Coupled Systems

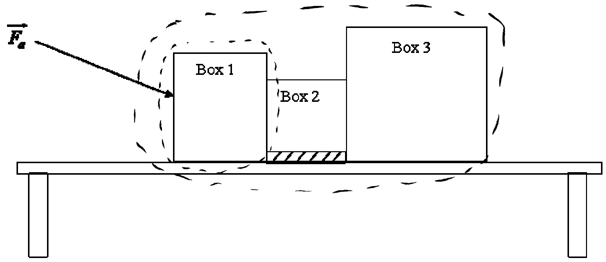
Sunday, July 6, 2014 2:43 PM

Multiple object systems: Coupled Systems

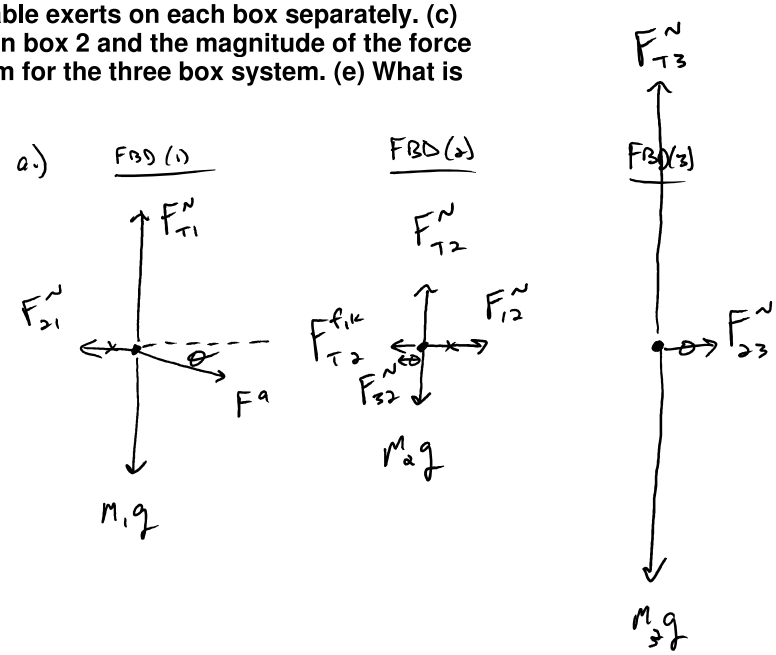
Method for Analysing

* Identify system(s) & draw FBD's
* " " 3rd Law F.P.s
* " " Constraints that connect systems
* 2nd Law & Solve, $\sum \vec{F}_{ext} = M_{sys}\,\vec{a}_{com}$

Three boxes are being accelerated on a horizontal table with a 60N force applied to box 1 at an angle 35° below the horizontal. (see figure) The bottom of box 1 and 3 experience no friction with the table surface. The coefficient of kinetic friction between box 2 and the table is 0.15. The mass of box 1 is 10 kg, the mass of box 2 is 5 kg, and the mass of box 3 is 20 kg. (a) Draw a free-body diagram for each box separately, identify all of Newton's 3rd law force pairs. (b) Find the normal force the table exerts on each box separately. (c) Find the magnitude of the force that box 1 exerts on box 2 and the magnitude of the force box 3 exerts on box 2. (d) Draw a free-body diagram for the three box system. (e) What is the acceleration of the three boxes?



a.)



b.) $\sum F_y = m a_y$

$\boxed{1}$ $F^N_{T1} - F^a \sin\theta - m_1 g = m_1 \cancel{a_{y_1}}^{\,0}$

$F^N_{T1} = F^a \sin\theta + m_1 g$

$\boxed{2}$ $F^N_{T2} = m_2 g$ $\qquad$ $\boxed{3}$ $F^N_{T3} = m_3 g$

Constraints

* F.P.'s
* $\vec{a}_1 = \vec{a}_2 = \vec{a}_3 \equiv \vec{a}$

c.) $\boxed{1}$ $\sum F_x \Rightarrow \overset{\checkmark}{F^a} \overset{\checkmark}{\cos\theta} - \overset{?}{F^N_{21}} = \overset{\checkmark}{m_1} \overset{?}{a_{1x}}$ (i)

$\boxed{2}$ $\sum F_x \Rightarrow \overset{?}{F^N_{12}} - \overset{\checkmark}{\mu_k}(\overset{\checkmark}{F^N_{T2}}) - \overset{?}{F^N_{32}} = \overset{\checkmark}{m_2} \overset{?}{a_{2x}}$ (ii) ... getting messy w/ unknowns

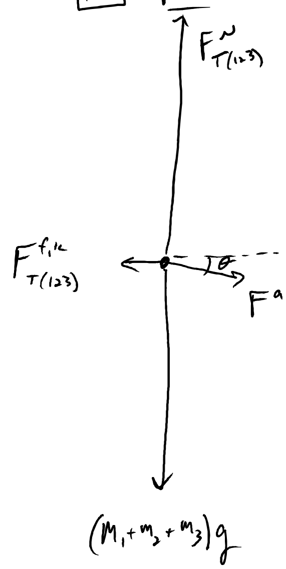
? / ?

$\boxed{2}$ $\sum F_x \Rightarrow F^N_{12} - \mu_k(F^N_{T2}) - F^N_{32} = m_2 a_{2x}$ ... getting messy w/ unknowns

$\boxed{3}$ $\sum F_x \Rightarrow \overset{?}{F^N_{23}} = \overset{\checkmark}{m_3} \overset{?}{a_{3x}}$ .... looks like too many unknowns.

... really 3 eq's, 3 unknowns $\longrightarrow F^N_{12}, F^N_{23}, a_x$

$\boxed{123}$ FBD



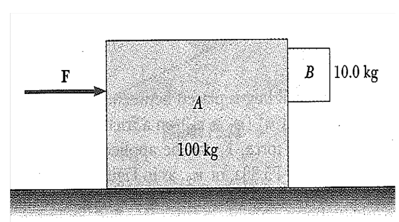
$\sum F_x \Rightarrow F^a\cos\theta - \underbrace{\mu_k(m_2 g)}_{F^{f,k}} = (m_1+m_2+m_3)a_x$

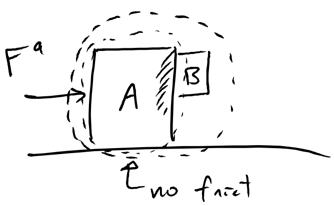
$\underline{a_x = 1.19\,m/s^2}$ w/ a_x known

use eq(i) + eq(ii) to solve for F^N_{12} + F^N_{23}

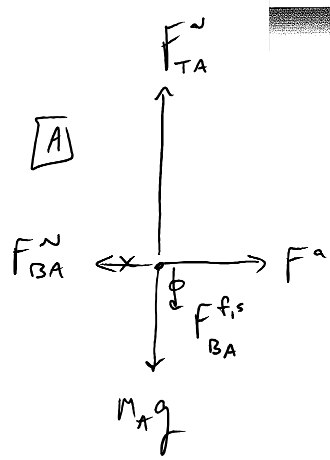
$|F^N_{21}| = 37.2N$, $|F^N_{23}| = 23.9N$

Example: What minimum force must be exerted on block A in order for block B not to fall? The coefficient for static friction between blocks A and B is 0.55 and the horizontal surface is frictionless. (Answer: 1960 N)

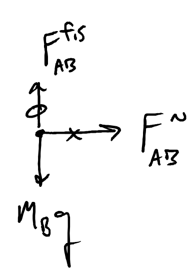




$\boxed{A}$



$\boxed{B}$



$\boxed{B}$ $\sum F_y \Rightarrow F^{f,s}_{AB} = m_B g \leftarrow a_{By} = 0$

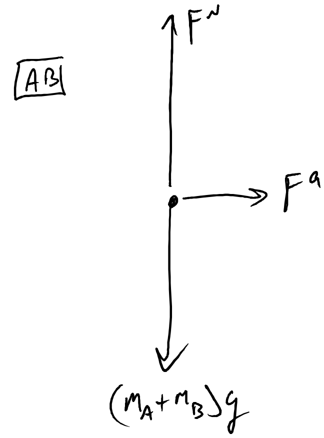
$\mu_s F^N_{AB} = m_B g \longrightarrow \mu_s(\cancel{m_B} a_{Bx}) = \cancel{m_B} g$

$$M_s (\cancel{M_B} a_{Bx}) = \cancel{M_B} g$$

$$a = \frac{g}{M_s}$$

$$\left.\begin{aligned} M_s F^N_{AB} &= M_B g \\ \Sigma F_x \Rightarrow \quad F^N_{AB} &= M_B a_{Bx} \end{aligned}\right\} \quad \text{Combine} \quad a_{Bx} = \frac{g}{M_s}$$

Same



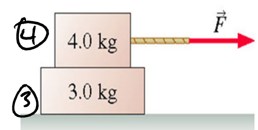
$$\Sigma F_x \Rightarrow \quad F^a = (M_A+M_B)\, a_{ABx}$$

Constraint

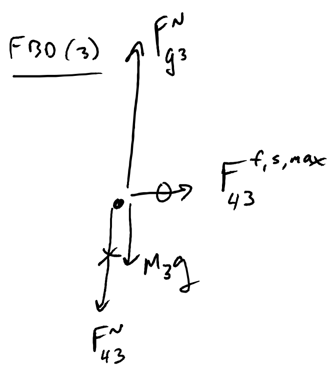
$$a_{Ax} = a_{Bx} = a_{ABx} \equiv a_x$$

$$F^a = (M_A+M_B)\frac{g}{M_s} = \boxed{1950\,N}$$

Example: A 4.0-kg block rests atop a 3.0-kg block. If the coefficient of static friction between the blocks is 0.4, and there is no friction between the 3.0-kg block and the bottom surface, what is the maximum horizontal force that can be applied to the 4.0-kg block and the two not slip relative to each other? (Answer: 36.6 N)

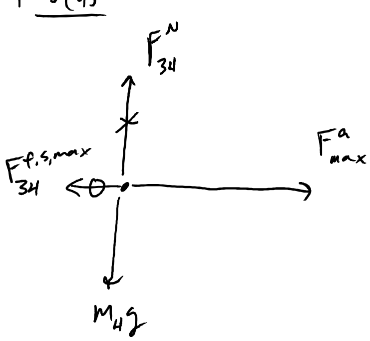


* for F^a_{max}, friction static is max



$$\Sigma F_y \Rightarrow \quad F^N_{g3} - M_3 g - F^N_{43} = M_3 \cancel{a_{3y}}^{\,0}$$

$$\Sigma F_x \Rightarrow \quad F^{f,s,max}_{43} = M_3 a_{3x}$$

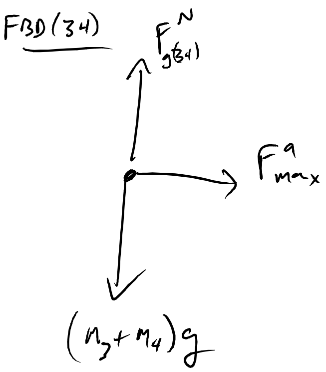


$$\Sigma F_y \Rightarrow \quad F^N_{34} = M_4 g \leftarrow a_{4y} = 0$$

$$\Sigma F_x \Rightarrow \quad \overset{?}{F^a_{max}} - \overset{\checkmark}{\mu_s} \overset{\checkmark}{F^N_{34}} = \overset{\checkmark}{M_4} \overset{?}{a_{4x}}$$

$m_4 g$

FBD(34)



Constraints

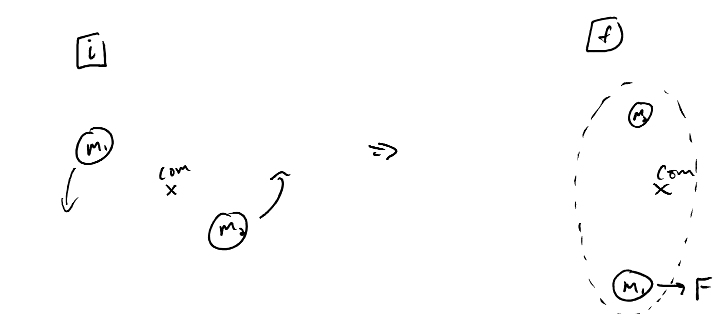
$\vec{a}_3 = \vec{a}_4 = \vec{a}_{34} = \vec{a}$

Same

$\Sigma F_x \Rightarrow \quad F^a_{max} = (m_3 + m_4) a_x$

Algebra Solve for $\boxed{F^a_{max} = 33.6\,N}$

Two asteroids, m_1 and m_2, orbit about their common stationary center of mass. If suddenly a constant force F, much smaller than the gravitational force attracting each asteroid, is applied in the positive x-direction to m_1, the two undergo a complicated motion and orbit about each other. Determine an expression for the change in position of the center of mass as a function of, the two masses (m_1 and m_2), the constant force (F), and time (t).



FBD(12)

$\Sigma F_x \Rightarrow \quad F = (m_1+m_2) a_{com_{12}x}$

$a_x = \frac{F}{m_1+m_2}$

$\Sigma F_y \Rightarrow \quad 0 = (m_1+m_2)\cancel{a_{com_{12}y}}^{0} \quad \Rightarrow \quad a_y = 0$

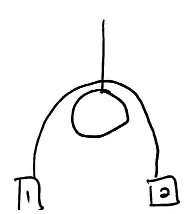
Center of Mass

12

$$\left.\begin{aligned} \Delta x &= \cancel{v_{ix}}^{0}\Delta t + \tfrac{1}{2}a_x \Delta t^2, \quad & \Delta x_{12} &= \tfrac{1}{2}\left(\frac{F}{m_1+m_2}\right)\Delta t^2 \\ \Delta y &= \cancel{v_{iy}}^{0}\Delta t + \cancel{\tfrac{1}{2}a_y \Delta t^2}^{0} & \Delta y_{12} &= 0 \end{aligned}\right\} \Delta \vec{r} = \left\langle \frac{1}{2}\frac{F}{m_1+m_2}t^2, 0 \right\rangle$$

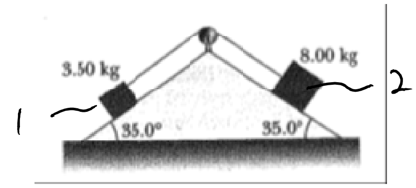
Pulleys

Assumptions: massless & Frictionless .... They only redirect force of tension

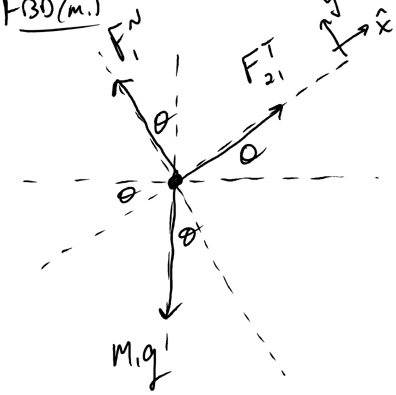


$|\vec{F}^T_{12}| = |\vec{F}^T_{21}|$

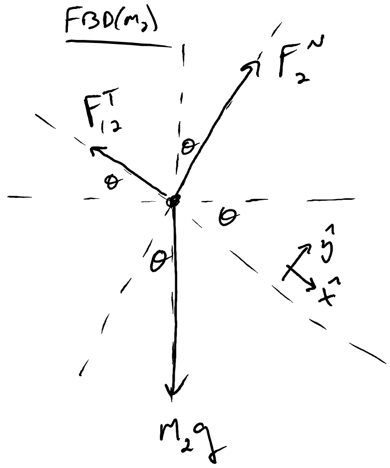
Two blocks of mass 3.5 kg and 8.0 kg are connected by a massless string that passes over a frictionless pulley. The inclines are frictionless. Find (a) the magnitude of acceleration of each block and (b) the tension in the string.



FBD(m_1)



FBD(m_2)



Constraint

$|\vec{F}_{12}^T| = |\vec{F}_{21}^T| \equiv F^T$

$\vec{a}_1 = \vec{a}_2 \equiv \vec{a}$

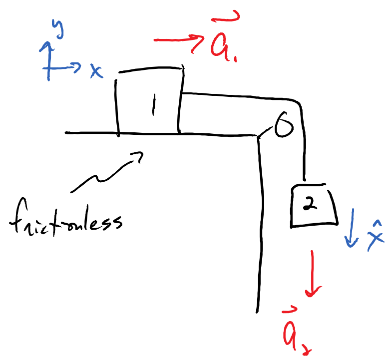
[1] $\sum F_x \Rightarrow \quad F^T - m_1 g \sin\theta = m_1 a_x$

[2] $\sum F_x \Rightarrow \quad m_2 g \sin\theta - F^T = m_2 a_x$

} 2 eq's, 2 unknowns

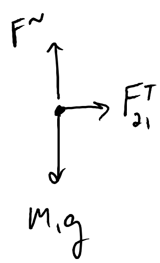
$$\boxed{\begin{array}{l} a_x = 2.2\ m/s^2 \\ F^T = 27.4\ N \end{array}}$$

ex.

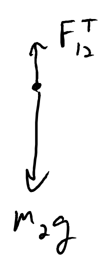


find $|\vec{F}^T|$ + $|\vec{a}_1|$, $m_1 = 1kg$, $m_2 = 2kg$

FBD(1)



FBD(2)



$F^T < m_2 g$

Constraints

$|\vec{F}_{12}^T| = |\vec{F}_{21}^T| \equiv F^T$

$|\vec{a}_1| = |\vec{a}_2|$

wise cord

$\vec{a}_1 = \vec{a}_2 \equiv \vec{a}$

[1] $\sum F_x \Rightarrow \quad \overset{?}{F^T} = m_1 \overset{?}{a_x}$

[2] $\sum F_x \Rightarrow \quad m_2 g - F^T = m_2 a_x$

} 2eq, 2unknowns

$a_x = \dfrac{m_2 g}{(m_1 + m_2)} = \dfrac{20}{3}\ m/s^2$] look @ $m_1 \to \infty$. $a \to 0$

$$a_x = \frac{m_2 g}{(m_1 + m_2)} = \frac{20}{3} \, m/s^2$$

$$F^T = m_1 \left(\frac{m_2 g}{m_1 + m_2} \right) \simeq \frac{20}{3} \, N$$

look @ $m_1 \to \infty, \quad a \to 0$

$m_2 \to \infty, \quad a \to g$