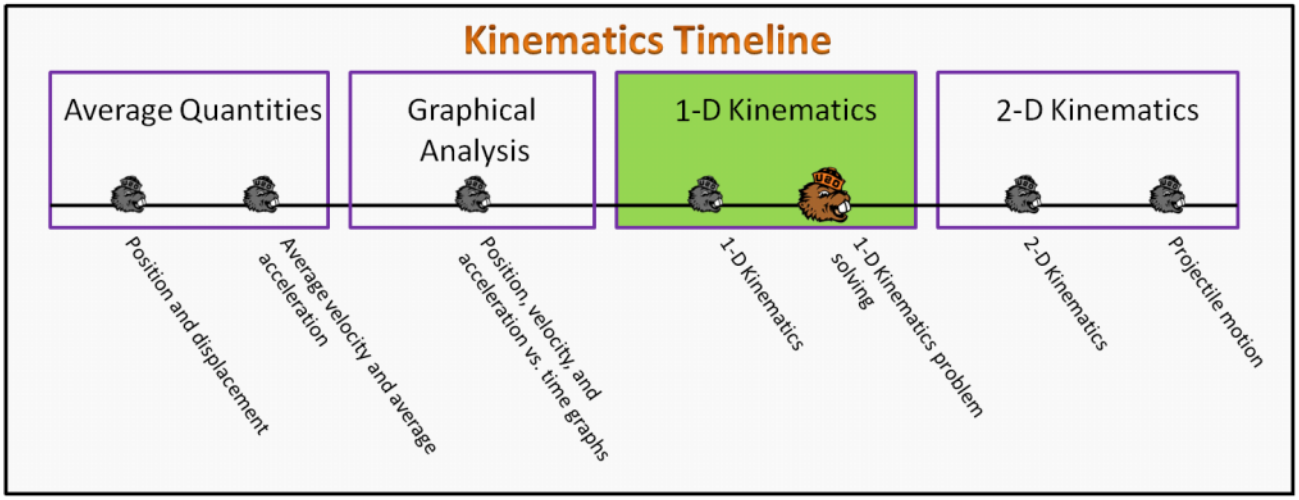
Kinematics: 1-D Problem Solving

Monday, January 22, 2018 5:44 PM

1-D Kinematics
Foundation Stage (K1.2)

lecture 2
1-D Kinematics problem solving



Textbook Chapters

- **BoxSand** :: KC videos (1D Kinematics)
- **Giancoli** (Physics Principles with Applications 7th) :: 2-5 ; 2-6 ; 2-7
- **Knight** (College Physics : A strategic approach 3rd) :: 2.5 ; 2.6 ; 2.7
- **Knight** (Physics for Scientists and Engineers 4th) :: 2.5

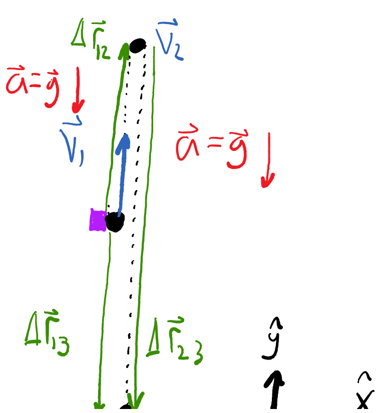
Warm up

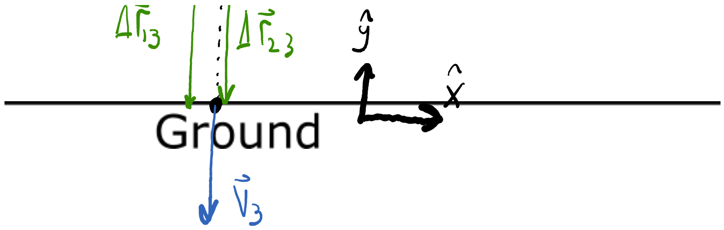
K1.2-1:

Description: Draw a physical representation for a 1-D kinematic problem.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: A helicopter is traveling upwards at a constant speed of 5.00 m/s. When the helicopter is 15.0 m above the level ground, and still moving upwards, a person releases a ball outside the window of the helicopter. Draw a physical representat ion for the ball if you were trying to find the total time of flight of the ball after it was released.





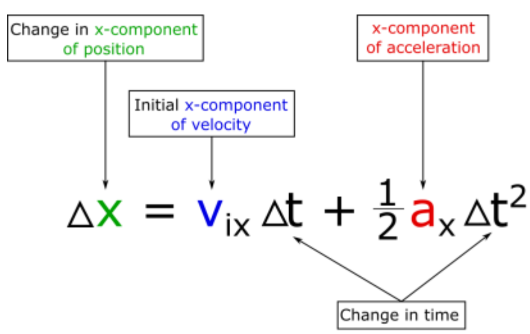
Selected Learning Objectives

1. Identify that the motion all occurs along a line and can be treated with a 1-dimensional analysis.
2. Define free-fall and identify when a free-fall analysis is appropriate.
3. Denote that objects speeding up have an acceleration that points in the same direction as their velocity, while those slowing down have an acceleration that points in the opposite direction of their velocity.
4. Translate a descriptive representation to an appropriate physical representation that includes a displacement vector, initial and final velocity vectors, an acceleration vector, and a coordinate system.
5. Draw an appropriate physical representation for a system that includes multiple stages or objects by including vector representations for each.
6. Identify known and unknown quantities for each stage or object.
7. Translate from the mathematical, physical, or descriptive representation to the graphical representation.
8. Translate to the mathematical representation with the help of the descriptive, physical, and graphical representations.
9. Identify the appropriate kinematic equation for constant acceleration to use when analyzing the problem.
10. Use one of the kinematic equations to find the value of an unknown, then use that value and another kinematic equation to sol ve for desired unknown.
11. Solve simultaneous equations when there are two or more equations with the same two unknowns.
12. Solve problems that involve multiple objects or multiple stages.
13. Apply any connections between stages or objects when appropriate, e.g. the geometric connection between two runners when they do not start at the same location.
14. Apply sign sense-making procedures to check their solutions.
15. Apply order of magnitude sense-making procedures to check their solutions.
16. Apply dimensional analysis sense-making procedures to check their solutions.

Key Terms

- Free-fall
- Multi-stage problem
- Connections
- Trajectory

Key Equations



In words: The change in the x-component of position is equal to the initial x-compoent of velocity multiplied by the change in time plus one-half of the x-component of acceleration multiplied by the change in time squared.

Final x-component of velocity — Initial x-component of velocity — x-component of acceleration — Change in time

$$v_{fx} = v_{ix} + a_x \Delta t$$

In words: The final x-compoent of velocity is equal to the initial x-component of velocity plus the x-component of acceleration multiplied by the change in time.

Final x-component of velocity — Initial x-component of velocity — x-component of acceleration — Change in x-component of position

$$v_{fx}^2 = v_{ix}^2 + 2\, a_x \Delta x$$

In words: The final x-component of velocity squared is equal to the initial x-component of velocity squared plus two times the x-component of acceleration multiplied by the change in the x-component of position.

Key Concepts

- Choosing a non-standard coordinate system can simplify any algebra involved in solving kinematic problems.
- Free-fall is defined as an object that only moves along the vertical direction and has an acceleration equal to the acceleration d ue to gravity.
- Kinematic problems require two "snap-shots" in time, one at the initial position and one at the final position. However, sometimes there are multiple stages which involved choosing more than 1 set of initial and final positions.
- When there are multiple stages, or multiple objects, there will be some quantity(s) that connect the stages, or object, toget her.

Act I: 1-D Kinematics

Questions

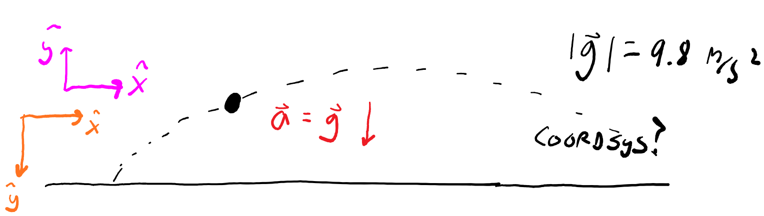
K1.2-2:

Description: Definition of acceleration for projectile-motion. (2 minutes)

Learning Objectives: [6]

Problem Statement: A ball is kicked into the air. What is the acceleration of the ball during its flight through the air?

(1) -9.80 m/s^2 $\hat{y}$
(2) 9.80 m/s^2 $\hat{y}$
(3) $-g\,\hat{y}$
(4) $g\,\hat{y}$
(5) Unable to determine



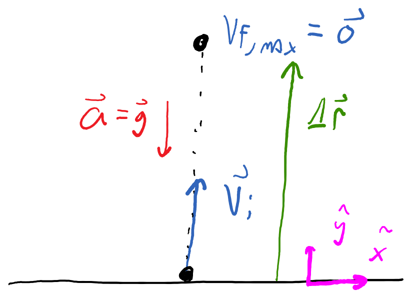
K1.2-3:

Description: Identify important quantities at the max height for free-fall problems. (2 minutes)

Learning Objectives: [1, 2, 6]

Problem Statement: A ball is kicked straight upwards in the positive direction. Which of the following statements are true regarding the instant the ball reaches its maximum height?

T (1) The velocity of the ball is zero.
F (2) The velocity of the ball points downwards.
F (3) The acceleration of the ball is zero.
T (4) The acceleration of the ball is -g.
F (5) The acceleration of the ball is g.



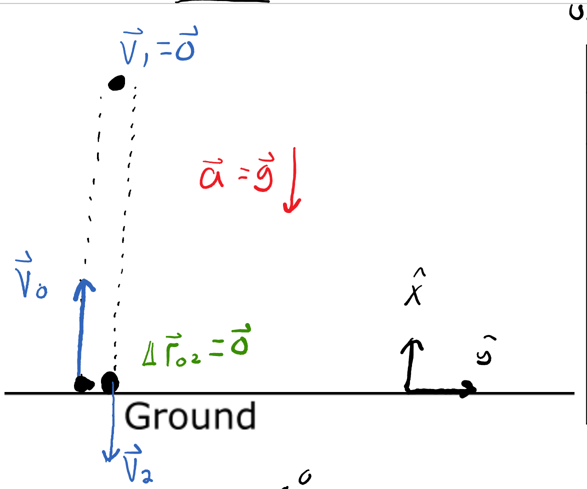
K1.2-4:

Description: Free-fall problem solving for time. (5 minutes)

Learning Objectives: [1, 2, 4, 6, 8, 9]

Problem Statement: A ball is kicked vertically upwards with an initial velocity of 15 m/s. How long does it take for the ball to come back down to the same level?

USE SNAPSHOTS 0 + 2 AS i AND f



Δx v_{ix} v_{fx} a_x Δt	
K	UK
$\Delta x = 0$	v_{fx}
$v_{ix} = 15\ m/s$	Δt
$a_x = -9.8\ m/s^2$	

$\Delta x = v_{ix}\Delta t + \frac{1}{2}a_x\Delta t^2$

$v_{fx} = v_{ix} + a_x\Delta t$

$v_{fx}^2 = v_{ix}^2 + 2\,a_x\Delta x$

$$\cancel{\Delta x}^{\,0} = V_{ix}\Delta t + \frac{1}{2}a_x\Delta t^2$$

$$0 = V_{ix}\Delta t - \frac{1}{2}g\,\Delta t^2$$

$$0 = V_{ix} - \frac{1}{2}g\,\Delta t$$

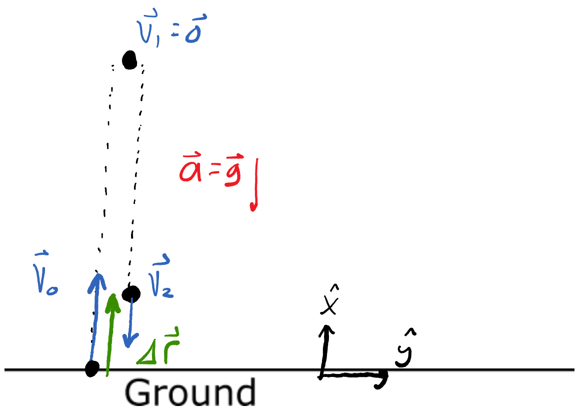
$$\Delta t = \frac{2V_{ix}}{g} = \frac{2(15\ m/s)}{9.8} \approx \boxed{3.06\ \text{SECONDS}}$$

K1.2-5:

Description: Free-fall kinematics problem solving for time. (6 minutes)

Learning Objectives: [1, 2, 4, 6, 8, 9, 15]

Problem Statement: A ball is kicked vertically upwards with an initial velocity of 15 m/s. How long does it take for the ball to come back down to 2 meters above where it was kicked?



Δx	v_{ix}	v_{fx}	a_x	Δt
K			UK	
$\Delta x = 2\,m$ $v_{ix} = 15\,m/s$ $a_x = -9.8\,m/s^2$			v_{fx} Δt	

$\Delta x = v_{ix}\Delta t + \frac{1}{2}a_x\Delta t^2$

$v_{fx} = v_{ix} + a_x\Delta t$

$v_{fx}^2 = v_{ix}^2 + 2\,a_x\Delta x$

$$\Delta x = v_{ix}\Delta t + \frac{1}{2}a_x\Delta t^2$$

$$\Delta x = v_{ix}\Delta t - \frac{1}{2}g\Delta t^2$$

$$\underbrace{-\frac{1}{2}g}_{a}\Delta t^2 + \underbrace{v_{ix}}_{b}\Delta t \underbrace{-\Delta x}_{c} = 0$$

QUADRATIC $\quad \Delta t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$-4.9\,\Delta t^2 + 15\,\Delta t - 2 = 0$$

$\Delta t = 0.140\ s$ OR $\boxed{2.92\ s}$

ON THE WAY UP

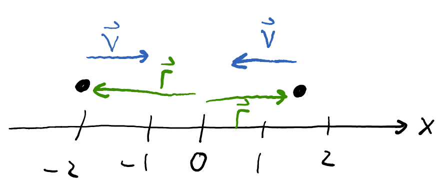
K1.2-6:

Description: Given information about position and velocity, determine what is known about the other kinematic quantities. (3 minutes)

Learning Objectives: [3, 4]

Problem Statement: An object moving along a straight line has it's velocity pointing in the opposite direction of its position. Which one of the following statements concerning the object is *necessarily* true?

? (1) The value of the acceleration is negative.
? (2) The direction of the acceleration is in the opposite direction as the displacement.
? (3) The direction of the acceleration is in the opposite direction of the velocity.
T (4) The object is moving towards the origin.
? (5) The object is slowing down.



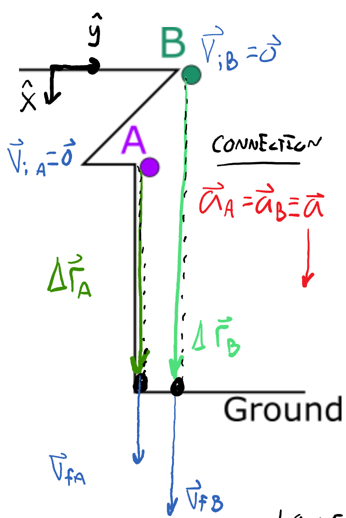
K1.2-7:

Description: Free-fall problem with two object solving for final velocity. (7 minutes + 3 minutes)

Learning Objectives: [1, ~~2, 5, 6, 8, 9, 10,~~ 12, 13]

Problem Statement: A ball is dropped from a high tower on a distant planet where the acceleration of gravity is unknown. In one case, it is noticed that it takes 3 seconds to hit the ground, traveling at 88.2 m/s upon impact. In another case, the ball is dropped from a higher height and it is noticed it takes 6 seconds to hit the ground.

(a) How fast is the ball traveling upon impact in the second case?



Object A	
Δx_A $\quad v_{iAx}$ $\quad v_{fAx}$ $\quad a_{Ax}$ $\quad \Delta t_A$	
K	UK
$V_{iAx} = 0$	Δx_A
$V_{fAx} = 88.2$ m/s	a_x
$\Delta t_A = 3$ s	

Object B	
Δx_B $\quad v_{iBx}$ $\quad v_{fBx}$ $\quad a_{Bx}$ $\quad \Delta t_B$	
K	UK
$V_{iBx} = 0$	Δx_B
$\Delta t_B = 6$ s	V_{fBx}
	a_x

?? ✓ ✓✓ ?

$\Delta x = v_{ix}\,\Delta t + \frac{1}{2} a_x \Delta t^2$

✓? ✓ ? ✓✓

$v_{fx} = v_{ix} + a_x \Delta t$

$v_{fx}^2 = v_{ix}^2 + 2\, a_x \Delta x$

LONG

$V_{FAx} = \cancel{V_{iAx}}^{\,0} + a_x \Delta t_A$

$a_x = \dfrac{V_{FAx}}{\Delta t_A}$ $\quad$ (29.4 m/s^2)

$V_{FBx} = \cancel{V_{iBx}}^{\,0} + a_x \Delta t_B$

$V_{FBx} = a_x \Delta t_B$

$V_{FBx} = \dfrac{V_{FAx}\,\Delta t_B}{\Delta t_A}$

$V_{FBx} = (88.2\text{ m/s})\,\dfrac{6}{3}$

$V_{FBx} = 2\,(88.2\text{ m/s})$

$\boxed{V_{FBx} = 176\text{ m/s}}$

QUICK

FOR BOTH A & B $\quad V_{Fx} = \cancel{V_{ix}}^{\,0} + a_x \Delta t$

$V_{Fx} = a_x \Delta t$

w/ a_x = CONST

$V_{Fx} \propto \Delta t$

IF $\Delta t \rightarrow 2\Delta t$

$V_{Fx} \rightarrow 2\,V_{Fx}$

(b) How much further did ball **B** travel compared to ball **A**?

(1) 1.41 times
(2) 2 times
(3) 3.2 times
(4) 3 times
(5) 4 times
(6) 4.8 times

LOOK @ $\Delta X = \cancel{V_{ix}}^{\,0}\Delta t + \frac{1}{2} a_x \Delta t^2$

$V_{ix} = 0$ FOR A & B

$\Delta X = \frac{1}{2} a_x \Delta t^2$

w/ a_x = CONST

$w/\ a_x = \text{const}$

$\Delta x \propto \Delta t^2$

If $\Delta t \rightarrow 2\Delta t$

$\Delta x \rightarrow 4\Delta x$

$\Delta x_B = 4\Delta x_A$

K1.2-8:

Description: Free-fall problem with two object comparing final speed. (5 minutes)

Learning Objectives: [1, 2, 5, 6, 8, 12, 13]

Problem Statement: A person standing at the edge of a cliff throws one ball straight up and another ball straight down at the same initial speed. Neglecting air resistance, which ball hits the ground below the cliff with a greater speed?

(1) The ball that was thrown up.
(2) The ball that was thrown down.
(3) Both hit the ground with the same speed.

CONNECTIONS

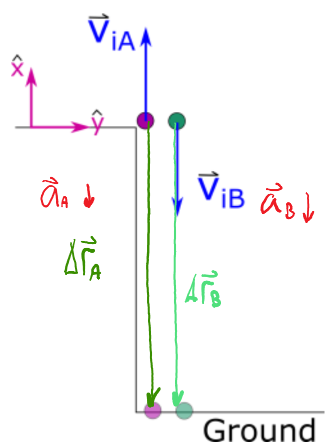
$\vec{a}_A = \vec{a}_B = \vec{g}$

$\Delta\vec{r}_A = \Delta\vec{r}_B$

$V_{iAx} = -V_{iBx}$

$* \ V_{iAx}^2 = V_{iBx}^2$

$\Delta t_A > \Delta t_B$



$\Delta x = V_{ix}\Delta t + \frac{1}{2}a_x\Delta t^2$

$V_{fx} = V_{ix} + a_x\Delta t$

$V_{fx}^2 = \underbrace{V_{ix}^2 + 2a_x\Delta x}_{\text{SAME}}$

SO $V_{fx} = \text{SAME}$

K1.2-9:

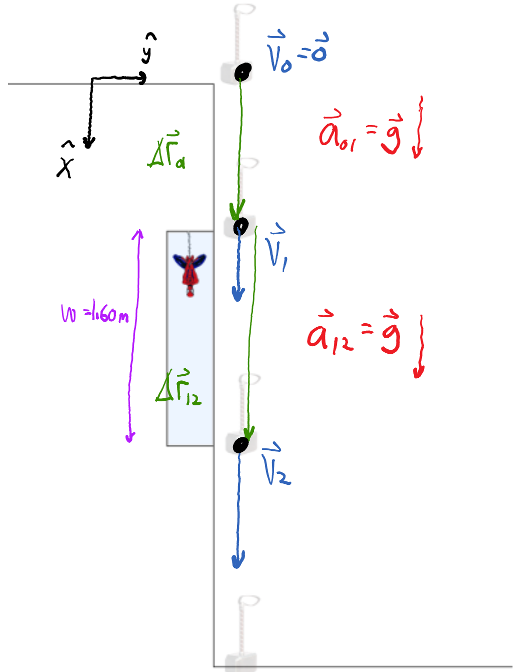
Description: Free-fall problem with one object and two stages solving for a distance. (10 minutes)

Learning Objectives: [1, 2, 5, 6, 8, 9, 10, 12, 13,]

Problem Statement: Thor's hammer falls from rest from the top of an unworthy building. Spiderman notices that it takes 0.20 s for the hammer to pass by his 1.60-m-tall window. Eventually we wish to determine how far above the top of this window is the roof of the building.

(a) This is a two stage problem, which of the following positions represent the beginning of the first stage?

(1) Above the top of the building.
(2) The top of the building.
(3) The top of the window.
(4) The bottom of the window.
(5) The ground.



(b) Which of the following positions represent the beginning of the second stage?

(1) Above the top of the building.
(2) The top of the building.
(3) The top of the window.
(4) The bottom of the window.
(5) The ground.

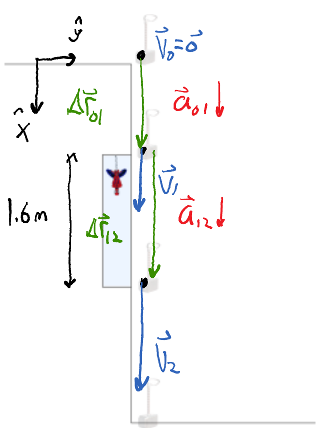
(c) Which of the following positions represent the end of the second stage?

(1) Above the top of the building.
(2) The top of the building.
(3) The top of the window.
(4) The bottom of the window.
(5) The ground.

(d) Which coordinate system will make all the quantities positive in this problem?

(1) x-axis pointing up.
(2) x-axis pointing down.
(3) Neither option will work.

(e) Thor's hammer falls from rest from the top of an unworthy building. Spiderman notices that it takes 0.20 s for the hammer to pass by his 1.60-m-tall window. **How far above the top of the window is the roof of the building**?



Stage A ($0 \to 1$)		Stage B ($1 \to 2$)	
Δx_A v_{iAx} v_{fAx} a_{Ax} Δt_A		Δx_B v_{iBx} v_{fBx} a_{Bx} Δt_B	
K	UK	K	UK
$V_0 = 0$	ΔX_{01}	$\Delta X_{12} = 1.6\,m$	V_{1x}
$a_{01} = g$	V_{1x}	$a_{12} = g$	V_{2x}
	Δt_{01}	$\Delta t_{12} = 0.2\,sec$	

$$\Delta x = v_{ix}\,\Delta t + \frac{1}{2} a_x \Delta t^2$$

$$v_{fx} = v_{ix} + a_x \Delta t$$

$$v_{fx}^2 = v_{ix}^2 + 2\,a_x \Delta x$$

CONNECTIONS

$\vec{a}_{01} = \vec{a}_{12} = \vec{g}$

$V_{fAx} = V_{iBx} = V_{1x}$

$$\Delta X_{12} = V_{1x}\,\Delta t_{12} + \frac{1}{2} a_{12x} \Delta t_{12}^2$$

$$1.6\,m = V_{1x}(0.2s) + \frac{1}{2}(9.8\,m/s^2)(0.2s)^2$$

$$V_{1x} = 7.02\,m/s$$

$$V_{1x}^2 = \cancel{V_{0x}^2}^{\,0} + 2 a_{01x} \Delta X_{01}$$

$$(7.02\,m/s)^2 = 2(9.8\,m/s^2)\Delta X_{01}$$

$V_{1x} = 7.02\ m/s$ $(7.02\ m/s)^2 = 2(9.8\ m/s^2)\Delta X_{01}$

$\boxed{\Delta X_{01} = 2.51\ m}$

K1.2-10:

Description: Free-fall problem with two objects solving for how the distance between the object changes as a function of time. (1 minute + 2 minutes + 2 minutes + 2 minutes + 2 minutes)

Learning Objectives: [1, 2, 5, 6, 8, 9, 12, 13]

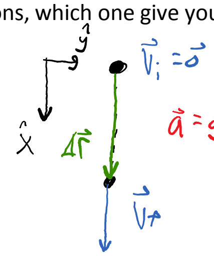
Problem Statement: Two balls are released from rest and undergo free-fall. Ball **A** is released first, followed by ball **B** after $\mathbf{T_0}$ seconds later. We've learned that the distance between the two increases through a graphical analysis. Let's use the mathematical representation to learn more about the nature of how they separate over time.

(a) Given the three kinematic equations, which one give you the change in position, as a function of time only?

(1) $\Delta \mathbf{x} = \mathbf{v}_{ix}\, \Delta \mathbf{t} + \frac{1}{2}\, \mathbf{a}_x\, \Delta \mathbf{t}^2$

(2) $\mathbf{v}_{fx} = \mathbf{v}_{ix} + \mathbf{a}_x\, \Delta \mathbf{t}$

(3) $\mathbf{v}_{fx}^2 = \mathbf{v}_{ix}^2 + 2\, \mathbf{a}_x\, \Delta \mathbf{x}$



(b) Using a coordinate system pointing downward, which of the following set of equations is a simplification of the answer to the previous question?

$\Delta X = \cancel{V_{ix}\Delta t}^{\,0} + \frac{1}{2} a_x \Delta t^2$

$\Delta X = \frac{1}{2} g \Delta t^2$

(1) $\Delta \mathbf{x}_A = -\frac{1}{2}\, \mathbf{g}\, \Delta \mathbf{t}_A^2$; $\Delta \mathbf{x}_B = -\frac{1}{2}\, \mathbf{g}\, \Delta \mathbf{t}_B^2$

(2) $\Delta \mathbf{x}_A = \frac{1}{2}\, \mathbf{g}\, \Delta \mathbf{t}_A^2$; $\Delta \mathbf{x}_B = \frac{1}{2}\, \mathbf{g}\, \Delta \mathbf{t}_B^2$

(3) $\Delta \mathbf{x}_A = \mathbf{v}_{ixA}\, \Delta \mathbf{t}_A - \frac{1}{2}\, \mathbf{g}\, \Delta \mathbf{t}_A^2$; $\Delta \mathbf{x}_B = \mathbf{v}_{ixB}\, \Delta \mathbf{t}_B - \frac{1}{2}\, \mathbf{g}\, \Delta \mathbf{t}_B^2$

(4) $\Delta \mathbf{x}_A = -\frac{1}{2}\, \mathbf{g}\, \Delta \mathbf{t}^2$; $\Delta \mathbf{x}_B = -\frac{1}{2}\, \mathbf{g}\, \Delta \mathbf{t}^2$

(c) Which of the following equations connects the elapsed time for ball **A** to that for ball **B**?

CLOCK B STARTS AFTER CLOCK A

SO $\Delta t_A = \Delta t_B + T_0$

(1) $\Delta \mathbf{t}_B = \Delta \mathbf{t}_A$

(2) $\Delta \mathbf{t}_B = \Delta \mathbf{t}_A + \mathbf{T}_0$

(3) $\Delta \mathbf{t}_A = \Delta \mathbf{t}_B + \mathbf{T}_0$

(d) Using the correct set of $\Delta \mathbf{x}_A$ and $\Delta \mathbf{x}_B$ from part (b), which of the following expressions represent $\Delta \mathbf{x}_A - \Delta \mathbf{x}_B$?

$\Delta(\Delta X) = \Delta X_A - \Delta X_B$

(1) $\frac{1}{2}\, \mathbf{g}\, \mathbf{T}_0^2 + \mathbf{g}\, \mathbf{T}_0\, \Delta \mathbf{t}_B$

(1) $\frac{1}{2}\ \mathbf{g}\ \mathbf{T_0^2} + \mathbf{g}\ \mathbf{T_0}\ \Delta\mathbf{t}_{\mathrm{B}}$

(2) $\frac{1}{2}\ \mathbf{g}\ (\Delta\mathbf{t}_{\mathrm{B}} + \mathbf{T}_0)^2\ \ -\frac{1}{2}\ \mathbf{g}\ \Delta\mathbf{t}_{\mathrm{B}}^2$

(3) $-\frac{1}{2}\ \mathbf{g}\ (\Delta\mathbf{t}_{\mathrm{A}} - \mathbf{T}_0)^2\ +\frac{1}{2}\ \mathbf{g}\ \Delta\mathbf{t}_{\mathrm{A}}^2$

(4) $-\frac{1}{2}\ \mathbf{g}\ \mathbf{T_0^2} + \mathbf{g}\ \mathbf{T_0}\ \Delta\mathbf{t}_{\mathrm{B}}^2$

(5) $-\frac{1}{2}\ \mathbf{g}\ \mathbf{T_0^2} + \mathbf{g}\ \mathbf{T_0}\ \Delta\mathbf{t}_{\mathrm{A}}$

$$\Delta(\Delta X) = \Delta X_A - \Delta X_B$$
$$= \tfrac{1}{2} g \Delta t_A^2 - \tfrac{1}{2} g \Delta t_B^2$$
$$= \tfrac{1}{2} g \left(\Delta t_A^2 - \Delta t_B^2\right)$$
$$= \tfrac{1}{2} g \left((\Delta t_B + T_0)^2 - \Delta t_B^2\right)$$
$$= \tfrac{1}{2} g \left(\cancel{\Delta t_B^2} + 2T_0\Delta t_B + T_0^2 - \cancel{\Delta t_B^2}\right)$$
$$\Delta(\Delta x) = \underbrace{\tfrac{1}{2} g T_0^2}_{\Delta X_A \text{ BEFORE B WAS DROPPED}} + \underbrace{g T_0 \Delta t_B}_{\text{DISTANCE BETWEEN INCREASES LINEARLY W/ TIME}}$$

$\Delta t_B'$

(e) Given the simplified expression above, how does the distance between the two balls increase?

(1) constant .
(2) linearly.
(3) quadratically.
(4) cubically.

T_0 IS A CONSTANT

Conceptual questions for discussion

1. Do you agree with the following statement? When solving a kinematic equation quadratic in time, negative time solutions do n ot make sense because negative time does not exist? Provide an example to support your thought.
2. In problem K1.2-8, and other free-fall problems where an object hits the ground, the final velocity of the object is not zero. Provide a reason(s) why we don't use a final velocity of zero even though it hits the ground?

Hints

K1.2-1: Hold a pencil in your hand and start with your hand on the floor next to a table. Briskly move your hand upwards at a con stant speed and release the pencil when your hand is level with the table making sure to keep the same upwards speed. Watch the mo tion of the pencil. Does it drop vertically downwards, or does it go a little bit above the table before it turns around and move s downwards?

K1.2-2: No hints.

K1.2-3: No hints.

K1.2-4: If you start and stop at the same location, what is your change in position?

K1.2-5: What is the mathematical definition of average velocity?.

K1.2-6: Sketch a physical representation.

K1.2-7: Fill out the knowns and unknown lists first, then identify which kinematic equation(s) is must useful. What kinematic var iable connects these two objects (i.e. what quantity is the same for both objects)? To answer quickly, try proportional reasoning.

K1.2-8: Identify which quantities are the same for both object A and object B. Then identify a kinematic equation that is helpful .

K1.2-9: Identify the connection between the stages.

K1.2-10: No hints.