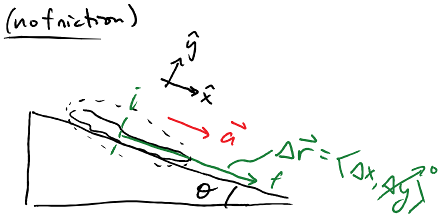
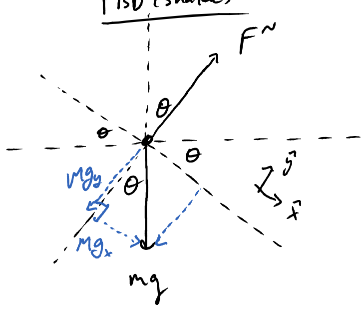
# Snakes on an Incline Plane

(no friction)




FBD (snake)




2nd Law

$$\sum F_y \Rightarrow F^N - \underbrace{mg\cos\theta}_{mg_y} = m\cancel{a_y}^{0}$$

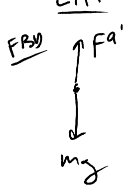
$$F^N = mg\cos\theta$$

$$\sum F_x \Rightarrow \cancel{m}g\sin\theta = \cancel{m}a_x \longrightarrow a_x = g\sin\theta$$

**Mechanical advantage is the ratio of the force required without the use of a machine (sometimes very simple machine) to that needed when using the machine. Compare the force to lift an object to that needed to slide the same object up a frictionless incline and show that the mechanical advantage of the inclined plane is the length of the incline divided by the height of the incline.**

Lift

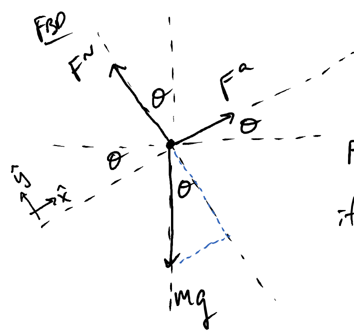
FBD

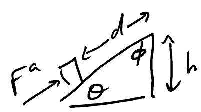



if $F^{a'} > mg$ it lifts

Sliding up a plane

FBD




$\sum F_x \Rightarrow$

$F^a - mg\sin\theta = ma_x$

if $F^a > mg\sin\theta$ accelerates up incline

Mech Advantage

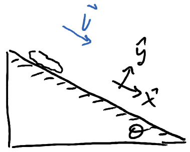
$$\frac{F^{a'}}{F^a} = \frac{mg}{mg\sin\theta} = \frac{1}{\sin\theta} \quad \ldots \text{ w/ Geometry } \sin\theta = \frac{h}{d}$$

$$+ \quad \boxed{\frac{F^{a'}}{F^a} = \frac{d}{h} \quad \therefore}$$

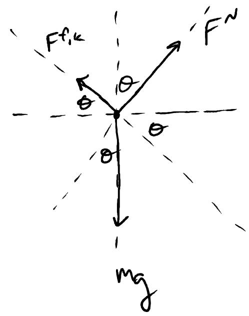
Snakes on a friction incline plane

FBD (snake)

## Snakes on a friction incline plane




FBD (snake)




$$\Sigma F_y \Rightarrow F^N = mg\cos\theta$$

$$\Sigma F_x \Rightarrow mg\sin\theta - \mu_k F^N = ma_x$$

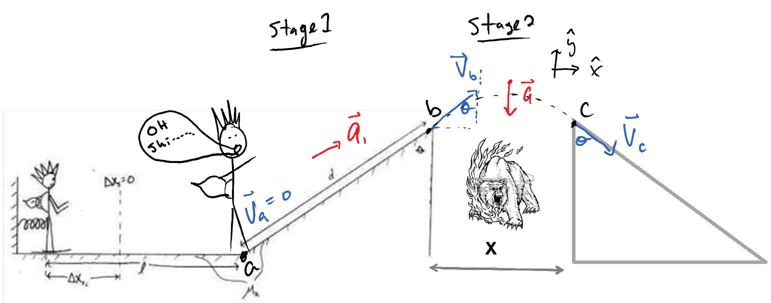
$$mg\sin\theta - \mu_k(mg\cos\theta) = ma_x$$

$$a_x = g(\sin\theta - \mu_k\cos\theta) \quad \ldots \text{ if equil. } a_x = 0$$

$$\sin\theta - \mu_k\cos\theta = 0$$

thus $\underline{\mu_k = \tan\theta}$

**Example: Our stuntman wishes to reach the top of an incline of length d and angle θ with the vertical, and still have enough speed to jump a gap of length X. Our stuntman will employ the aid of a jet pack that pushes on him, parallel to the surface he resides on. He starts at the bottom of the hill (ignore the spring and the flat ground) from rest and must overcome the friction (coefficient $\mu_k$) between the ramp and his skis. What must the thrust from the jetpack be to achieve this great stunt if his jetpack shuts off at the top of the ramp?**




Stage 2

FBD


$$\Sigma F_y \Rightarrow -\cancel{m}g = \cancel{m}a_y$$

$$a_y = -g$$

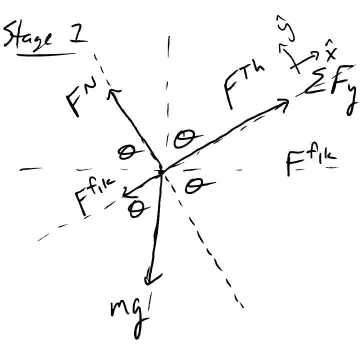
Kinematics $\cancel{\Delta y}^{0} = \overbrace{V_b\cos\theta}^{V_{by}}\Delta t - \frac{g}{2}\Delta t^2$

… rearrange $\Delta t = \frac{2V_b\cos\theta}{g}$ ; $\frac{gX}{2\cos\theta\sin\theta}$

$$\Delta X = X = V_b\sin\theta\,\Delta t + \frac{1}{2}\cancel{g_x}^{0}\Delta t^2$$

$\left(\underline{\qquad}\right)^{1/2}$

Stage 1




$$\Sigma F_y \Rightarrow F^N = mg\sin\theta$$

$F^{fik}$

$a_y = 0$

$$\Sigma F_x \Rightarrow F^{Th} - \mu_k\overbrace{(mg\sin\theta)} - mg\cos\theta = ma_x$$

$$a_x = \frac{F^{Th}}{m} - g(\mu_k\sin\theta + \cos\theta)$$

Kinematics $V_1^2 = \cancel{V}^{2\,0} + 2a_x\Delta x$ ← d

Combine & Solve for $F^{Th}$

$$F^{Th} = \left[\frac{gX}{\qquad} + g(\mu_k\sin\theta + \cos\theta)\right]m$$

Kinematics $V_b^2 = \cancel{V_a^2}^{\,0} + 2a_x \Delta x \leftarrow d$

$V_b^2 = 2d\left(\frac{F^{Th}}{m} - g(\mu_k \sin\theta + \cos\theta)\right)$

$F^{Th} = \left[\frac{g x}{4d\cos\theta\sin\theta} + g(\mu_k \sin\theta + \cos\theta)\right]m$