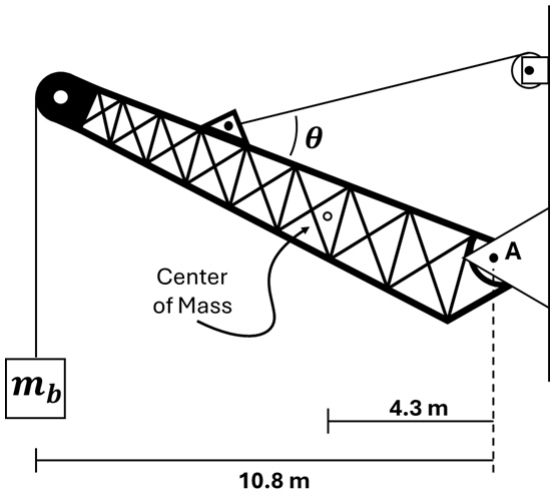
Question 1:

A **1200 kg** crane is supporting a box of mass, $\mathbf{m_b = 890\ kg}$. The crane pivots about a point labeled "A". A cable, holding the system in static equilibrium, attaches and pulls on the crane arm at a point **7.8 m** from the pivot point, A. The angle made between the cable and the crane arm is $\boldsymbol{\theta = 42°}$. The horizontal distance from the pivot point, A, to the center of mass of the crane is **4.3 m**, and the horizontal distance from the pivot point to the box is **10.8 m**.



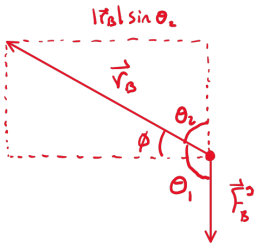
(a) Draw an extended free body diagram with location A as the reference axis.

(b) What do you expect the *Order of Magnitude* of your answer to question (c) to be? (hint: what are the *Orders of Magnitude* of the torques from the box and the center of mass of the crane?)

(c) Find the magnitude of the force of tension in the cable between the wall and the crane.

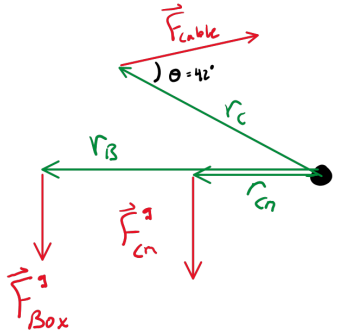
(d) Complete your *Order of Magnitude* sense-making by comparing your found answer in part (c) to your predicted answer in part (b).

1) a) for the box and the center of mass torques, we don't have θ or $\vec{r}$. But, we do know the value of $|\vec{r}|\sin\theta$! This is the portion of $\vec{r}$ that is perpendicular to $\vec{F}$. Here is one way to show this:



$$|\vec{\tau}_B| = |\vec{r}_B||F_B^g|\sin\theta_1$$
$$= |\vec{r}_B||\vec{F}_B^g|\sin\theta_2$$
$$= |\vec{r}_B||F_B^g|\cos\phi$$

therefore, we can draw our eFBD like this



b) $\tau_{Box} = |\vec{r}_B| \sin\theta_B \, |\vec{F}_B|$

$= (10.8\,m)(890\,kg)(9.8\,m/s^2)$

$= 94,197\ N\text{-}m \approx \underbrace{100,000\ N\text{-}m}_{\text{order of magnitude}}$

$\tau_{cm} = |\vec{r}_{cm}| \sin\theta_{cm} \, |\vec{F}_{cm}|$

$= (4.3\,m)(1200\,kg)(9.8\,m/s^2)$

$= 50,568\ N\text{-}m \sim \underbrace{10,000\ N\text{-}m}_{\text{O. of M.}}$

because the order of magnitude of $\tau_{Box} + \tau_{cm}$ is 100,000 N-m (some write O. of M. as $100,000 = 10^5 \Rightarrow$ O. of M. = 5), I expect τ_{cable} to be the same order of magnitude. $\tau_{cable} + \tau_{cm} + \tau_{box} = 0$ b/c system is in static equilibrium. This means that $\tau_{cable} = -(\tau_{cm} + \tau_{box})$.

(τ_{cable}: same O. of M.; $(\tau_{cm} + \tau_{box})$: same O. of M.)

$\tau_{cable} = |\vec{r}_c||\vec{F}_c| \sin(\theta)$

$\sim 100,000\ N\text{-}m = (\sim 10\,m)(|\vec{F}_c|)(\sim 1)$

$\Rightarrow$ $\boxed{|\vec{F}_c| \sim 10,000\ N \quad \text{or} \quad \text{O. of M. } 10^4 \text{ or } 4}$

c) $\Sigma\tau = I\alpha^{\,0}$

$\Sigma\tau = 0$

$\tau_{cable} + \tau_{cm} + \tau_{box} = 0$

$\tau_{cable} = -(94,197\ N\text{-}m + 50,568\ N\text{-}m)$

$|\vec{r}_c||\vec{F}_c| \sin\theta = -144,765\ N\text{-}m$

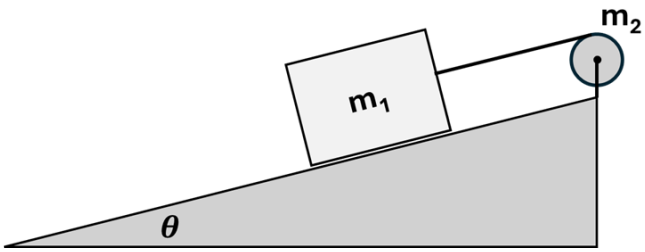
$\boxed{|\vec{F}_c| = 2.77 \times 10^4\ N\text{-}m}$

d) My prediction matches my found answer.

Both are O. of M. 10^4 or 4.

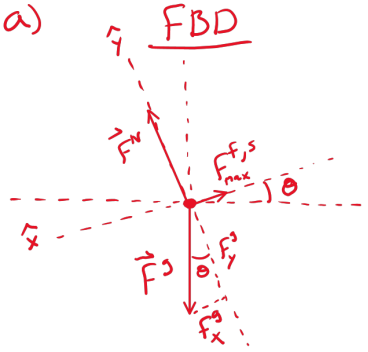
Question 2:

A block of mass, $\mathbf{m_1 = 7\ kg}$, is on a ramp and is connected to a light string, the other end of which is wound around a solid disk-shaped pulley of mass, $\mathbf{m_2 = 1.3\ kg}$, and radius, $\mathbf{r = 20\ cm}$. Assume the pulley rotates about its axle without friction. The angle of the ramp is $\boldsymbol{\theta = 25^o}$. The static and kinetic coefficients of friction between the block and ramp are **0.4** and **0.3**, respectively.



(a) Show that if the block is placed at rest, it will begin to slide. (hint: what is the maximum force of static friction?)
(b) Draw a free body diagram for the block and an extended free body diagram for the pulley.
(c) What is the angular acceleration of the pulley?
(d) If the system starts from rest, how far does the block travel in 3 s.
(e) After 3 s what is the angular velocity of the pulley?

a) FBD



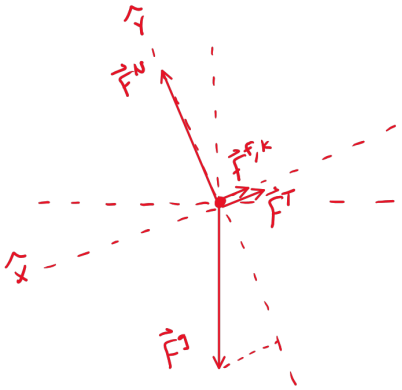
if $f_x^g > f^{f,s}$, then the block will slide down the ramp.

$$f_x^g = |F^g| \sin\theta = mg\sin\theta = 29.0\ N$$

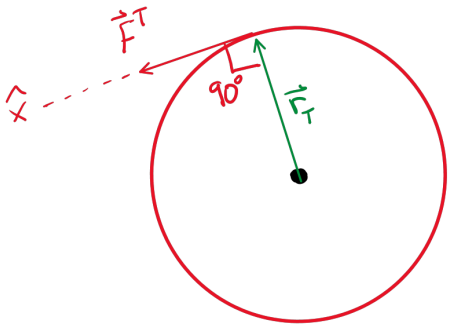
$$f_{max}^{f,s} = \mu_s F^N = \mu_s F_y^g = \mu_s mg\cos\theta = 24.9\ N$$

The x-component of F^g is indeed larger than the maximum strength of static friction. The block will slide.

b) FBD - block



eFBD - pulley



c) Force analysis - block

y: $\Sigma f_y = m_b a_y^{\,0}$

$|F^N| - |F_y^g| = 0$

$\Rightarrow |F^N| = m_b g \cos\theta$

x: $\Sigma f_x = m a_x$

$+|F_x^g| - |F^{f,k}| - |F^T| = m_b a_x$

$+ m_b g \sin\theta - \mu_k |F^N| - |F^T| = m_b a_x$

$+ m_b g \sin\theta - \mu_k m_b g \cos\theta - |F^T| = m_b a_x$

Torque analysis - pulley

$\Sigma \tau = I \alpha$

$|\vec{F}^T||\vec{r}_p| \sin(90^{\circ\,1}) = \left(\frac{1}{2} m_p r_p^2\right)\left(\frac{a_x}{r_p}\right)$

$|F^T| = \frac{1}{2} m_p a_x$

$I_{disk} = \frac{1}{2} m_p r_p^2$

$\alpha = \frac{a_t}{r}$

a_t for the pulley is a_x for the box!

Combining force & torque analyses

$+ m_b g \sin\theta - \mu_k m_b g \cos\theta - \frac{1}{2} m_p a_x = m_b a_x$

$m_b g (\sin\theta - \mu_k \cos\theta) = a_x (m_b + \frac{1}{2} m_p)$

$\Rightarrow a_x = g \frac{m_b}{(m_b + \frac{1}{2} m_p)} (\sin\theta - \mu_k \cos\theta)$

you could do some really cool sensemaking here!

Covariation → what should (conceptually) happen if $\theta\uparrow$ or $\theta\downarrow$?

Does the symbolic solution support that conceptual understanding?

what about $m_p\uparrow$ or $m_p\downarrow$?

$a_x = 1.35\ m/s^2$

$\alpha_p = \frac{a_x}{r_p} = 6.76\ \frac{rad}{s^2}$

d) $\Delta t = 3s \quad v_{ix} = 0 \quad a_x = 1.35\ m/s^2$

$\Delta x = v_{ix}^{\,0} \Delta t + \frac{1}{2} a_x \Delta t^2$

$\Delta x = \frac{1}{2} (1.35 m/s^2)(3s)^2 = 6.08\ m$

e) $\omega_f = \omega_i^{\,0} + \alpha \Delta t$

$\omega_f = (6.76\ rad/s^2)(3s) = 20.3\ rad/s$