Question 1:

You are a world-famous physicist-lawyer defending a client who has been charged with murder. It is alleged that your client, Mr. Lawton, shot the victim, Mr. Cray. The detective who investigated the scene of the crime, Mr. Dibny, found a second bullet, from a shot that missed Mr. Cray, that had embedded itself into a chair. You arise to cross-examine the detective.

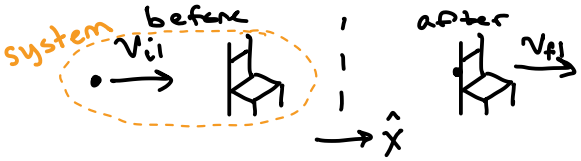
You: In what type of chair did you find the bullet?
Dibny: A **wooden** chair.
You: How massive was this chair?
Dibny: It had a mass of 20 kg.
You: How did the chair respond to being struck with a bullet?
Dibny: It slid across the floor.
You: How far?
Dibny: Three centimeters. The slide marks on the dusty floor are quite distinct.
You: What kind of floor was it?
Dibny: A **wood** floor.
You: What was the mass of the bullet you retrieved from the chair?
Dibny: Its mass was 10 g.
You: Have you tested the gun you found in Mr. Lawton's possession?
Dibny: I have.
You: What is the muzzle velocity of bullets fired from that gun?
Dibny: The muzzle velocity is 450 m/s.

With only slight hesitation, you turn confidently to the jury and proclaim, "My client's gun did not fire those shots!"

(a) How are you going to convince the jury and judge?
(b) Choose one part of your solution and perform a sense-making analysis. Choose between *Sign*, *Units / Dimensionality*, *Related Quantities*, or *Proportionality / Covariation* sensemaking techniques. Clearly indicate which you are employing. You will likely want to make a clear prediction, explain the prediction, then compare the prediction to your found result.

(a) I will show that given the muzzle velocity of my client's gun and conservation of momentum, the chair would have slid a different distance.

Stage 1: Collision



k
$v_i = 450\,m/s$
$m_b = 0.01\,kg$
$m_c = 20\,kg$

vk
v_{f1}

Relevant eqns.
$\Delta\vec{p} = 0$
$\Sigma\vec{p}_i = \Sigma\vec{p}_f$

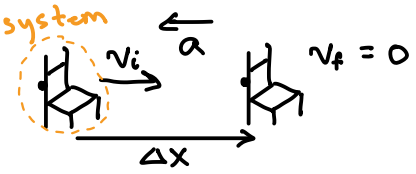
$\Sigma F_{ext} \neq 0$ (friction)
but $\Delta t_{collision} \approx 0$ so $\Sigma\vec{F}_{ext}\Delta t \approx 0 \rightarrow \Delta\vec{p} \approx 0$

$$\vec{p}_i = \vec{p}_f$$

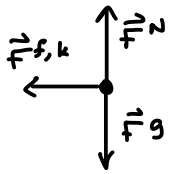
$$m_b v_i = (m_c + m_b) v_{f1}$$

$$v_{f1} = \frac{m_b v_{i1}}{m_c + m_b}$$

Stage 2 : Sliding to rest



FBD



k

$m = m_b + m_c = 20.01\ kg$

$v_{i2} = v_{f1} = 0.225\ m/s$

$v_{f2} = 0$

$\mu_k \approx 0.2$ (google μ_k wood on wood)

Uk

a

Δt

Relevant equations

$\Sigma \vec{F} = m\vec{a}$

$\Delta x = v_i \Delta t + \frac{1}{2} a \Delta t^2$

$v_f^2 = v_i^2 + 2a\Delta x$

$v_f = v_i + a\Delta t$

Forces analysis

$\Sigma \vec{F} = m\vec{a}$

$-F^{fk} = ma_x$

$-\mu_k |F^N| = ma_x$

$a_x = -\frac{\mu_k m g}{m}$

$a_x = -\mu_k g$

$F^N - F^g = ma_y$ ($a_y = 0$)

$F^N - mg = 0$

$F^N = mg$

kinematics analysis

$v_f^2 = v_i^2 + 2a\Delta x$ ($v_f = 0$)

$-\frac{v_i^2}{2a} = \Delta x$

from stage 1

$\Delta x = \frac{v_i^2}{2\mu_k g} = \frac{m_b v_{i1}^2}{(m_b + m_c)} \frac{1}{2\mu_k g} = 0.0258\ m$

$= 2.58\ cm$

< 3 cm — my client's gun couldn't have pushed it as far

- or -

$v_{i2} = \sqrt{2\mu_k g \Delta x} = \frac{m_b v_{i1}}{m_c + m_b}$

$v_{i1} = \frac{(m_c + m_b)\sqrt{2\mu_k g \Delta x}}{m_b} = 686.2\ m/s$

> 450, muzzle velocity of the perpetrator's gun was greater than that of my client's.

(c) Unit check

$$\Delta x = \frac{m_b v_{ii}^2}{(m_b + m_c)} \frac{1}{2\mu_k g} \qquad \frac{\cancel{kg} \cdot \left(\frac{m}{s}\right)^2}{\cancel{kg}} \frac{1}{m/s^2} = \frac{m^2}{s^2} \cdot \frac{s^2}{m} = m \checkmark$$

$$v_{ii} = \frac{(m_c + m_b)\sqrt{2\mu_k g \Delta x}}{m_b} \qquad \frac{\cancel{kg}\sqrt{\frac{m}{s^2} \cdot m}}{\cancel{kg}} = \sqrt{\frac{m^2}{s^2}} = \frac{m}{s} \checkmark$$

Proportionality

$$\Delta x = \frac{m_b v_{ii}^2}{(m_b + m_c)} \frac{1}{2\mu_k g}$$

$\Delta x \propto v_{ii}$ if the bullet has a greater v_i, it will push the chair further. This makes sense because The momentum will be greater, so the chair starts sliding faster, so it slides further before stopping.

$\Delta x \propto \frac{1}{\mu_k}$ if the coefficient of friction is larger, Δx is smaller. This makes sense because the friction force would be larger, creating a larger acceleration, so it would stop the chair in a shorter distance.