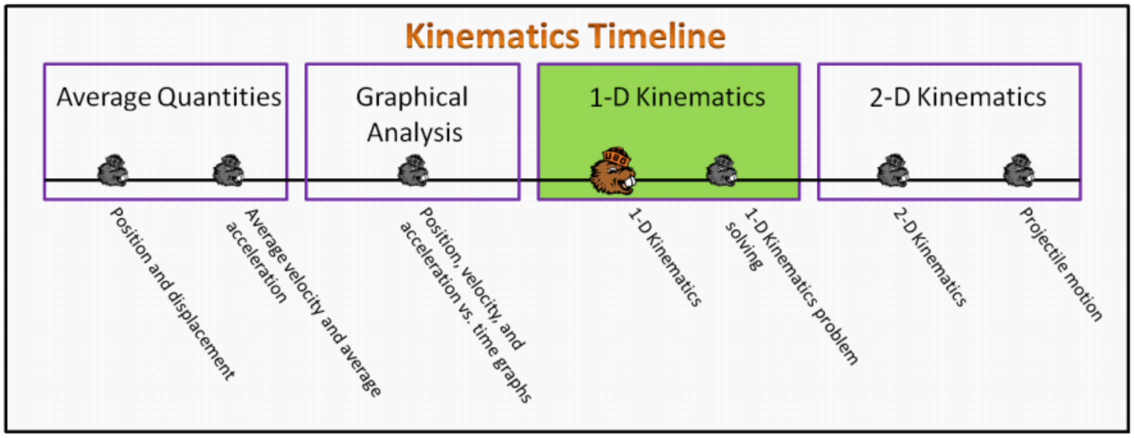
Kinematics: 1-D

Monday, January 22, 2018 5:44 PM

1-D Kinematics
Foundation Stage (K1.2)

lecture 1
1-D Kinematics



Textbook Chapters

- **BoxSand** :: KC videos ([1D Kinematics])
- **Giancoli** (Physics Principles with Applications 7^{th}) :: 2-5 ; 2-6 ; 2-7
- **Knight** (College Physics : A strategic approach 3^{rd}) :: 2.5 ; 2.6 ; 2.7
- **Knight** (Physics for Scientists and Engineers 4^{th}) :: 2.1 ; 2.4

Warm up

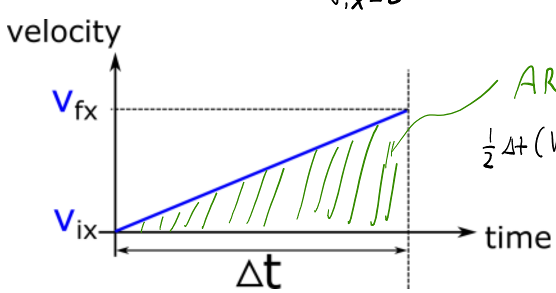
K1.2-1:

Description: Given a position vs time graph, calculate displacement.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: Wally the Walrus's velocity as a function of time is plotted below as he swims along a straight path. It takes Wally 1 second to start from rest and speed up uniformly to a final velocity of 4 mph (1.79 m/s). During this time, how far does Wally travel?

Δt $V_{ix}=0$ V_{fx}



AREA $= \Delta X$

$\frac{1}{2}\Delta t\,(V_{fx}-V_{ix}) = \Delta X$

$\frac{1}{2}(1\,sec)(1.79\,m/s - 0\,m/s) = \Delta X$

$\boxed{\Delta X = 0.895\ m}$

AREA $\begin{cases} X(t) \\ V_x(t) \end{cases}$

Selected Learning Objectives

1. Identify that the motion all occurs along a line and can be treated with a 1-dimensional analysis.
2. Define free-fall and identify when a free-fall analysis is appropriate.
3. Denote that objects speeding up have an acceleration that points in the same direction as their velocity, while those slowing down have an acceleration that points in the opposite direction of their velocity.
4. Translate a descriptive representation to an appropriate physical representation that includes a displacement vector, initial and final velocity vectors, an acceleration vector, and a coordinate system.

5. Draw an appropriate physical representation for a system that includes multiple stages or objects by including vector representations for each.
6. Identify known and unknown quantities for each stage or object.
7. Translate from the mathematical, physical, or descriptive representation to the graphical representation.
8. Translate to the mathematical representation with the help of the descriptive, physical, and graphical representations.
9. Identify the appropriate kinematic equation for constant acceleration to use when analyzing the problem.
10. Use one of the kinematic equations to find the value of an unknown, then use that value and another kinematic equation to solve for desired unknown.
11. Solve simultaneous equations when there are two or more equations with the same two unknowns.
12. Solve problems that involve multiple objects or multiple stages.
13. Apply any connections between stages or objects when appropriate, e.g. the geometric connection between two runners when they do not start at the same location.
14. Apply sign sense-making procedures to check their solutions.
15. Apply order of magnitude sense-making procedures to check their solutions.
16. Apply dimensional analysis sense-making procedures to check their solutions.

Key Terms

- Acceleration
- Deceleration
- Physical representation
- Knowns and Unknowns
- Kinematic equations
- Trajectory

Key Equations

Change in x-component of position

x-component of acceleration

Initial x-component of velocity

$$\Delta x = v_{ix}\,\Delta t + \frac{1}{2} a_x \Delta t^2$$

Change in time

In words: The change in the x-component of position is equal to the initial x-compoent of velocity multiplied by the change in time plus one-half of the x-component of acceleration multiplied by the change in time squared.

Final x-component of velocity

x-component of acceleration

Initial x-component of velocity

Change in time

$$v_{fx} = v_{ix} + a_x \Delta t$$

In words: The final x-compoent of velocity is equal to the initial x-component of velocity plus the x-component of acceleration multiplied by the change in time.

Final x-component of velocity

x-component of acceleration

Initial x-component of velocity

Change in x-component of position

$$v_{fx}^2 = v_{ix}^2 + 2\,a_x \Delta x$$

In words: The final x-compoent of velocity squared is equal to the initial x-compoent of velocity squared plus two times the x-component of acceleration multiplied by the change in the x-component of position.

Key Concepts

- Acceleration is a physical quantity, and deceleration is not a physical quantity.
- A physical representation should include information about displacement, initial velocity, final velocity, the constant acceleration between two locations, the time interval between the two locations, and a coordinate system.
- Known and unknown lists help organize kinematic information as well as your thoughts.
- It is highly recommended to not attempt to do algebra (i.e. re-arrange kinematic equations and/or plug them into each other) until you have identified the same number of equations as you have unknowns.

Act I: 1-D Kinematics

Questions

K1.2-2:

Description: Connect acceleration to change in velocity. (2 minutes)

Learning Objectives: [3]

Problem Statement: Which of following situations is a car accelerating?

(1) Moving in the positive direction and speeding up.
(2) Moving in the positive direction and slowing down.
(3) Moving in the negative direction and speeding up.
(4) Moving in the negative direction and slowing down.

K1.2-3:

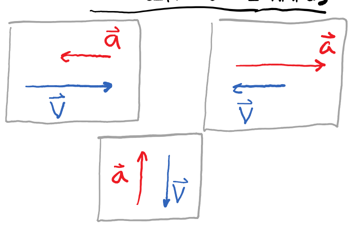
Description: Differentiate between acceleration and deceleration. (4 minutes)

Learning Objectives: [3]

Problem Statement: Which of the following situations is a car decelerating?

(1) Moving in the positive direction and accelerating in the negative direction.
(2) Moving in the positive direction and accelerating in the positive direction.
(3) Moving in the negative direction and accelerating in the negative direction.
(4) Moving in the negative direction and accelerating in the positive direction.
(5) Moving in the positive direction and slowing down.
(6) Moving in the negative direction and speeding up.

DECELERATION EXAMPLES



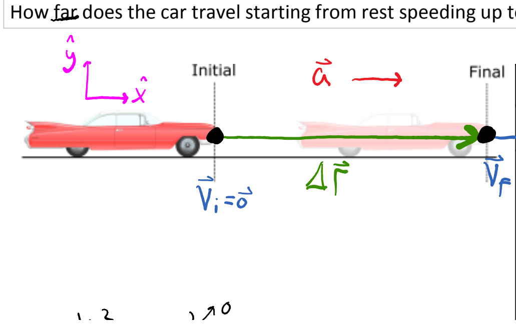
NOT A PHYSICAL QUANTITY UNLIKE ACCELERATION
DECELERATION REFERS TO SITUATIONS WHERE THE ACCELERATION POINTS IN THE OPPOSITE DIRECTION OF VELOCITY. THIS MEANS THAT THE OBJECT IS SLOWING DOWN

K1.2-4:

Description: 1-D kinematics problem solving for displacement. (6 minutes)

Learning Objectives: [6, 8, 9]

Problem Statement: A car starts from rest and accelerates at a constant rate of 10.70 m/s² until it reaches 60.0 mph (26.82 m/s). How far does the car travel starting from rest speeding up to 60 mph?



Δx v_{ix} v_{fx} a_x Δt	
K	UK
$V_{ix} = 0$	Δx
$V_{fx} = 26.82\ m/s$	Δt
$a = 10.7$ ...	

$$\Delta x = v_{ix}\Delta t + \tfrac{1}{2} a_x \Delta t^2$$

$$v_{fx} = v_{ix} + a_x \Delta t$$

2 EQNS
2 UNKNS

$$v_{fx}^2 = v_{ix}^2 + 2 a_x \Delta x$$

1 EQN
1 UNKN

$V_{fx} = 26.82\ m/s$ $a_x = 10.7\ m/s^2$	Δt

$v_{fx}^2 = v_{ix}^2 + 2\,a_x\,\Delta x$ } 1 EQN, 1 UNKNOWN

$$V_{fx}^2 = \cancel{V_{ix}^2}^{\,0} + 2a_x\Delta x$$

$$V_{fx}^2 = 2a_x\Delta x$$

$$\Delta x = \frac{V_{fx}^2}{2a_x} = \frac{(26.82)^2}{2(10.7)}\,m \approx \boxed{33.6\ m}$$

K1.2-5:

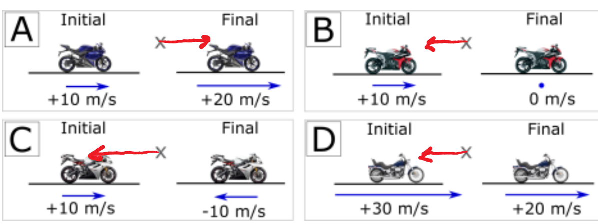
Description: Connect acceleration with change in velocity. (7 minutes)

Learning Objectives: [3]

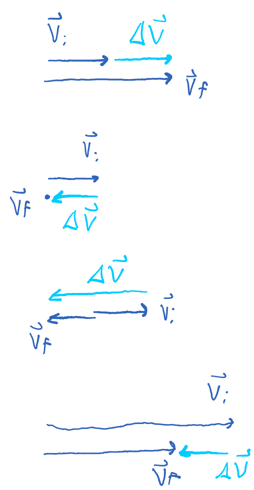
Problem Statement: In each figure below, a motorcycle's velocity is shown at some initial instant of time and again at some later time which is labeled as final.

(a) Rank the magnitude of the change in velocity during the time interval.

$$|\Delta V_C| > |\Delta V_A| = |\Delta V_B| = |\Delta V_D|$$



Math	Vector-op
$\Delta V = V_{fx} - V_{ix}$	TAIL-TO-TAIL $\Delta\vec{V}$ POINTS FROM I TO F
A: $= 20 - 10$ $\Delta V_A = 10\ m/s$	
B: $= 0 - 10$ $\Delta V_B = -10\ m/s$	
C: $= -10 - 10$ $\Delta V_C = -20\ m/s$	
D: $= 20 - 30$ $\Delta V_D = -10\ m/s$	



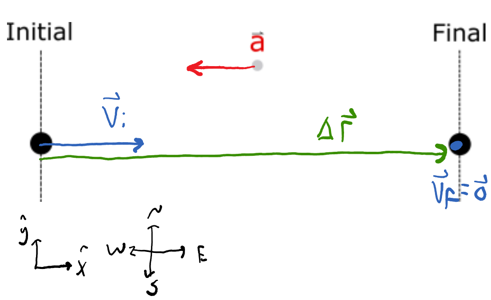
(b) Sketch a vector to represent the acceleration of the motorcycle on the spots marked by an x.

K1.2-6:

Description: 1-D kinematics problem solving for displacement. (9 minutes)

Learning Objectives: [4, 5, 6, 8, 9, 10]

Problem Statement: An airplane lands traveling due east. A passenger used their cell phone to record the time it took to stop which was about 45 seconds, and the magnitude of acceleration which was about 2 m/s². How far did the plane travel from touch-down to a complete stop. This plane's motion is all along a straight line.



Δx v_{ix} v_{fx} a_x Δt	
K	UK
$V_{fx} = 0$ $a_x = -2\ m/s^2$ $\Delta t = 45\ s$	Δx V_{ix}

$\Delta x = v_{ix}\,\Delta t + \frac{1}{2}\,a_x\,\Delta t^2$ } 2 EQNS, 2 UNKNS

$v_{fx} = v_{ix} + a_x\,\Delta t$

$v_{fx}^2 = v_{ix}^2 + 2\,a_x\,\Delta x$ } 2 EQNS, 2 UNKN

$$V_{fx}^{\,0} = V_{ix} + a_x\Delta t$$

$\Delta t = 45\ s$	

$$\cancel{V_{fx}}^{\,0} = V_{ix} + a_x \Delta t$$

$$0 = V_{ix} + a_x \Delta t$$

$$V_{ix} = -a_x \Delta t$$

$$V_{ix} = -(-2)(45)\ m/s$$

$$V_{ix} = 90\ m/s$$

$$\Delta X = V_{ix}\Delta t + \tfrac{1}{2} a_x \Delta t^2 = (90\ m/s)(45\ s) + \tfrac{1}{2}(-2\ m/s^2)(45\ s)^2 \approx \boxed{2025\ m}$$

$$\Delta X = -a_x \Delta t^2 + \tfrac{1}{2} a_x \Delta t^2$$

$$\Delta X = -\tfrac{1}{2} a_x \Delta t^2$$

$$\Delta X = -\tfrac{1}{2}(-2\ m/s^2)(45\ s)^2 \approx \boxed{2025\ m}$$

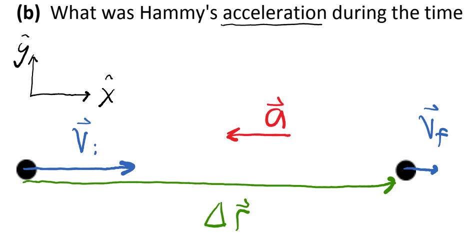
K1.2-7:

Description: 1-D kinematics problem solving for initial velocity and acceleration. (10 minutes)

Learning Objectives: [4, 5, 6, 8, 9, 11]

Problem Statement: Hammy the Hamster is at full sprint when all of a sudden he spots a cat in front of him. Within 0.10 seconds and 1 cm Hammy has uniformly reduced his speed to 0.50 cm/s, and is about to turn around and run the other way.

(a) How fast was Hammy running initially when he saw the cat?
(b) What was Hammy's acceleration during the time he uniformly slowed down?



Δx v_{ix} v_{fx} a_x Δt	
K	UK
$\Delta X = 1\ cm$	V_{ix}
$V_{fx} = 0.5\ cm/s$	a_x
$\Delta t = 0.1\ s$	

$$\left.\begin{aligned} \Delta x &= v_{ix}\Delta t + \tfrac{1}{2} a_x \Delta t^2 \\ v_{fx} &= v_{ix} + a_x \Delta t \end{aligned}\right\} \text{2 Eqns, 2 unknowns}$$

$$v_{fx}^2 = v_{ix}^2 + 2\, a_x \Delta x$$

a)

$$\Delta X = V_{ix}\Delta t + \tfrac{1}{2} a_x \Delta t^2 \qquad V_{fx} = V_{ix} + a_x \Delta t$$

$$\frac{V_{fx} - V_{ix}}{\Delta t} = a_x$$

$$\Delta X = V_{ix}\Delta t + \tfrac{1}{2}\left(\frac{V_{fx} - V_{ix}}{\Delta t}\right)\Delta t^2$$

$$\Delta X = V_{ix}\Delta t + \tfrac{1}{2} V_{fx}\Delta t - \tfrac{1}{2} V_{ix}\Delta t$$

$$\Delta X - \tfrac{1}{2} V_{fx}\Delta t = V_{ix}\Delta t - \tfrac{1}{2} V_{ix}\Delta t$$

$$\Delta X - \tfrac{1}{2} V_{fx}\Delta t = \tfrac{1}{2} V_{ix}\Delta t$$

$$V_{ix} = \frac{2(\Delta X - \tfrac{1}{2} V_{fx}\Delta t)}{\Delta t}$$

$$V_{ix} = \frac{2\Delta X}{\Delta t} - V_{fx} = \frac{2(1\ cm)}{0.1\ s} - (0.5)\ cm/s = \boxed{19.5\ cm/s}$$

b)

$$a_x = \frac{V_{fx} - V_{ix}}{\Delta t} = \frac{0.5\ cm/s - 19.5\ cm/s}{0.1\ s} \approx -190\ cm/s^2$$

$$\boxed{\vec{a} = \langle -190, 0 \rangle\ cm/s^2}$$

K1.2-8:

Description: Multiple select true or false statements connecting acceleration to change in velocity. (1 minute + 2 minutes + 2 minutes + 2 minutes + 3 minutes + 3 minutes + 3 minutes)

Learning Objectives: [3]

Problem Statement: Below are 6 conceptual questions. Determine whether each statement is true or false.

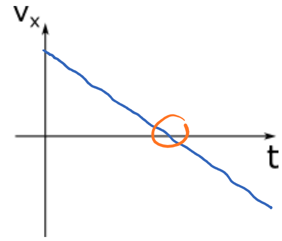
F (1) True or False: An object at rest can have a constant non-zero acceleration and stay stopped. If true, come up with an example, if false, explain your reasoning. NON-ZERO ACCEL MEANS $\frac{\Delta V}{\Delta t} \neq 0$ SO $\Delta V \neq 0$ ... V MUST CHANGE

T (2) True or False: An object can have a non-zero velocity and its acceleration be zero. If true, come up with an example, if false, explain your reasoning. CAR @ CONSTANT 60 MPH, NORTH

T (3) True or False: An object can have zero velocity and non-zero acceleration at the same time. If true, come up with an example, if false, explain your reasoning. TOSS A BALL VERTICALLY UPWARDS. @ MAX HEIGHT $\vec{V} = \vec{0}$ BUT $\vec{a} \neq \vec{0}$

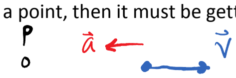
(4) Sketch on the velocity vs time plot a situation where an object is momentarily at rest but it's still accelerating.

$V_x(t)$ SLOPE $a_x(t)$



T (5) True or False: An object can be increasing in speed as it's acceleration is decreasing. If true, come up with an example, if false explain your reasoning. INITIALLY PRESS GAS PEDAL TO THE FLOOR THEN SLOWLY LET PEDAL COME BACK OUT.

F (6) True or False: If an object has an acceleration toward a point, then it must be getting closer and closer to that point.



OBJ MOVING AWAY FROM P BUT IS SLOWING DOWN

Conceptual questions for discussion

1. Can you think of a scenario in your life where drawing a physical representation is useful? Make a sketch of this physical representation. Remember that a physical representation includes a basic picture of key objects and labeled quantities. This does not have to be a formal kinematics physical representation or even anything physics related.
2. Do you agree with the following statement? *The direction of acceleration that an object has dictates the direction of motion of that object.* Provide examples that support your thoughts.

Hints

K1.2-1: Find an expression for the area under the curve given in terms of the variables labeled on the graph. Also recall that area under velocity vs time is equal to Δx. Now plug in your known quantities.

K1.2-2: No hints.

K1.2-3: No hints.

K1.2-4: Fill out the knowns and unknown lists first, then identify which kinematic equation(s) is must useful.

K1.2-5: What is the mathematical definition of average velocity?.

K1.2-6: Fill out the knowns and unknown lists first, then identify which kinematic equation(s) is must useful.

K1.2-7: Fill out the knowns and unknown lists first, then identify which kinematic equation(s) is must useful. Recall that if you have 2 equations and 2 unknowns there are multiple ways to solve for each unknown.

K1.2-8: No hints.