes into play whenever the waves overlap. The waves are approaching each other at a speed of 1 m/s, that is, each part of each wave is moving 1 m every second.

Solve: The graph at $t = 1\text{ s}$ differs from the graph at $t = 0\text{ s}$ in that the left wave has moved to the right by 1 m and the right wave has moved to the left by 1 m. This is because the distance covered by the wave pulse in 1 s is 1 m. The snapshot graphs at $t = 2\text{ s}$, 3 s, 4 s, 5 s, and 6 s are a superposition of the left- and the right-moving waves. The overlapping parts of the two waves are shown by the dotted lines.



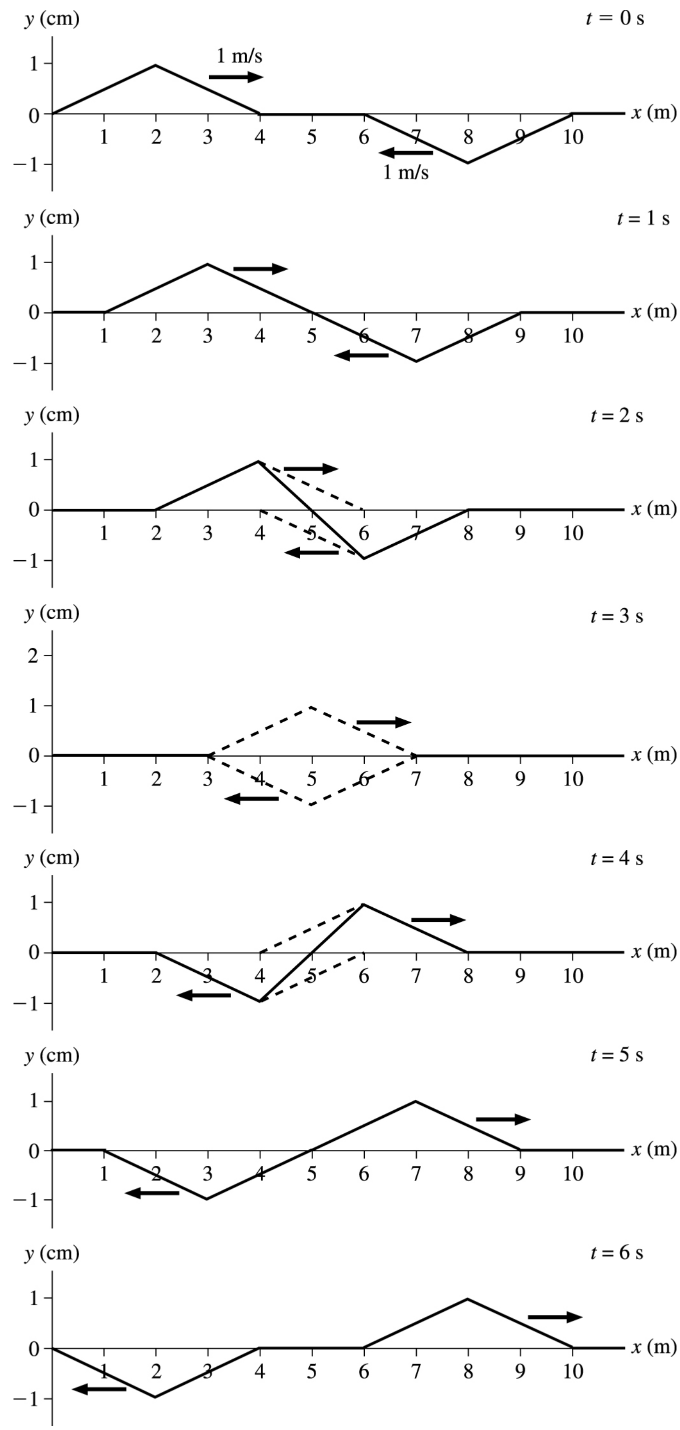
Solve: From the figure, the values of the displacement at $x = 5.0\text{ cm}$ are

t (s)	y (cm)
0	0
1	0
2	0
3	0
4	0
5	0
6	0

Assess: The point at $x = 5.0\text{ m}$ is a special point.

P16.5. Prepare: The principle of superposition comes into play whenever the waves overlap. The waves are approaching each other at a speed of 1 m/s, that is, each part of each wave is moving 1 m every second.

Solve: The graph at $t = 1\text{ s}$ differs from the graph at $t = 0\text{ s}$ in that the left wave has moved to the right by 1 m and the right wave has moved to the left by 1 m. This is because the distance covered by the wave pulse in 1 s is 1 m. The snapshot graphs at $t = 2\text{ s}$, 3 s, 4 s, 5 s, and 6 s are a superposition of the left- and the right-moving waves. The overlapping parts of the two waves are shown by the dotted lines.

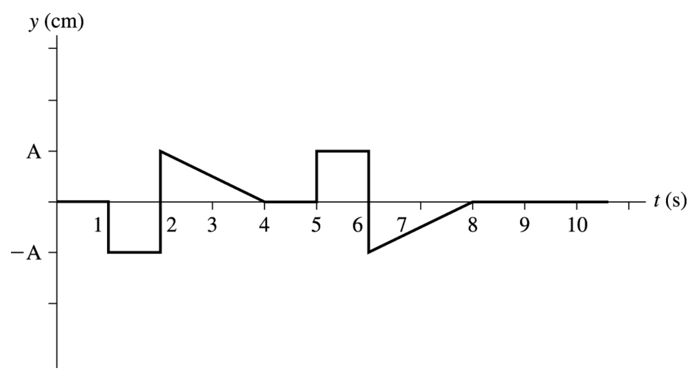

y (cm)
t = 0 s
1 m/s
1 m/s
x (m)
t = 1 s
t = 2 s
t = 3 s
t = 4 s
t = 5 s
t = 6 s

Solve: From the figure, the values of the displacement at $x = 5.0$ cm are

t (s)	*y* (cm)
0	0
1	0.5
2	1
3	0
4	–1
5	–0.5
6	0

Assess: The point at $x = 4.0$ m moves first up, then down, then up again to end at zero displacement.

P16.6. Prepare: Look at the original graph and concentrate on the point $x = 3$ m while you mentally let the pulse travel to the right at 1 m/s. After 1 s the leading edge of the pulse will reach the point of interest.
Remember that since the string is fixed at 5 m the pulse will invert on reflection.
Solve:



Assess: You can also track the trailing edge of the pulse (at the end of the triangular section). It will take 4 s to get to $x = 3$ m and 4 more seconds to get back to the same point $x = 3$ m after reflection.

P16.7. Prepare: The pulse will have to travel to the end of the string (4.0 m), be reflected back to the other end of the string (10.0 m), and then be reflected again and travel to the same position (6.0 m) in order to have the same appearance as at $t = 0$ s. This means the total distance traveled is 20.0 m and that the pulse is traveling at a speed of 4.0 m/s.
Solve: The time required is $\Delta t = \Delta x/v_x = 20.0 \text{ m}/(4.0 \text{ m/s}) = 5.0 \text{ s}$.
Assess: This is basically a straightforward kinematics problem.

P16.8. Prepare: Knowing the frequency and wavelength of the traveling wave disturbance, we can determine how fast it is traveling along the string $(v = f\lambda)$. Standing waves will be established along the entire length of the string once the disturbance has traveled to the end of the string and back. The time for the traveling wave disturbance to travel to the end of the string and back may be determined by $t = 2L/v$.
Solve: The time for the traveling wave to travel to the end of the string and back is

$$t = \frac{2L}{v} = \frac{2L}{f\lambda} = \frac{2(3.0 \text{ m})}{(3.5 \text{ Hz})(1.0 \text{ m})} = 1.7 \text{ s}$$

Assess: This is a reasonable time for the disturbance to travel 6.0 m.

P16.9. Prepare: Reflections at both ends of the string cause the formation of a standing wave. Figure P16.9 indicates that there are three full wavelengths on the 2.0-m-long string and that the wave speed is 40 m/s. We will use Equation 15.10 to find the frequency of the standing wave.

Solve: The wavelength of the standing wave is $\lambda = \frac{1}{3}(2.0 \text{ m}) = 0.667 \text{ m}$. The frequency of the standing wave is thus

$$f = \frac{v}{\lambda} = \frac{40 \text{ m/s}}{0.667 \text{ m}} = 60 \text{ Hz}$$

Assess: The units are correct and this is a reasonable frequency for a vibrating string.

P16.10. Prepare: Reflections at the string boundaries cause a standing wave on the string. The oscillating frequency of the wave on the string is 100 Hz.

Solve: Figure P16.10 indicates one and a half full wavelengths on the string. Hence $\lambda = \frac{2}{3}(60 \text{ cm}) = 40 \text{ cm}$. Thus $v = \lambda f = (0.40 \text{ m})(100 \text{ Hz}) = 40 \text{ m/s}$.

Assess: The units are correct and this is a reasonable speed for the wave to be traveling along the string.

P16.11. Prepare: We assume that the string is tied down at both ends so there are nodes there. This means the length of the string is $L = \frac{1}{2}\lambda$ in the fundamental mode (there are no nodes between the ends). $\lambda = 2L = 2(0.89 \text{ m}) = 1.78 \text{ m}$.

Solve: Use the fundamental relationship for periodic waves: $v = \lambda f = (1.78 \text{ m})(30 \text{ Hz}) = 53 \text{ m/s}$.

Assess: Remember, we are talking about the speed of the wave on the string, not the speed of sound in air. These numbers are reasonable for bass guitar strings.

P16.12. Prepare: For a string fixed at both ends, the possible vibration frequencies are related to the tension by

$$f = (m/(2L))\sqrt{T/\mu}$$

Solve: If we have a new tension $T_{\text{new}} = T/2$, we will have a new frequency:

$$f_{\text{new}} = (m/(2L))\sqrt{T_{\text{new}}/\mu} = (m/(2L))\sqrt{(T/2)/\mu} = (1/\sqrt{2})f = (1/\sqrt{2})(384 \text{ Hz}) = 272 \text{ Hz}$$

Assess: We could solve this problem by just inspecting the above expression for the frequency as a function of the tension. We see the frequency is proportional to the square root of the tension. So if the tension is reduced by a factor of 2, the frequency will be reduced by a factor of $\sqrt{2}$. This will give a new frequency of $f_{\text{new}} = f/\sqrt{2} = 384 \text{ Hz}/\sqrt{2} = 272 \text{ Hz}$.

P16.13. Prepare: A string fixed at both ends supports standing waves. A standing wave can exist on the string only if its wavelength is given by Equation 16.1, that is, $\lambda_m = \frac{2L}{m}$, $m = 1, 2, 3\ldots$ The length L of the string is 240 cm.

Solve: (a) The three longest wavelengths for standing waves will therefore correspond to $m = 1, 2, \text{and } 3$. Thus,

$$\lambda_1 = \frac{2(2.40 \text{ m})}{1} = 4.80 \text{ m} \qquad \lambda_2 = \frac{2(2.40 \text{ m})}{2} = 2.40 \text{ m} \qquad \lambda_3 = \frac{2(2.40 \text{ m})}{3} = 1.60 \text{ m}$$

(b) Because the wave speed on the string is unchanged from one m value to the other,

$$f_2\lambda_2 = f_3\lambda_3 \Rightarrow f_3 = \frac{f_2\lambda_2}{\lambda_3} = \frac{(50.0 \text{ Hz})(2.40 \text{ m})}{1.60 \text{ m}} = 75.0 \text{ Hz}$$

Assess: The units on each determination are correct and the values are reasonable. The maximum wavelength of a standing wave in a string is twice the length of the string and all other possible wavelengths are fractions of this value.

P16.14. Prepare: A string fixed at both ends forms standing waves. The wavelengths of standing wave modes of a string of length L are given by Equation 16.1, so we can easily determine the third harmonic wavelength. With a known frequency of 180 Hz we can find the wave speed using Equation 15.10. Equation 15.2 will allow us to find the tension in the wire. Mass density μ of the wire is equal to the ratio of its mass and length.

Solve: **(a)** The wavelength of the third harmonic is calculated as follows:

$$\lambda_m = \frac{2L}{m} \Rightarrow \lambda_3 = \frac{2L}{3} = \frac{2.42\text{ m}}{3} = 0.807\text{ m}$$

(b) The speed of the waves on the string is $v = \lambda_3 f_3 = (0.807\text{ m})(180\text{ Hz}) = 145.3\text{ m/s}$. The speed is also given by $v = \sqrt{T_S/\mu}$, so the tension is

$$T_S = \mu v^2 = \frac{m}{L}v^2 = \frac{4.00\times10^{-4}\text{ kg}}{1.21\text{ m}}(145.3\text{ m/s})^2 = 70.0\text{ N}$$

Assess: You must remember to use the linear density in SI units of kg/m. Also, the speed is the same for all modes, but you must use a matching λ and *f* to calculate the speed.

P16.15. Prepare: A string fixed at both ends forms standing waves. Three antinodes means the string are vibrating as the $m = 3$ standing wave. The wavelengths of standing wave modes of a string of length *L* are given by Equation 16.1.

Solve: **(a)** The frequency is $f_3 = 3f_1$, so the fundamental frequency is $f_1 = \frac{1}{3}(420\text{ Hz}) = 140\text{ Hz}$. The fifth harmonic will have the frequency $f_5 = 5f_1 = 700\text{ Hz}$.

(b) The wavelength of the fundamental mode is $\lambda_1 = 2\text{L} = 1.20\text{ m}$. The wave speed on the string is $v = \lambda_1 f_1 = (1.20\text{ m})(140\text{ Hz}) = 168\text{ m/s}$. Alternatively, the wavelength of the $n = 3$ mode is $\lambda_3 = \frac{1}{3}(2L) = 0.40\text{ m}$, from which $v = \lambda_3 f_3 = (0.40\text{ m})(420\text{ Hz}) = 168\text{ m/s}$. The wave speed on the string, given by Equation 15.2, is

$$v = \sqrt{\frac{T_S}{\mu}} \Rightarrow T_S = \mu v^2 = (0.0020\text{ kg/m})(168\text{ m/s})^2 = 56\text{ N}$$

Assess: You must remember to use the linear density in SI units of kg/m. Also, the speed is the same for all modes, but you must use a matching λ and *f* to calculate the speed.

P16.16. Prepare: A string fixed at both ends forms standing waves.

Solve: A simple string sounds the fundamental frequency $f_1 = v(2L)$. Initially, when the string is of length $L_A = 32.8\text{ cm}$, the note has the frequency $f_{1A} = v/(2L)_A$. For a different length, $f_{1C} = v/(2L)_C$. Taking the ratio of each side of these two equations gives

$$\frac{f_{1A}}{f_{1C}} = \frac{v/(2L)_A}{v/(2L)_C} = \frac{L_C}{L_A} \Rightarrow L_C = \frac{f_{1A}}{f_{1C}}L_A$$

We know that the second frequency is desired to be $f_{1B} = 523\text{ Hz}$. The string length must be

$$L_C = \frac{440\text{ Hz}}{523\text{ Hz}}(32.8\text{ cm}) = 27.6\text{ cm}$$

The question is not how long the string must be, but where must the violinist place his finger. The full string is 32.8 cm long, so the violinist must place his finger 5.2 cm from the end.

Assess: A fingering distance of 5.2 cm from the end is reasonable.

P16.17. Prepare: Reflections at the string boundaries cause a standing wave on a stretched string. The wavelengths of standing wave modes of a string of length *L* are given by Equation 16.1, so we can easily determine the wavelength from the vibrating length of the string, which is 1.90 m. With a known frequency of 27.5 Hz we can find the wave speed using Equation 15.10. Equation 15.2 will now allow us to find the tension in the wire. Mass density μ of the wire is equal to the ratio of its mass and length.

Solve: Because the vibrating section of the string is 1.9 m long, the two ends of this vibrating wire are fixed, and the string is vibrating in the fundamental harmonic. The wavelength is

$$\lambda_m = \frac{2L}{m} \Rightarrow \lambda_1 = 2L = 2(1.90\text{ m}) = 3.80\text{ m}$$

The wave speed along the string is $v = f_1\lambda_1 = (27.5\text{ Hz})(3.80\text{ m}) = 104.5\text{ m/s}$. The tension in the wire can be found as follows:

$$v = \sqrt{\frac{T_S}{\mu}} \Rightarrow T_S = \mu v^2 = \left(\frac{\text{mass}}{\text{length}}\right)v^2 = \left(\frac{0.400\text{ kg}}{2.00\text{ m}}\right)(104.5\text{ m/s})^2 = 2180\text{ N}$$

Assess: You must remember to use the linear density in SI units of kg/m. Also, the speed is the same for all modes, but you must use a matching λ and f to calculate the speed.

P16.18. Prepare: For a string fixed at both ends, successive resonant frequencies occur at

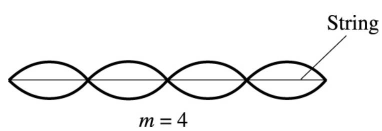
$$f_m = mf_1 \quad \text{and} \quad f_{(m+1)} = (m+1)f_1$$

Solve: **(a)** Inserting given information into the above expressions, obtain

$$24\text{ Hz} = mf_1 \quad \text{and} \quad 32\text{ Hz} = (m+1)f_1$$

Dividing the second expression by the first and for m, obtain $m = 3$.
Knowing that $m = 3$, we can use the first expression to obtain the fundamental frequency: $f_1 = 24\text{ Hz}/m = 24\text{ Hz}/3 =$ 8.0 Hz. Or using the second expression, we obtain the same value: $f_1 = 33\text{ Hz}/(m+1) = 32\text{ Hz}/(3+1) = 8.0\text{ Hz}$.

(b) When the string is vibrating at 32 Hz, it is vibrating in the $m = 4$ mode, or the fourth harmonic, and four half wavelengths or two wavelengths will fit into the string as shown in the following figure.



Assess: It is good practice to look for simple checks on your work. Calculating the fundamental frequency using both expressions is a simple check.

P16.19. Prepare: Assuming we want the next antinode closest to the one at the wall, it is ½ wavelength away.
Solve: The antinodes are a half-wavelenth apart, or in this case $\lambda/2 = 26\text{ m}/2 = 13\text{ m}$. The period of her motion is related to the speed and wavelength.

$$T = \frac{1}{f} = \frac{\lambda}{v} = \frac{26\text{ m}}{4.4\text{ m/s}} = 5.9\text{ s}$$

Assess: These seem like reasonable answers.

P16.20. Prepare: Expressions for the fundamental frequency for an open-open tube and an open-closed tube are respectively given by

$$f_{oo} = v/(2L)_{oo} \quad \text{and} \quad f_{oc} = v/(4L)_{oc}$$

Solve: Solving these for the length of each tube, we have
(a) Length for open-open tube: $L_{oo} = v/(2f)_{oo} = (343\text{ m/s})/(2(20\text{ Hz})) = 8.6\text{ m}$
(b) Length for open-closed tube: $L_{oc} = v/(4f)_{oc} = (343\text{ m/s})/(4(20\text{ Hz})) = 4.3\text{ m}$
Assess: When the air in a tube is resonating in the fundamental mode, half of a wavelength fits into an open-open tube and one fourth of a wavelength fits into an open-closed tube. Since the wavelength is the same (v and f are the same, so λ is the same) for each case, the open-closed tube only needs to be half as long as the open-open tube.

P16.21. Prepare: We want to use the fundamental relationship for periodic waves. But first, convert the length to SI units (use Table 1.3).

$$L = 18\text{ ft}\left(\frac{0.30\text{ m}}{1\text{ ft}}\right) = 5.49\text{ m}$$

Solve: **(a)** For an open-closed pipe in the fundamental mode $L = \frac{1}{4}\lambda$ (one-quarter of a wavelength fits in the pipe),

$$\lambda = 4L = 4(5.49 \text{ m}) = 21.9 \text{ m}$$

$$f = \frac{v}{\lambda} = \frac{350 \text{ m/s}}{21.9 \text{ m}} = 16 \text{ Hz}$$

This is below the arbitrary lower limit of the range of human hearing.

(b) We notice that the true value (27.5 Hz) is different from the answer we got in part **(a)**. To find the "effective length" of the instrument with a fundamental frequency of 27.5 Hz using the open-closed tube model, we simply do the previous calculations in reverse order. First find λ for the fundamental mode.

$$\lambda = \frac{v}{f} = \frac{350 \text{ m/s}}{27.5 \text{ Hz}} = 12.7 \text{ m}$$

Now

$$L = \frac{1}{4}\lambda = \frac{1}{4}(12.7 \text{ m}) = 3.18 \text{ m} = 10.4 \text{ ft} \approx 10 \text{ ft}$$

Assess: The "effective length" is just over half of the real length. This would lead us to conclude that the open-closed tube model is not a very accurate model for this situation. A good contrabassoon gives foundation and body to the orchestra.

P16.22. **Prepare:** We have an open-open tube that forms standing sound waves.

Solve: The gas molecules at the ends of the tube exhibit maximum displacement, making antinodes at the ends. There is another antinode in the middle of the tube. Thus, this is the $m = 2$ mode and the wavelength of the standing wave is equal to the length of the tube, that is, $\lambda = 0.80$ m. Since the frequency $f = 500$ Hz, the speed of sound in this case is $v = f\lambda = (500 \text{ Hz})(0.80 \text{ m}) = 400 \text{ m/s}$.

Assess: The experiment yields a reasonable value for the speed of sound.

P16.23. **Prepare:** For the open-open tube, the two open ends exhibit antinodes of a standing wave. The possible wavelengths for this case are $\lambda_m = 2L/m$, where $m = 1, 2, 3\ldots$ On the other hand, in the case of an open-closed tube $\lambda_m = 4L/m$, where $m = 1, 3, 5\ldots$ The length of the tube is 121 cm.

Solve: **(a)** The three longest wavelengths are

$$\lambda_1 = \frac{2(1.21 \text{ m})}{1} = 2.42 \text{ m} \qquad \lambda_2 = \frac{2(1.21 \text{ m})}{2} = 1.21 \text{ m} \qquad \lambda_3 = \frac{2(1.21 \text{ m})}{3} = 0.807 \text{ m}$$

(b) The three longest wavelengths are

$$\lambda_1 = \frac{4(1.21 \text{ m})}{1} = 4.84 \text{ m} \qquad \lambda_2 = \frac{4(1.21 \text{ m})}{3} = 1.61 \text{ m} \qquad \lambda_3 = \frac{4(1.21 \text{ m})}{5} = 0.968 \text{ m}$$

Assess: It is clear that the end of the air column, whether open or closed, changes the possible modes.

P16.24. **Prepare:** For an open-open pipe, the length of the pipe is an integral number of half wavelengths. For an open-closed organ pipe, the length of the pipe is an integral number of quarter wavelengths $L = m\lambda/4$, where $m = 1, 3, 5,$ etc. The expression $v = f\lambda$ connects speed with the wavelength and frequency.

Solve: For an open-open pipe with a fundamental frequency of $f_1 = 27.5$ Hz, the length of the pipe is

$$L = \frac{\lambda_1}{2} = \frac{1}{2}\left(\frac{v_{\text{sound}}}{f_1}\right) = \frac{1}{2}\left(\frac{343 \text{ m/s}}{27.5 \text{ Hz}}\right) = 6.24 \text{ m}$$

For an open-closed pipe with a fundamental frequency of $f_1 = 27.5$ Hz, the length of the pipe is

$$L = \frac{\lambda_1}{4} = \frac{1}{4}\left(\frac{v_{\text{sound}}}{f_1}\right) = \frac{1}{4}\left(\frac{343 \text{ m/s}}{27.5 \text{ Hz}}\right) = 3.12 \text{ m}$$

Assess: While this is a long pipe, it is a reasonable length for the longest pipe in the "rack of pipes."

P16.25. Prepare: The length of the tube is unchanged, so the wavelength is also unchanged. Use ratios.
Solve: For the fundamental mode, $m=1$ in $\lambda_m = 2L/m$, so using Equation 15.10,

$$\frac{f_{\text{He}}}{f_{\text{air}}} = \frac{v_{\text{He}}/\lambda}{v_{\text{air}}/\lambda} \Rightarrow f_{\text{He}} = \frac{v_{\text{He}}}{v_{\text{air}}} f_{\text{air}} = \frac{1010 \text{ m/s}}{343 \text{ m/s}}(315 \text{ Hz}) = 928 \text{ Hz}$$

Assess: Note that the length of the tube is one-quarter the wavelength, whether the tube is filled with helium or air.

P16.26. Prepare: For an open-closed organ pipe, the length of the pipe is an integral number of quarter wavelengths $L = m\lambda/4$, where $\text{m} = 1, 3, 5, \text{etc}$. The expression $v = f\lambda$ connects speed with the wavelength and frequency. We will use 350 m/s as the speed of sound inside the body.
Solve: A general expression for the allowed frequiences is

$$f_m = \frac{v}{\lambda_m} = \frac{v}{(4L/m)} = m\frac{v}{4L} \quad \text{where } m = 1, 3, 5, \text{etc}$$

For the fundamental $(m=1)$ we have

$$f_1 = (1)\frac{v}{4L} = \frac{v}{4L} = \frac{350 \text{ m/s}}{4(1.5 \text{ m})} = 58 \text{ Hz}$$

Since all other frequencies are multiples of this frequency we have

$$f_3 = 3f_1 = 170 \text{ Hz} \text{ and } f_5 = 5f_1 = 292 \text{ Hz}$$

Assess: These are reasonable frequencies and they are multiples of the fundamental frequency.

P16.27. Prepare: The frequencies at which resonance will occur in an open-open pipe are determined by $f_m = m(v/(2L))$.
Solve: (a) Knowing the speed of sound, the fundamental frequency is determined by

$$f_1 = (1)(v/(2L)) = (340 \text{ m/s})/(2(30.0 \text{ m})) = 5.67 \text{ Hz}$$

(b) The lowest frequency we can hear is about 20 Hz. For an open-open tube the frequency of the harmonics are related to the fundamental frequency by $f_m = mf_1$, hence the lowest harmonic that would be audible to the human ear is

$$m = f_m/f_1 = 20 \text{ Hz}/(5.67 \text{ Hz}) = 3.5$$

But m must be an integer, so we must take $m = 4$ and calculate the frequency.

$$4f_1 = 4(5.67 \text{ Hz}) = 22.7 \text{ Hz}$$

(c) As the air cools the speed of sound will decrease. Inspecting the function given in the previous Prepare step, we see that this would in turn result in a decrease in frequency.
Assess: This is a straightforward example of resonance in an open-open pipe.

P16.28. Prepare: We are given that $v = 343$ m/s. 32 ft $= 9.75$ m.
Solve: For an open-open tube we use

$$f_1 = \frac{343 \text{ m/s}}{2(9.75 \text{ m})} = 17.6 \text{ Hz} \approx 18 \text{ Hz}$$

Solve the open-closed equation for L and use $f = 17.6$ Hz.

$$L = m\frac{v}{4f_m} = (1)\frac{343 \text{ m/s}}{4(17.6 \text{ Hz})} = 4.88 \text{ m} = 16 \text{ ft}$$

Assess: We expected the length of the open-closed pipe to be half the length of the open-open pipe for the same fundamental frequency.

P16.29. Prepare: For an open-closed tube we need Equation 16.7

$$f_m = m\frac{v}{4L}$$

where we are given that $m = 1$ (for the fundamental frequency) and that $f_1 = 200\text{ Hz}$. We are also given that $v = 350\text{ m/s}$.

Solve: Solve the equation for L.

$$L = m\frac{v}{4f_m} = (1)\frac{350\text{ m/s}}{4(200\text{ Hz})} = 0.44\text{ m} = 44\text{ cm}$$

Assess: Since the 200 Hz is a "typical" fundamental frequency we don't really expect the length obtained to be the exact length of *your* vocal tract; but we do note that it is in the right ballpark (put a meter stick next to your stretched neck and guesstimate the distance from your mouth to your diaphragm).

P16.30. Prepare: Assume the shower is a closed-closed system, for which we need an equation like Equation 16.6:

$$f_m = m\frac{v}{2L} \qquad m = 1, 2, 3, 4, \ldots$$

where we are given that $L = 0.75\text{ cm}$ for the short axis and $L = 1.5\text{ m}$ for the long axis. We assume that $v = 343\text{ m/s}$.

Solve: Plug in $m = 1$ for the two lengths and obtain the corresponding frequencies. For the short axis;

$$f_1 = (1)\frac{343\text{ m/s}}{2(0.75\text{ m})} = 230\text{ Hz} \qquad f_2 = (2)\frac{343\text{ m/s}}{2(0.75\text{ m})} = 460\text{ Hz}$$

For the longer axis;

$$f_1 = (1)\frac{343\text{ m/s}}{2(1.5\text{ m})} = 110\text{ Hz} \qquad f_2 = (2)\frac{343\text{ m/s}}{2(1.5\text{ m})} = 230\text{ Hz}$$

Assess: All of these frequencies are in the range of human singing.

P16.31. Prepare: Follow Example 16.6 very closely. Assume the ear canal is an open-closed tube, for which we need Equation 16.7

$$f_m = m\frac{v}{4L} \qquad m = 1, 3, 5, 7\ldots$$

where we are given that $L = 1.3\text{ cm} = 0.013\text{ m}$. We assume that $v = 350\text{ m/s}$ in the warm ear canal.

Take the audible range as $20\text{ Hz–}20\text{ kHz}$.

Solve: Plug in various values of m and obtain the corresponding frequencies.

$$f_1 = (1)\frac{350\text{ m/s}}{4(0.013\text{ m})} = 6730\text{ Hz} \approx 6700\text{ Hz}$$

Higher frequencies are odd multiples of this fundamental.

$$f_3 = 3(6730\text{ Hz}) = 20200\text{ Hz}$$

Already f_3 is out of the audible range. So the only one in the audible range is $f_1 = 6700\text{ Hz}$.

P16.32. Prepare: The quiet spot will be at a node of the standing wave. The distance between an antinode (at the wall) and an adjacent node in a standing wave is only $\frac{1}{4}\lambda$. So all we have to do is find the wavelength and divide by 4.

$$\frac{\lambda}{4} = \frac{v}{4f} = \frac{340\text{ m/s}}{4(50\text{ Hz})} = 1.7\text{ m}$$

Assess: This is a reasonable distance for a low pitch sound.

P16.33. Prepare: We can determine the frequency of the first formant from Figure 16.23. Developing an expression for the resonant frequencies of an open-closed pipe, we can determine the length of the tube.

Solve: Notice in the graph that the first 1000 Hz is divided into seven equal sections and the desired frequency is 2/7 of the way to 1000 Hz or $(2/7)(1000) = 290$ Hz. The relationship between the speed of sound, the frequency of the sound, and the length of the open-closed pipe is

$$v = f\lambda = f(4L/m) \quad \text{or} \quad L = mv/4f = v/4f = \frac{343 \text{ m/s}}{4(290 \text{ Hz})} = 0.30 \text{ m}$$

Assess: This is a reasonable length for the vocal system.

P16.34. Prepare: We model both tubes as open-closed. For an open-closed tube we need Equation 16.7.

$$f_m = m\frac{v}{4L}$$

We also assume that $m = 1$ (for the fundamental frequency) for these formants. We are also given that $v = 350$ m/s.

Solve: Solve the equation for L. For the throat

$$L = m\frac{v}{4f_m} = (1)\frac{350 \text{ m/s}}{4(800 \text{ Hz})} = 11 \text{ cm}$$

For the mouth

$$L = m\frac{v}{4f_m} = (1)\frac{350 \text{ m/s}}{4(1500 \text{ Hz})} = 5.8 \text{ cm}$$

Assess: These lengths sound reasonable for a throat and mouth.

P16.35. Prepare: First assume the speed of sound is 350 m/s in the vocal tract. The relationship between speed, wavelength, and frequency for a traveling wave disturbance in any medium is $v = f\lambda$. The frequency of vibration in air is caused by and is the same as the frequency of vibration of the vocal cords. The length of the vocal tract is an integral number of half-wavelengths $L = m\lambda/2$. The length of the vocal tract and hence the wavelengths that cause standing wave resonance do not change as the diver descends. However, since the speed of the sound waves changes, the frequency will also change.

Solve: When a sound of frequency 270 Hz is coming out of the vocal tract, the wavelength of standing waves established in the vocal tract associated with this frequency is

$$\lambda = v/f = (350 \text{ m/s})/(270 \text{ Hz}) = 1.296 \text{ m}$$

As the diver descends, the vocal tract does not change length and hence this wavelength for standing wave resonance will not change. However since the sound is now travelling through a helium-oxygen mixture with a speed of 750 m/s, the frequency of the sound will change to

$$f = v/\lambda = (750 \text{ m/s})/(1.296 \text{ m}) = 580 \text{ Hz}$$

Going through the same procedure for sound at a frequence of 2300 Hz we get the frequency in the helium-oxygen mixture to be $f = 4900$ Hz.

Assess: We are aware that the sound should be at a higher frequency and the frequencies obtained have higher values.

P16.36. Prepare: The two waves will produce destructive interference any time their path-length difference is a whole number of wavelengths plus half a wavelength. Equation 16.9 describes this situation. Since the frequency of the sound waves is given to be 686 Hz, we will use Equation 15.10 to find the wavelength.

Solve: (a) Let Δd represent the path-length difference. Using $m = 0$ for the smallest Δd and the condition for destructive interference, we get

$$\Delta d = \left(m + \frac{1}{2}\right)\lambda = \frac{1}{2} \Rightarrow \Delta d = \frac{1}{2}\left(\frac{v}{f}\right) = \frac{1}{2}\left(\frac{343 \text{ m/s}}{686 \text{ Hz}}\right) = 0.250 \text{ m}$$

Assess: This is a reasonable number. If you have two speakers, you can set them up in such a manner as to produce quiet spots.

P16.37. Prepare: Interference occurs as a result of the path difference between the path lengths of the sound from the two speakers. A separation of 20 cm between the speakers leads to maximum intensity on the *x*-axis, but a separation of 30 cm leads to zero intensity.

Solve: (a) When the waves are in phase and lead to constructive interference, $(\Delta d_1) = m\lambda = 20 \text{ cm}$. For destructive interference, $(\Delta d)_2 = (m + \frac{1}{2})\lambda = 30 \text{ cm}$. Thus, for the same value of m

$$(\Delta d)_2 - (\Delta d)_1 = \frac{\lambda}{2} \Rightarrow \lambda = 2(30 \text{ cm} - 20 \text{ cm}) = 20 \text{ cm}$$

(b) If the distance between the speakers continues to increase, the intensity will again be a maximum when the separation between the speakers that produced a maximum has increased by one wavelength. That is, when the separation between the speakers is $20 \text{ cm} + 20 \text{ cm} = 40 \text{ cm}$.

Assess: The distances obtained are reasonable. As a check on our work we might want to determine the frequency of sound associated with the wavelength. A wavelength of 40 cm is associated with a frequency of 860 Hz, which is in the audible range.

P16.38. Prepare: Interference occurs as a result of the path length. To cancel the noise we want the speaker to be half a wavelength farther or closer to the worker than the noisy machine is, so we must calculate the wavelength.

Solve:

$$\lambda = \frac{v}{f} = \frac{340 \text{ m/s}}{80 \text{ Hz}} = 4.25 \text{ m} \Rightarrow \frac{\lambda}{2} = 2.125 \text{ m}$$

So the speaker could be placed half a wavelength closer than the machine, which is 5.0 m – 2.125 m = 2.9 m away from the worker.

Assess: This strategy does work, but only for a small region. Because of conservation of energy, the sound will be louder somewhere else.

P16.39. Prepare: We assume that the speakers are identical and that they are emitting in phase. Since you don't hear anything, the separation between the two speakers corresponds to the condition of destructive interference.

Solve: Equation 16.9 for destructive interference is

$$\Delta d = \left(m + \frac{1}{2}\right)\lambda \Rightarrow \Delta d = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}$$

Since the wavelength is

$$\lambda = \frac{v}{f} = \frac{340 \text{ m/s}}{170 \text{ Hz}} = 2.0 \text{ m}$$

three possible values for *d* are 1.0 m, 3.0 m, and 5.0 m.

Assess: The units worked out and these are reasonable distances.

P16.40. Prepare: The circular wave fronts emitted by the two sources show that the two sources are in phase. This is because the wave fronts of each source have moved the same distance from their sources.

Solve: Let us label the top source as 1 and the bottom source as 2. For the point *P*, $d_1 = 3\lambda$ and $d_2 = 4\lambda$. Thus, $\Delta d = d_2 - d_1 = 4\lambda - 3\lambda = \lambda$. This corresponds to constructive interference.

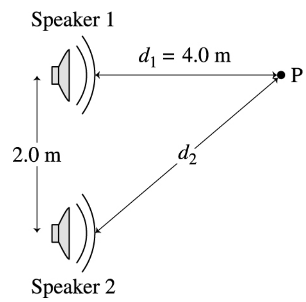
For the point *Q*, $d_1 = \frac{7}{2}\lambda$ and $d_2 = 2\lambda$, so $\Delta d = |d_2 - d_1| = |2\lambda - 7\lambda/2| = 3\lambda/2$. This corresponds to destructive interference.

For the point *R*, $d_1 = \frac{5}{2}\lambda$ and $d_2 = \frac{7}{2}\lambda$, $\Delta d = d_2 - d_1 = \lambda$. This corresponds to constructive interference.

	r_1	r_2	Δr	C/D
P	3λ	4λ	λ	C
Q	$\frac{7}{2}\lambda$	2λ	$\frac{3}{2}\lambda$	D
R	$\frac{5}{2}\lambda$	$\frac{7}{2}\lambda$	λ	C

Assess: When the path difference Δr is an integral number of whole wavelengths (cases P and R), constructive interference occurs. When the path difference Δr is an integral number of half wavelengths (case Q), destructive interference occurs.

P16.41. Prepare: The two speakers are identical, and so they are emitting circular waves in phase. The overlap of these waves causes interference. An overview of the problem follows.



Solve: The wavelength of the sound waves is

$$\lambda = \frac{v}{f} = \frac{340 \text{ m/s}}{1800 \text{ Hz}} = 0.1889 \text{ m}$$

From the geometry of the figure,

$$d_2 = \sqrt{d_1^2 + (2.0 \text{ m})^2} = \sqrt{(4.0 \text{ m})^2 + (2.0 \text{ m})^2} = 4.472 \text{ m}$$

So, $\Delta d = d_2 - d_1 = 4.472 \text{ m} - 4.0 \text{ m} = 0.472 \text{ m}$.

Because $\Delta d/\lambda = 0.472 \text{ m}/0.1889 \text{ m} = 2.5 = 5/2 = 2 + \frac{1}{2}$ or $\Delta d = 2 + \frac{1}{2}\lambda$, the interference is perfectly destructive.

Assess: Destructive interference (for two waves in phase) will occur when the path difference is an integral number of half wavelengths.

P16.42. Prepare: For destructive interference we need the path length difference to be a half wavelength. The path length difference is twice the length of the resonator tube because the sound travels down and back.

Solve:

$$\frac{\lambda}{2} = 2L \Rightarrow L = \frac{\lambda}{4} = \frac{v}{4f} = \frac{360 \text{ m/s}}{4(120 \text{ Hz})} = 75 \text{ cm}$$

Although just a bit long, this is a reasonable length for a tube in a car.

Assess: Destructive interference (for two waves in phase) will occur when the path difference is an integral number of half wavelengths.

P16.43. Prepare: The beat frequency is the difference of the two frequencies. We know the flat flute's frequency is lower.

Solve: $f_{\text{untuned}} = f_{\text{tuned}} - f_{\text{beat}} = 440 \text{ Hz} - 2 \text{ Hz} = 438 \text{ Hz}$.

Assess: Trained ears are quite sensitive to such differences in frequency.

P16.44. Prepare: Let's treat this problem as a blinker analog to two speakers emitting sound at different frequencies. Knowing the period of one of the blinkers, we can determine its frequency. Knowing the time interval between the instances when the blinkers are in sync, we can determine the frequency of occurrence of this event, that is, the beat frequency. Knowing the beat frequency, the frequency of the student blinker, and that the other blinker is faster, we can determine the frequency of the other blinker.

Solve: The frequency for the student blinker is $f_s = 1/T_s = 1/(0.85 \text{ s}) = 1.1765 \text{ Hz}$.

The beat frequency is $\Delta f = 1/\Delta t = 1/17 \text{ s} = 0.05882 \text{ Hz}$.

Knowing that the other blinker is faster, it's frequency is $f_0 = \Delta f + f_s = 0.0588 \text{ Hz} + 1.1765 \text{ Hz} = 1.2353 \text{ Hz}$.

The period for the blinker of the other car is $T_0 = 1/f_0 = 1/1.2353 \text{ Hz} = 0.81 \text{ s}$.

Assess: The other blinker is faster so it should have a smaller period. Note that the student blinker will flash 20 times and the other blinker will flash 21 times in the 17 s time interval.

P16.45. Prepare: Knowing the following relationships for a vibrating string, $v=\sqrt{T/\mu}$, $v=f\lambda$, and $L=m\lambda/2$, we can establish what happens to the frequency as the tension is increased. Knowing the relationship $f_{\text{beat}}=f_1-f_2$, we can determine the frequency of the string with the increased tension.

Solve: Combining the first three expressions above, obtain $f=\dfrac{m}{2L}\sqrt{\dfrac{T}{\mu}}$, which allows us to determine that the frequency will increase as the tension increases. Using the expression for the beat frequency, we know that the difference frequency between the two frequencies is 3 Hz. Combining this with our knowledge that the frequency increases, we obtain

$$f_{\text{beat}}=f_1-f_2 \Rightarrow 3\text{ Hz}=f_1-200\text{ Hz} \Rightarrow f_1=203\text{ Hz}$$

Assess: f_1 is larger than f_2 because the increased tension increases the wave speed and hence the frequency.

P16.46. Prepare: Assume each tube is in the fundamental frequency. It is straightforward to find the two frequencies from $f=v/2L$. The beat frequency will simply be the difference between the two.

Solve:

$$f_{\text{beat}}=f_2-f_1=\frac{v}{2L_2}-\frac{v}{2L_1}=\frac{350\text{ m/s}}{2(0.11\text{ m})}-\frac{350\text{ m/s}}{2(0.12\text{ m})}=1590\text{ Hz}-1460\text{ Hz}=130\text{ Hz}$$

Assess: The answer is in the range of human hearing, so it can be perceived as a "difference tone."

P16.47. Prepare: Knowing the expression for the beat frequency, we can determine by which the amount the frequency will change. But at this point we don't know if the frequency increases or decreases. Examining the expression for the frequency of a flute (modeled as an open-open pipe) as a function of its length, we can establish if the frequency increases or decreases when the "tuning joint" is removed.

Solve: Using the expression for the beat frequency, the flute player's initial frequency is either $523\text{ Hz}+4\text{ Hz}=527\text{ Hz}$ or $523\text{ Hz}-4\text{ Hz}=519\text{ Hz}$. Modeling the flute as an open-open pipe we see that $v=f\lambda$ and $L=m\lambda/2$, which may be combined to obtain $f=mv/(2L)$

This expression allows us to see that as the length increases, the frequency decreases. As a result we know that the initial frequency of sound from the flute was 527 Hz.

Assess: Since she matches the tuning fork's frequency by lengthening her flute, she is increasing the wavelength of the standing wave in the flute. A wavelength increase means a decrease of frequency because $v=f\lambda$. This confirms that the initial frequency was greater than the frequency of the tuning fork.

P16.48. Prepare: We'll use the fundamental relationship for periodic waves ($v=\lambda f$, where $f=440\text{ Hz}$) and we'll compute λ from Equation 16.1 ($\lambda_1=2L=2.0\text{ m}$, where $m=1$) because the problem is about the fundamental frequency.

Solve:

$$v=\lambda f=(2.0\text{ m})(440\text{ Hz})=880\text{ m/s}$$

Assess: This speed is over twice the speed of sound in air, but this string might be thin (small μ) and under a lot of tension in order to make the wave speed that high.

P16.49. Prepare: Since when a sound wave hits the boundary between soft tissue and air, or between soft tissue and bone, most of the energy is reflected, the situation is like a vibrating string with reflections at both ends. If that is the case, standing wave resonance will be established (and heating will occur) when twice the thickness of the soft tissue is an integral number of wavelengths. The speed of ultrasound in soft tissue is 1540 m/s.

Solve: For a standing wave,

$$f_m = m\left(\frac{v}{2L}\right) \Rightarrow m = \frac{f_m(2L)}{v} = \frac{(0.70\times 10^6\ \text{Hz})(2)(0.0055\ \text{m})}{1540\ \text{m/s}} = 5.0$$

Since this is an integer, then yes, there will be standing waves and heating of the tissue.
Assess: Since the frequency of the ultrasound and the thickness of the soft tissue are reasonable, we expect heating to occur.

P16.50. Prepare: According to Equation 16.6, the standing wave on a guitar string, vibrating at its fundamental frequency, has a wavelength λ equal to twice the length L. We will first calculate the frequency of the wave that the string produces using Equations 15.2 and 15.10. The wave created by the guitar string travels as a sound wave with the same frequency but with a speed of 343 m/s in air.
Solve: The wave speed on the stretched string is

$$v_{\text{string}} = \sqrt{\frac{T_S}{\mu}} = \sqrt{\frac{200\ \text{N}}{0.001\ \text{kg/m}}} = 447.2\ \text{m/s}$$

The wavelength of the wave on the string is $\lambda = 2L = 2(0.80\ \text{m}) = 1.60\ \text{m}$. Thus, the frequency of the wave is

$$f = \frac{v_{\text{string}}}{\lambda} = \frac{447.2\ \text{m/s}}{1.60\ \text{m}} = 279.5\ \text{Hz}$$

Finally, the wavelength of the sound wave that reaches your ear is

$$\lambda_{\text{air}} = \frac{v_{\text{sound}}}{f} = \frac{343\ \text{m/s}}{279.5\ \text{Hz}} = 1.2\ \text{m}$$

Assess: This is a reasonable wavelength and the units are correct.

P16.51. Prepare: The relationship between the velocity, frequency, and wavelength of a traveling wave disturbance is $v = f\lambda$. The relationship between the velocity, tension, and linear mass density for a traveling wave disturbance in a string is $v = \sqrt{T/\mu}$. The relationship that must be satisfied to create standing wave resonance in a stretched tendon is $L = m\lambda/2$. Finally, the relationship between linear and volume mass density is $\mu = \rho A$.
Solve: Start with $v = \sqrt{T/\mu}$ and insert $\mu = \rho A$ to obtain $v = \sqrt{T/(\rho A)}$. Express the wavelength as $\lambda = 2L/m$. Insert both v and λ into $v = f\lambda$ and solve for the frequency.

$$\sqrt{T/(\rho A)} = f_m(2L/m) \Rightarrow f_m = (m/(2L))\sqrt{T/(\rho A)} \quad \text{where } m = 1, 2, 3, \ldots$$

Inserting values, obtain the fundamental frequency:

$$f_1 = (1/(2(0.20\ \text{m})))\sqrt{500\ \text{N}/((1100\ \text{kg/m}^3)(1.00\times 10^{-4}\ \text{m}^2))} = 160\ \text{Hz}$$

Other possible frequencies are multiples of the fundamental

$$f_2 = 2f_1 = 320\ \text{Hz},\quad f_3 = 3f_1 = 480\ \text{Hz},\ \ldots\ \text{etc.}$$

Assess: These are reasonable frequencies for the vibration of a tendon.

P16.52. Prepare: For a string fixed at both ends, successive resonant frequencies occur at

$$f_m = mf_1 \quad \text{and} \quad f_{(m+1)} = (m+1)f_1$$

Solve: Inserting given information into the previous expressions, obtain

$$325\ \text{Hz} = mf_1 \quad \text{and} \quad 390 = (m+1)f_1$$

Dividing the second expression by the first and for m, obtain $m = 5$.
Knowing that $m = 5$, we can use the first expression to obtain the fundamental frequency.

$$f_1 = 325\ \text{Hz}/m = 325\ \text{Hz}/5 = 65\ \text{Hz}$$

Or using the second expression, we obtain the same value.

$$f_1 = 390\ \text{Hz}/(m+1) = 390\ \text{Hz}/(5+1) = 65\ \text{Hz}$$

Assess: It is good practice to look for simple checks on your work. Calculating the fundamental frequency using both expressions is a simple check.

P16.53. Prepare: The relationship between the velocity, frequency, and wavelength of a traveling wave disturbance is $v = f\lambda$. The relationship between the velocity, tension and linear mass density for a traveling wave disturbance in a string is $v = \sqrt{T/\mu}$. The relationship that must be satisfied to create standing wave resonance in a stretched tendon is $L = m\lambda/2$. Finally, the relationship between linear and volume mass density is $\mu = \rho A$.

Solve: First determine the velocity of the traveling wave disturbance of the struggling insect

$$v = f\lambda = f(2L/m) = 2fL = 2(100\,\text{Hz})(0.12\,\text{m}) = 24\,\text{m/s}$$

Next determine the linear mass density of the web

$$\mu = \rho A = \rho\pi d^2/4 = (1.300\times10^3\,\text{kg/m}^3)(\pi/4)(2.0\times10^{-6})^2 = 4.08\times10^{-9}\,\text{kg/m} \approx 4.1\times10^{-9}\,\text{kg/m}$$

Finally, determine the tension in the web

$$T = v^2\mu = (24\,\text{m/s})^2(4.08\times10^{-9}\,\text{kg/m}) = 2.4\times10^{-6}\,\text{N}$$

Assess: This is a very small tension; however, it is several orders of magnitude greater than the weight of a small insect that the spider might wish to capture.

P16.54. Prepare: The wave on a stretched string with both ends fixed is a standing wave. For vibration at its fundamental frequency, $\lambda = 2L$ (see Equation 16.6).

Solve: The wavelength of the wave reaching your ear is $39.1\,\text{cm} = 0.391\,\text{m}$. So the frequency of the sound wave is

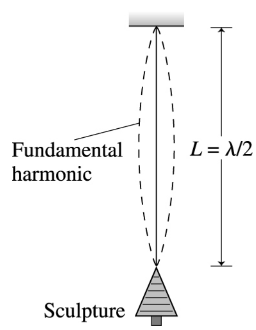
$$f = \frac{v_{\text{air}}}{\lambda} = \frac{344\text{ m/s}}{0.391\text{ m}} = 879.8\text{ Hz}$$

This is also the frequency emitted by the wave on the string. Thus,

$$879.8\text{ Hz} = \frac{v_{\text{string}}}{\lambda} = \frac{1}{\lambda}\sqrt{\frac{T_S}{\mu}} = \frac{1}{\lambda}\sqrt{\frac{150\text{ N}}{0.0006\text{ kg/m}}} \Rightarrow \lambda = 0.568\text{ m} \Rightarrow L = \frac{1}{2}\lambda = 28\text{ cm}$$

Assess: Since 28 cm is less than the length of a violin, it is a reasonable length for the vibrating section of the string.

P16.55. Prepare: For the stretched wire vibrating at its fundamental frequency, the wavelength of the standing wave from Equation 16.1 is $\lambda_1 = 2L$. From Equation 15.2, the wave speed is equal to $\sqrt{T_S/\mu}$, where $\mu = \text{mass/length} = 5.0\times10^{-3}\text{ kg}/0.90\text{ m} = 5.555\times10^{-3}\text{ kg/m}$. The tension T_S in the wire equals the weight of the sculpture or Mg.



Solve: The wave speed on the steel wire is

$$v_{\text{wire}} = \sqrt{\frac{T_s}{\mu}} = \sqrt{\frac{Mg}{\mu}} = \sqrt{\frac{(12\text{ kg})(9.8\text{ m/s}^2)}{5.55\times10^{-3}\text{ kg/m}}} = 145.6\text{ m/s}$$

Now we can solve for frequency.

$$v_{\text{wire}} = f\lambda \Rightarrow f = \frac{v_{\text{wire}}}{\lambda} = \frac{v_{\text{wire}}}{2L} = \frac{145.6\text{ m/s}}{2(0.90\text{ m})} = 81\text{ Hz}$$

Assess: A frequency of 81 Hz for the wire is reasonable.

P16.56. Prepare: According to Figure P16.56, a half wavelength is 400 km. From the study of standing wave resonance we know that a standing wave goes from maximum displacement to normal water level in one fourth of a period, which in this case is 3 hours. Finally we know that the velocity, wavelength, and frequency of a traveling wave disturbance are related by $v = f\lambda$.

Solve: (a) According to the figure, a half wavelength is 400 km, so the wavelength is $\lambda = 800$ km. **(b)** Knowing that a standing wave goes from maximum displacement to normal water level in one fourth of a period, which in this case is 3 hours, we get the period to be $\tau = 12\text{ h} = 4.32\times10^4\text{ s}$ or a frequency of $f = 1/\tau = 1/4.32\times10^4\text{s} = 2.3\times10^{-5}\text{ Hz}$.

(c) Using the relationship $v = f\lambda$, we obtain a wave speed of $v = f\lambda = (2.3\times10^{-5}\text{Hz})(8.0\times10^5\text{ m}) = 18\text{ m/s}$.

Assess: The wavelength is large and the frequency is small, however, they result in a reasonable wave velocity.

P16.57. Prepare: There are reflectors at both ends, so the electromagnetic standing wave acts just like the standing wave on a string that is tied at both ends. The electromagnetic waves of all frequencies travel with the speed of light *c*.

Solve: (a) The frequencies of the standing waves, using Equation 16.2, are

$$f_m\ m\frac{v_{\text{light}}}{2L} = m\frac{c}{2L} = m\frac{3.0\times10^8\text{ m/s}}{2(0.10\text{ m})} = m(1.5\times10^9\text{Hz}) = 1.5m\text{ GHz}$$

The generator can produce standing waves at any frequency between 10 GHz and 20 GHz. These are

m	*fm* (GHz)
7	10.5
8	12.0
9	13.5
10	15.0
11	16.5
12	18.0
13	19.5

(b) There are 7 different standing wave frequencies. Even-numbered values of *m* create a node at the center, and odd-numbered values of *m* create an antinode at the center. So the frequencies where the midpoint is an antinode are 10.5 GHz, 13.5 GHz, 16.5 GHz, and 19.5 GHz.

Assess: All of the frequencies are in the appropriate range and a quick sketch will verify that antinodes occur at the midpoint of the cavity for odd values of m.

P16.58. Prepare: Since light is reflected at both ends of a laser, there is a node at both ends of the cavity and an integral number of wavelengths must fit into the cavity. The speed of light, frequency, and wavelength are related by $c = f\lambda$. Finally, the length of the cavity is related to the wavelength of the light by $L = m\lambda/2$.

Solve: (a) The information that the laser is oscillating in the $m = 100\,000$ mode tells us that $100\,000$ half-wavelengths or $50\,000$ wavelengths fit into the Laser cavity. The length of the cavity is

$$L = m\lambda = 50\,000(10.6\times10^{-6}\text{ m}) = 0.530\text{ m}$$

(b) Knowing the wavelength and speed of light, we can determine its frequency.

$$f = c/\lambda = (3.00\times10^8\text{ m/s})/10.6\times10^{-6}\text{ m} = 2.83\times10^{13}\text{ Hz}$$

(c) In a time of one second, light will travel a distance of

$$d = ct = (3.00\times10^8\text{ m/s})(1.00\text{s}) = 3.00\times10^8\text{ m}$$

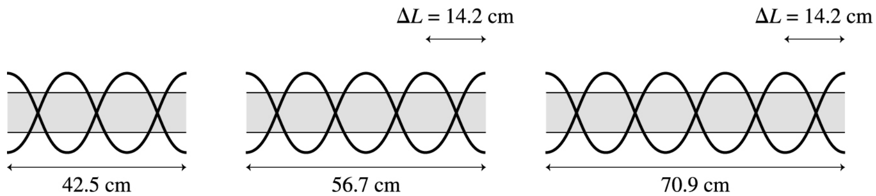
For a round trip, the light must travel $2\text{L} = 1.06\text{ m}$.

As a result the number of round trips the light will make in 1.00 s is

$$n = d/(2L) = 3.00\times10^8\text{ m}/1.06\text{ m} = 2.83\times10^8\text{ round trips}$$

Assess: The one thing that is easy to check is the length of the laser cavity. A value of 0.530 m is reasonable.

P16.59. Prepare: The nodes of a standing wave are spaced $\lambda/2$ apart. The wavelength of the *m*th mode of an open-open tube from Equation 16.6 is $\lambda_m = 2L/m$. Or, equivalently, the length of the tube that generates the *m*th mode is $L = m(\lambda/2)$. Here λ is the same for all modes because the frequency of the tuning fork is unchanged.



Solve: Increasing the length of the tube to go from mode *m* to mode $m+1$ requires a length change:

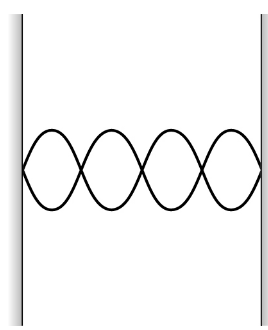
$$\Delta L = (m+1)(\lambda/2) - m\lambda/2 = \lambda/2$$

That is, lengthening the tube by $\lambda/2$ adds an additional antinode and creates the next standing wave. This is consistent with the idea that the nodes of a standing wave are spaced $\lambda/2$ apart. This tube is first increased by $\Delta L = 56.7\text{ cm} - 4.25\text{ cm} = 14.2\text{ cm}$, then by $\Delta L = 70.9\text{ cm} - 56.7\text{ cm} = 14.2\text{ cm}$. Thus $\lambda/2 = 14.2\text{ cm}$ and $\lambda = 28.4\text{ cm} = 0.284\text{ m}$. Therefore, the frequency of the tuning fork, using Equation 15.10, is

$$f = \frac{v}{\lambda} = \frac{343\text{ m/s}}{0.284\text{ m}} = 1210\text{ Hz}$$

Assess: This is a reasonable value for the frequency of a tuning for k in the audible range and the units are correct.

P16.60. Prepare: The nodes of a standing wave are spaced $\lambda/2$ apart.



Solve: As seen in the figure the width of the oven is 2λ.

$$2\lambda = 2(12\text{ cm}) = 24\text{ cm}$$

Assess: This is not a very big microwave oven; most microwave ovens are big enough to hold a dinner plate. Because there are cold spots it is a good idea to rotate the food through the nodes and antinodes.

P16.61. Prepare: The waves constructively interfere when speaker 2 is located at 0.75 m and 1.00 m, but not in between. Assume the two speakers are in phase (helpful for visualization, but the result will be generally true as long as the two frequencies are the same). For constructive interference the path length difference must be an integer number of wavelengths, $0.75\text{ m} = n\lambda$, and $1.00\text{ m} = (n+1)\lambda$. Subtracting the two equations gives $\lambda = 0.25\text{ m}$.

Solve:

$$f = \frac{v}{\lambda} = \frac{340\text{ m/s}}{0.25\text{ m}} = 1360\text{ Hz} \approx 1400\text{ Hz}$$

Assess: 1400 Hz is near the "middle" of the range of human hearing, so it is probably right.

P16.62. Prepare: Assume there are loud spots at the speakers. The wavelength of these sound waves is $\lambda = v/f = (345 \text{ m/s})/(115 \text{ Hz}) = 3.00 \text{ m}$. There are an integer number of wavelengths between the speakers $(42.0 \text{ m}/3.0 \text{ m} = 14.0)$, so there are standing waves.

Solve: If Susan is 19.5 m from one speaker then she is 6.5 wavelengths from the speaker. There is a loud spot every half wavelength, or every 1.5 m, so she is at a loud spot.

Assess: Halfway between the loudest spots are the quite spots.

P16.63. Prepare: The changing sound intensity is due to the interference of two overlapped sound waves. Minimum intensity implies destructive interference. Destructive interference occurs where the path length difference for the two waves is $\Delta d = (m + \frac{1}{2})\,\lambda$.

Solve: The wavelength of the sound is $\lambda = v_{\text{sound}}/f = (343 \text{ m/s})/(686 \text{ Hz}) = 0.500 \text{ m}$. Consider a point that is a distance d in front of the top speaker. Let d_1 be the distance from the top speaker to the point and d_2 the distance from the bottom speaker to the point. We have

$$d_1 = x \quad d_2 = \sqrt{x^2 + (3.00 \text{ m})^2}$$

Destructive interference occurs at distances d such that

$$\Delta d = \sqrt{x^2 + 9 \text{ m}^2} - x = \left(m + \frac{1}{2}\right)\lambda$$

To solve for x, isolate the square root on one side of the equation and then square:

$$x^2 + 9 \text{ m}^2 = \left[x + \left(m + \frac{1}{2}\right)\lambda\right]^2 = x^2 + 2\left(m + \frac{1}{2}\right)\lambda x + \left(m + \frac{1}{2}\right)^2 \lambda^2 \Rightarrow x = \frac{9 \text{ m}^2 - (m + \frac{1}{2})^2 \lambda^2}{2(m + \frac{1}{2})\lambda}$$

Evaluating x for different values of m:

m	x (m)
0	17.88
1	5.62
2	2.98
3	1.79

Because you start at $x = 2.5$ m and walk *away* from the speakers, you will only hear minima for values $x > 2.5$ m. Thus, minima will occur at distances of 2.98 m, 5.62 m, and 17.88 m.

Assess: These are reasonable distances and the units are correct.

P16.64. Prepare: The two loudspeakers actively push air back and forth, changing its pressure, so the two speakers are antinodes of pressure. Thus the air between them acts as if it is in a closed-closed tube. Further, since both speakers are acting identically, when the pressure at one speaker is high, so is the pressure at the other. This means that mode m is always an even number, and that there is always an antinode at the center (see Figure 16.19).

Solve: (a) The distance from a point where the intensity is maximum (an antinode) to where it is a minimum (a node) is one quarter of a wavelength. If Δr is this distance, then we have $\Delta r = 0.25 \text{ m} = \lambda/4$ so that $\lambda = 1.0 \text{ m}$. Then

$$f = \frac{v}{\lambda} = \frac{343 \text{ m/s}}{1.0 \text{ m}} = 340 \text{ Hz}$$

(b) As discussed above, at the halfway point the pressure is an antinode. The distance to the next antinode is half a wavelength, so now $\Delta r = 0.25 \text{ m} = \lambda/2$ so that $\lambda = 0.5 \text{ m}$. Then

$$f = \frac{v}{\lambda} = \frac{343 \text{ m/s}}{0.5 \text{ m}} = 690 \text{ Hz}$$

Assess: These frequencies are reasonable speaker frequencies.

P16.65. Prepare: The superposition of two slightly different frequencies gives rise to beats.

Solve: The third harmonic of note A and the second harmonic of note E are

$$f_{3\text{A}} = 3f_{1\text{A}} = 3(440 \text{ Hz}) = 1320 \text{ Hz} \quad f_{2\text{E}} = 2f_{1\text{E}} = 2(659 \text{ Hz}) = 1318 \text{ Hz} \Rightarrow f_{3\text{A}} - f_{2\text{E}} = 1320 \text{ Hz} - 1318 \text{ Hz} = 2 \text{ Hz}$$

The beat frequency between $f_{3\text{A}}$ and $f_{2\text{E}}$ is 2 Hz. It therefore emerges that the tuner looks for a beat frequency of 2 Hz.

Assess: It would be impossible to tune a piano without a good understanding of beat frequency and harmonics.

P16.66. Prepare: The superposition of two slightly different frequencies creates beats. The wavelength of the A note is determined by the length of the flute rather than the temperature of air or the increased sound speed.
Solve: (a) The wavelength of the sound initially created by the flutist is

$$\lambda = \frac{342 \text{ m/s}}{440 \text{ Hz}} = 0.77727 \text{ m}$$

When the speed of sound inside her flute has increased due to the warming of the air, the new frequency of the A note is

$$f' = \frac{346 \text{ m/s}}{0.77727 \text{ m}} = 445 \text{ Hz}$$

Thus the flutist will hear beats at the following frequency: $f' - f = 445 \text{ Hz} - 440 \text{ Hz} = 5 \text{ beats/s}$.
(b) The initial length of the flute is $L = \frac{1}{2}\lambda = \frac{1}{2}(0.7772 \text{ m}) = 0.3886 \text{ m}$. The new length to eliminate beats needs to be

$$L' = \frac{\lambda'}{2} = \frac{1}{2}\left(\frac{v'}{f}\right) = \frac{1}{2}\left(\frac{346 \text{ m/s}}{440 \text{ Hz}}\right) = 0.3932 \text{ m}$$

Thus, she will have to extend the "tuning joint" of her flute by $0.3932 \text{ m} - 0.3886 \text{ m} = 0.0046 \text{ m} = 4.6 \text{ mm}$.
Assess: This is a reasonable distance to have the move the "tuning joint" for a flute.

P16.67. Prepare: Frequencies for the Doppler shift in the microwave range may be summarized by:

$$f_{\pm} = f_s \frac{(c \pm v_{\text{O}})}{(c \mp v_s)}$$

Where $c =$ the speed of light, $v_s =$ speed of the source and $v_{\text{O}} =$ speed of the observer. In the numerator and the denominator, the top sign is used when the observer and source are moving toward each other and the bottom sign is used if the observer and source are moving away from each other. A speed of 55 mph is essentially 25 m/s.
Solve: The frequency sent out by the radar unit is $f_s = 10.5\times10^9$ Hz.
The frequency of waves observed by the moving car is

$$f_{\text{c}} = f_{\text{s}} \frac{(c + v_{\text{c}})}{c} = 10.5\times10^9 \text{ Hz} \frac{(3.00\times10^8 + 25) \text{ m/s}}{3.00\times10^8 \text{ m/s}} = 10.50000088\times10^9 \text{ Hz}$$

Now the car acts like a source of this frequency and since it is moving towards the police unit, the frequency of the reflected waves arriving at the unit may be determined by

$$f_{\text{u}} = f_{\text{c}} \frac{c}{c - v_{\text{c}}} = (10.50000088\times10^9 \text{ Hz}) \frac{3.00\times10^8 \text{ m/s}}{(3.00\times10^8 - 25) \text{ m/s}} = 10.500001750\times10^9 \text{ Hz}$$

As a result, the beat frequency determined by the unit is 1750 Hz.
When the unit is switched to calibration mode, a tuning fork vibrating in front of the unit at a frequency of 1750 Hz will register on the unit as 55 mph.
Assess: A frequency of 1750 Hz for a tuning fork is in the range that we hear very well. According to the operations manual that comes with the police radar unit, to ensure accuracy every unit is supposed to be calibrated with tuning forks before each working shift. Using several different tuning forks would ensure that the unit is accurate over a range of speeds. Finally, the problem could be solved in one step by inserting values into the first equation, but it is instructive to solve the problem in two steps as shown previously.

P16.68. Prepare: We will need concepts from Chapter 15 to solve this problem. The speed of ultrasound waves in human tissue is given in Table 15.1 as $v = 1540$ m/s. The frequency shift for a wave reflected from a moving object, such as blood, is given by Equation 15.18: $\Delta f = \pm 2\, f_{\text{s}} v_{\text{O}}/v$.
Solve:

$$\Delta f = \pm 2 f_{\text{s}} \frac{v_{\text{O}}}{v} = \pm 2(5.0\times10^6 \text{ Hz}) \frac{0.15 \text{ m/s}}{1540 \text{ m/s}} = 970 \text{ Hz}$$

This shift in frequency is also the difference from the original frequency, and hence the number of beats per second, or the beat frequency. Hence the beat frequency is 970 Hz.
Assess: This frequency is easily detectable and tells the ultrasound machine how fast the blood is moving.

P16.69. Prepare: We will need concepts from Chapter 15 to solve this problem. The speed of ultrasound waves in human tissue is given in Table 15.1 as $v = 1540\ \text{m/s}$. The frequency of the reflected wave must be $2.0\ \text{MHz} \pm 520\ \text{Hz}$, that is, $f_+ = 2000520\ \text{Hz}$ and $f_- = 1999480\ \text{Hz}$.

There are a couple of mathematical paths we could take, but it is probably easiest to carefully review Example 15.13 and use the mathematical result there, as it is very similar to our problem and gives an expression for the speed of the source v_s.

Solve:

$$v_s = \frac{f_+ - f_-}{f_+ + f_-} v = \frac{2000520\ \text{Hz} - 1999480\ \text{Hz}}{2000520\ \text{Hz} + 1999480\ \text{Hz}} 1540\ \text{m/s} = \frac{1040\ \text{Hz}}{4000000\ \text{Hz}} 1540\ \text{m/s} = 0.40\ \text{m/s}$$

Assess: We may not have a good intuition about how fast heart muscles move, but 0.40 m/s seems neither too fast nor too slow for a maximum speed.

P16.70. Prepare: Knowing that the frequency of G is 392 Hz, we can determine the frequency of its second harmonic. Knowing that the frequency of C is 262 Hz, we can determine the frequency of its third harmonic. Knowing these two frequencies, we can determine the beat frequency.

Solve: The frequency of the second harmonic of G is $(f_G)_{m=2} = 2(392\ \text{Hz}) = 784\ \text{Hz}$.

The frequency of the third harmonic of C is $(f_C)_{m=3} = 3(262\ \text{Hz}) = 786\ \text{Hz}$.

The beat frequency is $\Delta f = 786\ \text{Hz} - 784\ \text{Hz} = 2\ \text{Hz}$. The correct choice is B.

Assess: If you have access to a set of tuning forks or a piano you can experimentally verify this result.

P16.71. Prepare: The harmonics of the G-flat will be the integer multiples of 370 Hz: 370 Hz, 740 Hz, 1110 Hz, 1480 Hz, 1850 Hz, etc.

Solve: Comparing these harmonics with those given for C in the figure shows that some of them have differences in the range that produces dissonance. For example, 740 Hz (the second harmonic of G-flat) is 46 Hz away from 786 Hz (the third harmonic of C), and 1110 Hz is 62 Hz (the third harmonic of G-flat) away from 1048 Hz (the fourth harmonic of C). Those differences may not be quite *maximally* dissonant, but most people agree that that musical interval is a dissonant one.

The correct answer is B.

Assess: It should be obvious that G-flat and G sounded together would also be dissonant since each corresponding harmonic is just different enough from the corresponding harmonic for the other note to produce dissonance.

Acoustics in general and music in particular are fascinating areas of physics. It should in no way reduce our enjoyment of the aesthetic pleasures of music to understand the physics of it all; much rather the understanding should increase our appreciation for the beauty of the music.

P16.72. Prepare: Knowing the fundamental frequency of G (392 Hz) and the speed of sound, we can determine the wavelength. Knowing that when the air in the pipe is resonating at the fundamental frequency and that the wavelength is twice the length of the pipe, we can determine the length of pipe.

Solve: The wavelength is $\lambda = v/f = (343\ \text{m/s})/(392\ \text{Hz}) = 0.875\ \text{m}$.

The length of the pipe is $L = \lambda/2 = 0.875\ \text{m}/2 = 0.44\ \text{m}$.

Assess: This is a reasonable length of pipe. The correct choice is A.

P16.73. Prepare: A pipe that is open on one end and closed on the other only produces the odd harmonics. See Equation 16.7 and the preceding discussion.

Solve: The odd harmonics are 262 Hz, 786 Hz, and 1310 Hz, corresponding to $m = 1$, $m = 3$, and $m = 5$.

The correct answer is B.

Assess: Because only the odd harmonics are present in an open-closed pipe, it sounds different from an open-open pipe sounded at the same fundamental frequency.