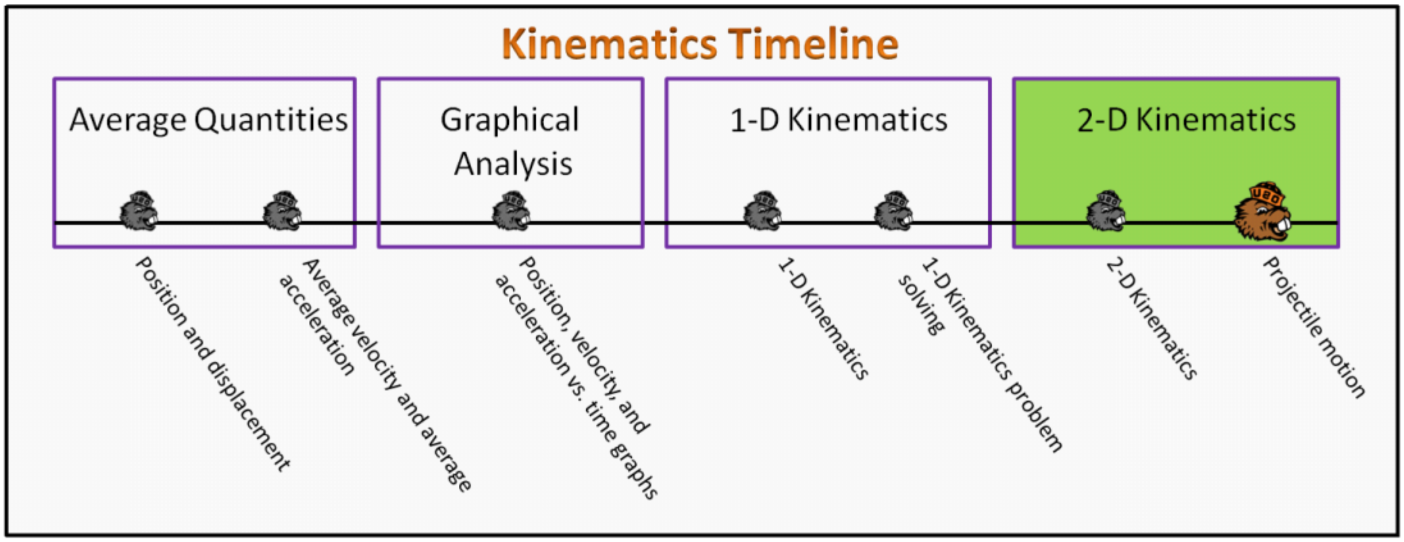
Kinematics: Projectile Motion

Monday, January 22, 2018 5:44 PM

2-D Kinematics
Foundation Stage (K1.2)

lecture 1
Projectile motion



Textbook Chapters

- **BoxSand** :: KC videos (Projectile motion)
- **Giancoli** (Physics Principles with Applications 7th) :: 3-5 ; 3-6 ; 3-7
- **Knight** (College Physics : A strategic approach 3rd) :: 3.6 ; 3.7
- **Knight** (Physics for Scientists and Engineers 4th) :: 4.1 ; 4.2

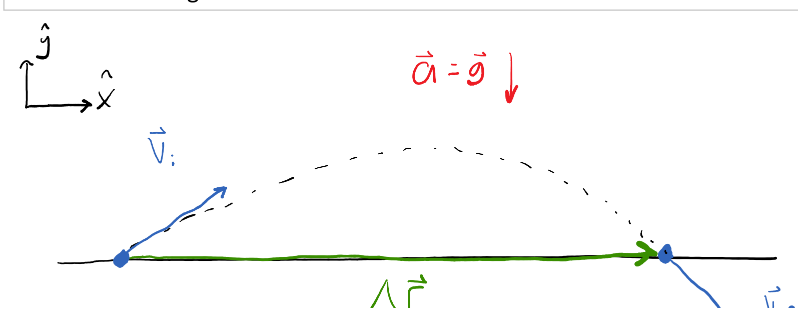
Warm up

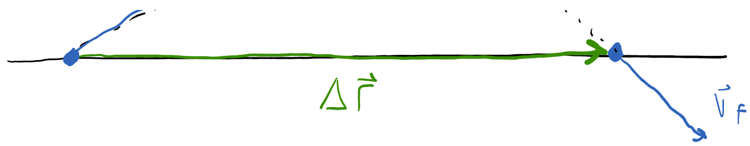
K1.2-1:

Description: Find the components of an initial velocity.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: Freya the Frog jumps with an initial speed of 6.00 m/s at an angle of 30° with respect to the horizontal ground. Sketch a physical representation, including the trajectory, for Freya the moment she leaves the ground to the moment she make s contact with the ground as she lands.





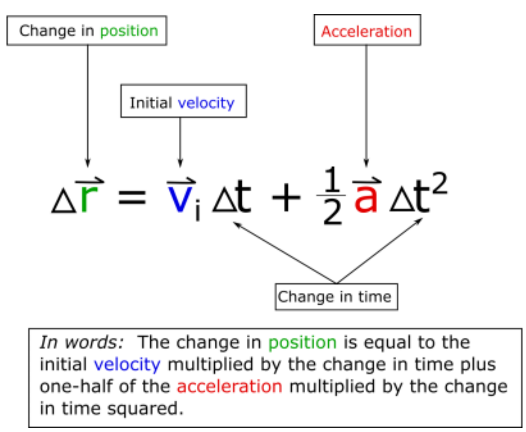
Selected Learning Objectives

1. Identify that the motion occurs in more than 1-dimension and requires a 2-D analysis.
2. Define a coordinate system that simplifies the complexity of the vector analysis.
3. Construct a physical representation the involves multiple dimensions and show a representation of the vector components.
4. Demonstrate the ability find the Cartesian components of a vector in the mathematical representation.
5. Identify known and unknown quantities for each object, stage, and dimension.
6. Solve for a desired unknown in the mathematical representation using a set of kinematic equations for each dimension. Use the problem solving skills developed in 1-D kinematics
7. Identify which quantities are the same when comparing two different dimensions, objects, or stages, e.g. elapsed time is the same for both x and y motion.
8. Define projectile motion.
9. Show that in projectile motion the acceleration has a magnitude of $g = 9.8\ m/s^2$ and points downward.
10. Show that in projectile motion time of flight is determined in an analysis of the vertical motion.
11. Show that in projectile motion the horizontal motion can be the same between two cases even when the vertical is not.
12. Show that in projectile motion range depends on both the horizontal speed and the time of flight, thus dependent on both the vertical and horizontal analysis.
13. Show that in projectile motion the range is the same for complementary angles.
14. Show that in projectile motion any system can be analyzed using only the fundamental kinematics equations for constant acceleration, e.g. you do not need specially derived equations like the *range* equation.
15. Apply limiting cases sense-making procedures to check their solutions.

Key Terms

- Projectile motion

Key Equations



Final velocity | Initial velocity | Acceleration | Change in time

$$\vec{v}_f = \vec{v}_i + \vec{a}\,\Delta t$$

In words: The final velocity is equal to the initial velocity plus the acceleration multiplied by the change in time.

Final x-component of velocity | Initial x-component of velocity | x-component of acceleration | Change in x-component of position

$$v_{fx}^2 = v_{ix}^2 + 2\,a_x\,\Delta x$$

In words: The final x-compoent of velocity squared is equal to the initial x-compoent of velocity squared plus two times the x-component of acceleration multiplied by the change in the x-component of position.

Final y-component of velocity | Initial y-component of velocity | y-component of acceleration | Change in y-component of position

$$v_{fy}^2 = v_{iy}^2 + 2\,a_y\,\Delta y$$

In words: The final y-compoent of velocity squared is equal to the initial y-compoent of velocity squared plus two times the y-component of acceleration multiplied by the change in the y-component of position.

Key Concepts

- By definition, an object undergoing projectile motion has an acceleration of 9.8 m/s^2 pointing down.
- Known and unknown lists help organize kinematic information as well as your thoughts.
- It is highly recommended to not attempt to do algebra (i.e. re-arrange kinematic equations and/or plug them into each other) until you have identified the same number of equations as you have unknowns.
- Recall that time is a scalar; there is no x-component of time or y-component of time, there is only one time which is the same value for both the x and y kinematic analysis.

Act I: Projectile motion

Questions

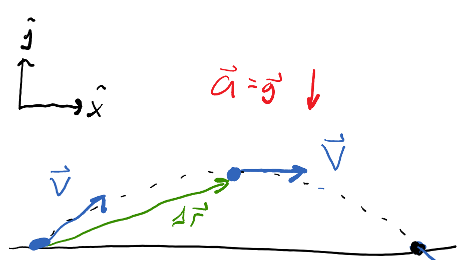
K2.2-2:

Description: Projectile motion maximum height conceptual question involving acceleration and velocity. (4 minutes)

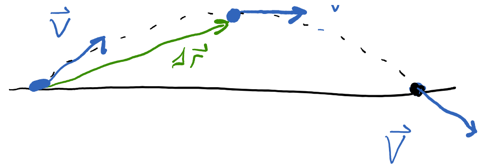
Learning Objectives: [8, 9]

Problem Statement: A baseball is thrown from 3rd base to 1st base. Assuming a standard coordinate system, which of the following statements are true regarding the instant the ball is at its maximum height.

F (1) The velocity of the ball is zero.
F (2) The acceleration of the ball in the horizontal direction is -9.8 m/s^2.
T (3) The acceleration of the ball in the horizontal direction is 0.
T (4) The acceleration of the ball in the vertical direction is -9.8 m/s^2.
F (5) The acceleration of the ball in the vertical direction is 0.
F (6) The velocity of the ball in the horizontal direction is 0.
T (7) The velocity of the ball in the horizontal direction is a non-zero value.
(8) The velocity of the ball in the vertical direction is 0.



F (5) The acceleration of the ball in the vertical direction is 0.
F (6) The velocity of the ball in the horizontal direction is 0.
T (7) The velocity of the ball in the horizontal direction is a non-zero value.
T (8) The velocity of the ball in the vertical direction is 0.



K2.2-3:

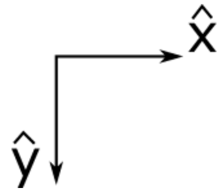
Description: Identify what the acceleration due to gravity is for projectile motion. (2 minutes)

Learning Objectives: [2, 8, 9]

Problem Statement: Given the coordinate system below, what is the acceleration due to gravity?

(1) <-9.8 , 0> m/s^2
(2) <9.8 , 0> m/s^2
(3) <0 , -9.8> m/s^2
(4) <0 , 9.8 > m/s^2

$\vec{a} = \vec{g} \downarrow$



K2.2-4:

Description: Conceptual projectile motion problem highlighting the de-coupling of horizontal and vertical motion. (4 minutes)

Learning Objectives: [9]

Problem Statement: A cart is rolling on a horizontal table when a ball is launched from the cart, vertically as seen with respect to the cart. Where does the ball land with respect to the cart?

(1) In front of the cart.
(2) Behind the cart.
(3) On the cart.

$\vec{a} = \vec{g} \downarrow$ ~ ONLY AFFECTS VERTICAL COMPONENT OF VELOCITY
DOES NOT AFFECT THE HORIZONTAL COMPONENT.

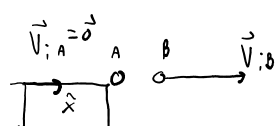
K2.2-5:

Description: Projectile motion time of flight conceptual question. (4 minutes)

Learning Objectives: [1, 8, 9, 10]

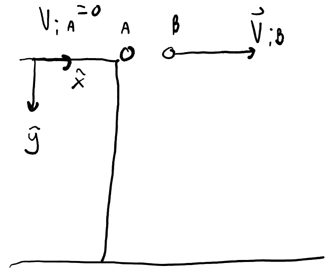
Problem Statement: Ball **A** is dropped at rest from a height **h** above the ground. At the same instant ball **B** is launched horizontally from the same height. Which ball hits the floor first?

(1) **A**
(2) **B**
(3) Both hit at the same time.



$\Delta y = \cancel{v_{iy}\Delta t} + \frac{1}{2} a_y \Delta t^2$

(1) **A**
(2) **B**
(3) Both hit at the same time.



$$\Delta y = \cancel{V_{iy}} \Delta t + \frac{1}{2} a_y \Delta t^2$$

$$\Delta y = \frac{1}{2} g \Delta t^2$$

SAME Δy SAME $a_y = g$ SO SAME Δt

K2.2-6:

Description: Projectile motion horizontal distance conceptual question. (2 minutes + 3 minutes)

Learning Objectives: [1, 8, 10, 11, 13]

Problem Statement: Consider the image below where two cannons each shoot a ball out with the same initial speed but different angle. For case **A**, the cannon shoots the ball at 20° with respect to the horizontal. It is observed that the cannonball in case **B** lands at the same location as case **A**. The angles are not drawn to scale.

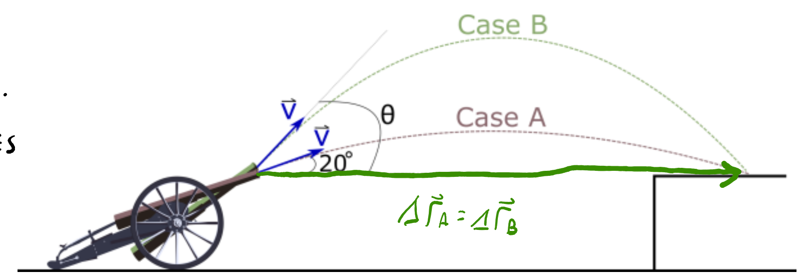
(a) What is the angle that the cannon ball is shot at in case **B**?

(1) 40°
(2) 50°
(3) 60°
(4) 70°

LOOK FOR SYMMETRIES...

COMPLEMENTARY ANGLES

20 + 70
30 + 60
40 + 50



(b) Considering both case **A** and case **B**, which quantities are different? Assume a standard coordinate system.

SAME (1) Δx
SAME (2) Δy
DIFF (3) Δt
DIFF (4) $\mathbf{v}_{iy}$
DIFF (5) $\mathbf{v}_{fy}$
SAME (6) final speed

K2.2-7:

Description: Projectile motion horizontal distance conceptual question. (7 minutes)

Learning Objectives: [1, 8, 10, 12]

Problem Statement: Cannonballs of different masses are shot from cannons at various angles above the horizontal. The velocity of each cannonball as it leaves the cannon is given, along with the horizontal component of that velocity. Rank the horizontal distance travelled by the cannonballs. Explain your reasoning.
Δx

X- COMPONENT

$\Delta x = V_{ix}\Delta t + \frac{1}{2}\cancel{a_x}^{\,0}\Delta t^2$

$\Delta X = V_{ix}\Delta t$

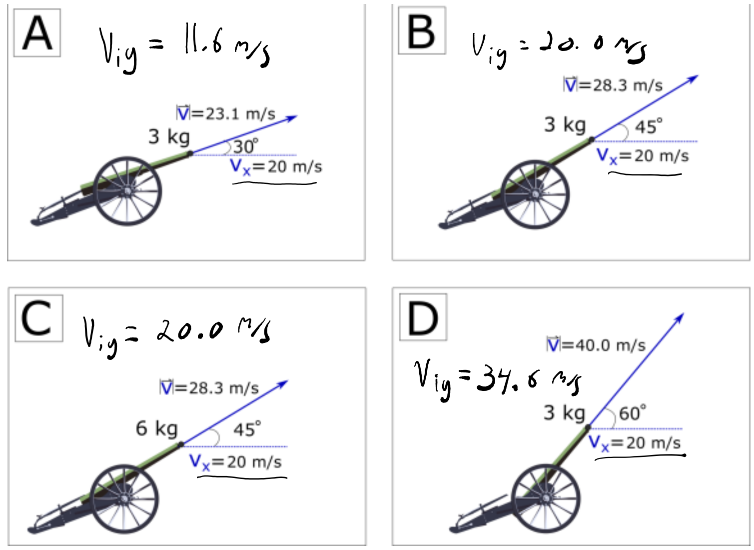
w/ SAME V_{ix} ..

$\Delta X \propto \Delta t$

So LONGEST $\Delta t \rightarrow$ LARGEST ΔX

y-COMPONENT

LARGEST $V_{iy} \rightarrow$ LARGEST Δt



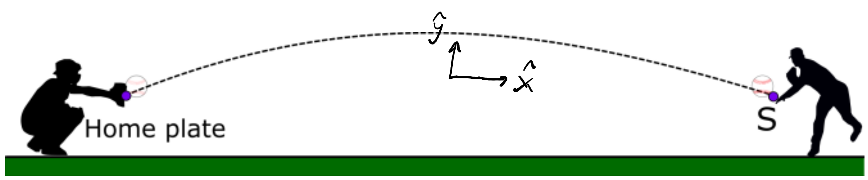
$$\boxed{\Delta X_D > \Delta X_B = \Delta X_C > \Delta X_A}$$

K2.2-8:

Description: Sketching graphs for velocity and acceleration as a function of time for projectile motion. (2 minutes + 3 minutes + 1 minute + 1 minute))

Learning Objectives: [1, 8, 9, 15]

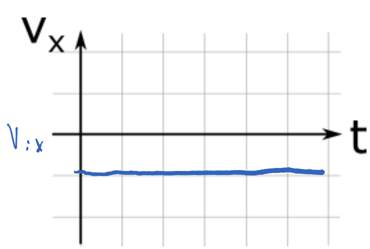
Problem Statement: A baseball is thrown from point **S** in the right field to home plate. The dashed line in the diagram shows the path of the ball.

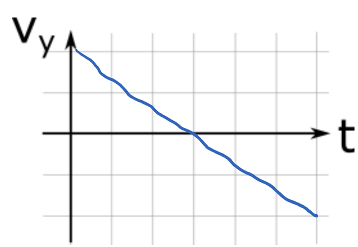


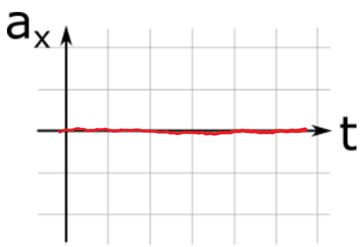
Use a coordinate system with up as the positive y-direction and to the right as the positive x-direction, with the origin at the point the ball was thrown from (point **S**).

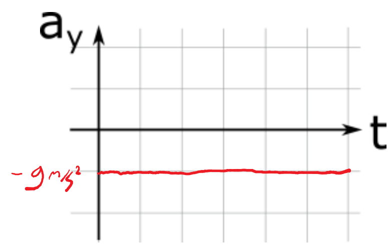
(a) Sketch the x-component of acceleration and the x-component of velocity as a function of time on the provided graphs.

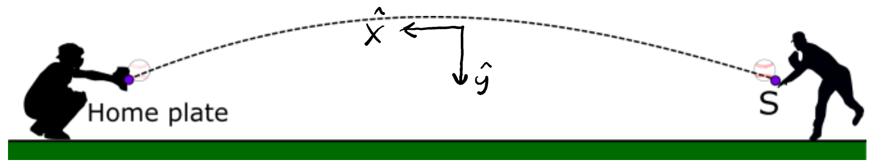
(b) Sketch the y-component of acceleration and the y-component of velocity as a function of time on the provided graphs.







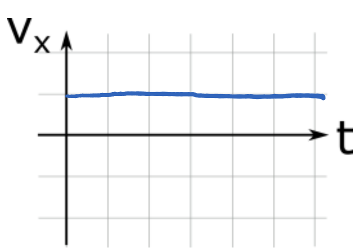


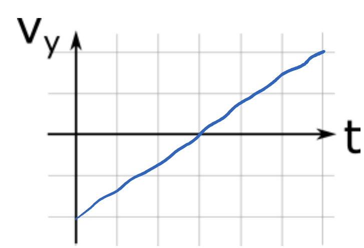


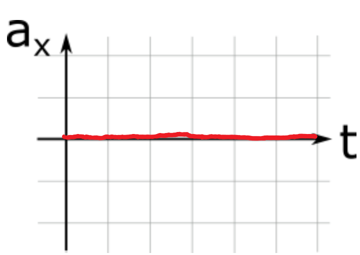
Now use a coordinate system with down as the positive y-direction and to the left as the positive x-direction, with the origin at the point the ball was thrown (point **S**).

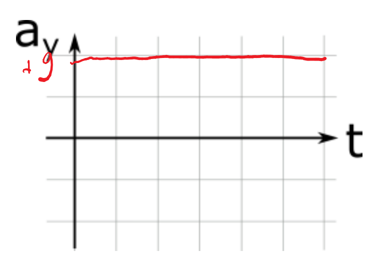
(c) Sketch the x-component of acceleration and the x-component of velocity as a function of time on the provided graphs.

(d) Sketch the y-component of acceleration and the y-component of velocity as a function of time on the provided graphs.









K2.2-9:

Description: Projectile motion problem solving for horizontal distance. (3 minutes + 4 minutes + 5 minutes + 8 minutes)

Learning Objectives: [1, 2, 3, 4, 5, 6, 7, 8, 14]

Problem Statement: A baseball is hit with an initial speed of 36.0 m/s at an angle of 66.0° above the horizontal. An outfielder perfectly positioned catches the ball at the same height it was hit without having to move. How far the outfielder is away f rom the location where the ball was hit.

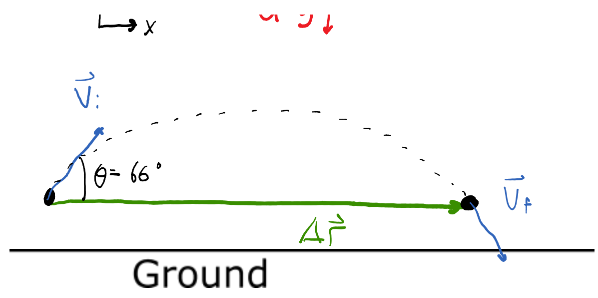
(a) Draw a physical representation.
(b) Identify knowns and unknowns in both the x and y components.

$\hat{y}$ $\hat{x}$ $\vec{a} = \vec{g}\downarrow$

$\vec{V}$:

X				
Δx	v_{ix}	v_{fx}	a_x	Δt

Y				
Δy	v_{iy}	v_{fy}	a_y	Δt



x		y	
$\Delta x \quad v_{ix} \quad v_{fx} \quad a_x \quad \Delta t$		$\Delta y \quad v_{iy} \quad v_{fy} \quad a_y \quad \Delta t$	
K	UK	K	UK
$V_{ix} = 14.6\ m/s$ $V_{fx} = 14.6\ m/s$ $a_x = 0$	Δx Δt	$\Delta y = 0$ $V_{iy} = 32.9\ m/s$ $a_y = -9.8\ m/s^2$	V_{fy} Δt

$V_{ix} = |\vec{V}_i| \cos\theta$ $\qquad$ $V_{iy} = |\vec{V}_i| \sin\theta$

$V_{ix} = 14.6\ m/s$ $\qquad$ $V_{iy} = 32.9\ m/s$

(c) Solve for how far away the outfielder is.

$$\Delta x = v_{ix}\,\Delta t + \tfrac{1}{2} a_x \Delta t^2$$

$$v_{fx} = v_{ix} + a_x\,\Delta t$$

$$v_{fx}^2 = v_{ix}^2 + 2\,a_x\,\Delta x$$

$$\Delta X = V_{ix}\Delta t + \tfrac{1}{2}\cancel{a_x}^{\,0}\Delta t^2$$

$$\Delta X = V_{ix}\,\Delta t$$

$$\Delta X = \frac{2 V_{ix} V_{iy}}{g} \approx \boxed{98.0\ m}$$

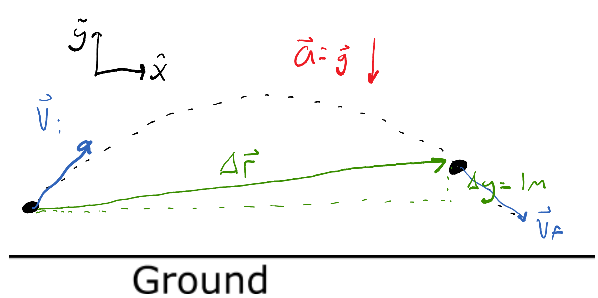
$$\cancel{\Delta y}^{\,0} = V_{iy}\Delta t + \tfrac{1}{2} a_y \Delta t^2$$

$$0 = V_{iy}\Delta t - \tfrac{1}{2} g \Delta t^2$$

$$0 = V_{iy} - \tfrac{1}{2} g \Delta t$$

$$\Delta t = \frac{2 V_{iy}}{g} \approx 6.71429\ s$$

(d) If the outfielder moved in slightly and raised their hand 1 meter above where the ball was hit to catch the ball, how long wa s the ball in the air?



x		y	
$\Delta x \quad v_{ix} \quad v_{fx} \quad a_x \quad \Delta t$		$\Delta y \quad v_{iy} \quad v_{fy} \quad a_y \quad \Delta t$	
K	UK	K	UK
$V_{ix} = 14.6\ m/s$ $V_{fx} = 14.6\ m/s$ $a_x = 0$	Δx Δt	$\Delta y = 1\ m$ $V_{iy} = 32.9\ m/s$ $a_y = -9.8\ m/s^2$	V_{fy} Δt

$$\Delta y = V_{iy}\Delta t + \tfrac{1}{2} a_y \Delta t^2$$

$$1m = 32.9\ m/s\ \Delta t - \tfrac{1}{2}(9.8\ m/s^2)\,\Delta t^2$$

$$\underbrace{-4.9\ m/s^2\ \Delta t^2}_{} + \underbrace{32.9\ m/s\ \Delta t}_{} \underbrace{-1m}_{} = 0$$

$$\Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\Delta t = 0.031$ sec or $\boxed{6.68\ \text{sec}}$

$$\underbrace{-4.9\,m/s^2}_{a}\,\Delta t^2 + \underbrace{32.9\,m/s}_{b}\,\Delta t\,\underbrace{-1\,m}_{c} = 0$$

$\Delta t = 0.031$ SEC OR $\boxed{6.68 \text{ SEC}}$

ON THE WAY UP

Conceptual questions for discussion

1. Do you agree with the following statement? *All objects that are thrown in the air can be modeled via projectile motion*. If you don't agree, provide an example. If you do agree, explain why the statement is true.
2. Do you agree with the following statement? *The acceleration due to gravity is -9.8 m/s^2 near the surface of the earth*. If you don't agree, fix the statement so that it is true. If you do agree, explain why the statement is true.
3. Does a model rocket undergo projectile motion?
4. If a ball is kicked from the level ground and lands a distance **d** away on a flat table of height **h** traveling horizontally the instant it lands. Is the angle that the ball was kicked at equal to the $\tan^{-1}$(**h**/**d**)?

Hints

K2.2-1: No hints.

K2.2-2: No hints.

K2.2-3: No hints.

K2.2-4: Are you making any assumptions (e.g. are you ignoring air resistance?).

K2.2-5: Identify which kinematic quantities are the same for both cases. Now look at the kinematic equations to help answer the problem.

K2.2-6: As per this problem, the range is proportional to sin(2 **θ**). After class, you should try to solve for the horizontal distance as a function of (**v**, **g**, **θ**).

K2.2-7: Which component (vertical or horizontal) determines how long a projectile is in the air?

K2.2-8: No hints.

K2.2-9: No hints.