

Week 6 Challenge Homework

Fluid Statics

Submission Details | Submit a digital copy (PDF, jpg, etc.) to Canvas. Please use the interface to associate each page of your submission with the corresponding question number! It makes grading much easier. Please clearly indicate which question is being solved. If data is needed to complete a problem, be sure to cite the source you've acquired your data from. See the course website for further details.

Group Submissions | You may submit a group collaboration to Canvas. Add each group member to the submission. Each group member should contribute to the work. Clearly indicate which part of the submission is written by each member (color or labels are preferable).

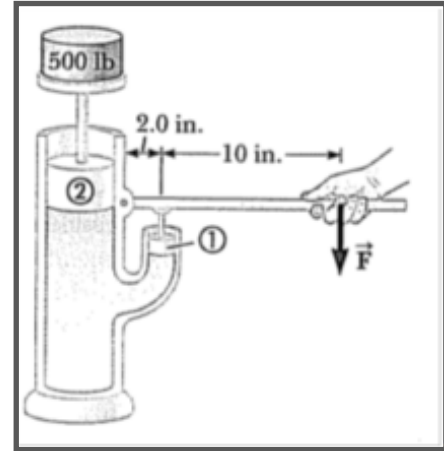
Sensemaking | You will be asked to apply sensemaking in some problems. More information about sensemaking can be found on the Boxsand [Sensemaking](#) page, which is linked on the Canvas homepage. There are many different types of sensemaking. In this course, we will focus on evaluative sensemaking. This is often phrased as checking if your answer is reasonable or not. Usually, the evaluative sensemaking process consists of making a prediction with an explanation, then a comparison. First, make a **prediction** of what you would expect your answer or solution to look like, based on one of the techniques below. Then, provide an **explanation** for the prediction using arguments based in physical reasoning. Finally, make a **comparison** between the answer that you did find and your prediction. Briefly explain why they do or do not match.

Short explanations of the various sensemaking techniques we will use in this course are included below. Please see the Boxsand sensemaking page for more complete details.

- *Sign*: Explain a prediction for the **sign** of your answer or a quantity in your solution. Compare your prediction with the found quantity.
- *Dimensionality*: Explain a prediction for the **units or dimensions** of your answer or a quantity in your solution. Compare your prediction with the found quantity.
- *Related Quantities*: Explain a prediction for the **relationship between two quantities** within your solution. Compare your prediction with the found relationship.
- *Proportionality*, also known as *Covariation*: Usually using a symbolic solution, explain a prediction for the **behavior of one quantity in your solution when another or others are changed**. Compare your prediction with the found quantity.
- *Order of Magnitude*: Explain a prediction for the **order of magnitude** of your answer or a quantity in your solution. Compare your prediction with the found quantity.
- *Graphical Analysis*: Explain a prediction for your answer or a quantity in your solution based on arguments made from **analysis of a graph**. Compare your prediction with the found quantity or relationship.
- *Special Cases*: Explain a prediction for your answer or a quantity in your solution by examining the behavior as another quantity is taken to a limit or **special case**, such as 0, infinity, 0 degrees, or 90 degrees. Compare your prediction with the found relationship.
- *Self-consistency*: Explain or show that your answer is **self-consistent**. Usually this involves using your found answer within an earlier part of your solution and showing that the result is as expected.
- *Known Values*: Explain a prediction of your answer or a quantity in your solution based on a **known value**, such as the speed of light or the density of water. This can involve research and citing a source. Compare your prediction with the found quantity.

Question 1:

Piston 1 in the figure to the right has a diameter of 0.250 in. Piston 2 has a diameter of 1.50 in and is 2 inches higher than piston 1. The hydraulic fluid is oil.



- In jacks like this (think changing a flat tire on your car), there are two forms of mechanical advantage simultaneously working to make the force applied much smaller than what it lifts. What are the two forms of mechanical advantage in this system?
- Determine the magnitude (F) of the force necessary to support the 500 lb. load in the absence of friction.
- Use *Order of Magnitude* sense-making to determine if not including the height difference of the fluid is a problem when calculating the force in part (b). Make sure to:
 - Compare the orders of magnitude* of your answers **with** the height difference factored in and **without**.
 - Explain whether or not it is reasonable to neglect the height difference in your answer to part (b).

*Wikipedia has a good article on order of magnitude, here: https://en.wikipedia.org/wiki/Order_of_magnitude

(a) There are 2 mechanical advantages, from the hydraulic system and from the lever. The hydraulic system uses the pressure transmitted all throughout the fluid by Pascal's principle, and because the area of piston 2 is larger than piston 1's area but the 2 have equal pressure, the force exerted by piston 2 will thus be much larger than the force you exert on piston 1.

The mechanical advantage from the lever comes from the fact that you are using a long lever arm, so the small force is exerted over a large distance, making it have larger torque than the force exerted between the 2 pistons over a tiny distance.

(b) KNOWNS:

$$d_1 = 0.25 \text{ in} = 6.35 \times 10^{-3} \text{ m} \Rightarrow r_1 = 3.175 \times 10^{-3} \text{ m}$$

$$d_2 = 1.5 \text{ in} = 3.81 \times 10^{-2} \text{ m} \Rightarrow r_2 = 1.905 \times 10^{-2} \text{ m}$$

$$m = 500 \text{ lbs} = 227 \text{ kg}$$

$$F_2 = mg = 227(9.8) = 2222.6 \text{ N}$$

$$\rho_{\text{oil}} = 700 \text{ kg/m}^3$$

$$d = 2 \text{ in} = 5.08 \times 10^{-2} \text{ m}$$

$$P_2 = P_1 + \rho g d$$

$$\frac{F_2}{A_2} = \frac{F_1}{A_1} + \rho g d$$

$$\frac{F_1}{\pi r_1^2} + \rho g d = \frac{F_2}{\pi r_2^2}$$

$$F_1 = \left(\frac{F_2}{\pi r_2^2} - \rho g d \right) (\pi r_1^2)$$

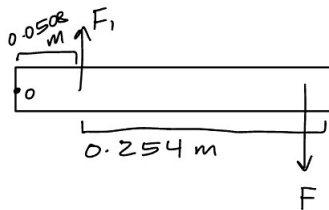
$$F_1 = \left(\frac{2222.6}{\pi (1.905 \times 10^{-2})^2} - 700(9.8)(5.08 \times 10^{-2}) \right) (\pi (3.175 \times 10^{-3})^2)$$

$$F_1 = 61.7 \text{ N}$$

* need to account for lever arm

$$10 \text{ in} = 0.254 \text{ m}$$

* torque is equal to zero b/c the lever arm is not rotating



$$\sum T = I \alpha^0$$

$$\sum T = 0$$

$$\tau = r \perp F \sin \theta$$

$$\sum T = T_1 - T$$

$$T_1 = T$$

$$(0.0508)(61.7) \sin 90 = (0.205)(F) \sin 90$$

$$F = 10.3 \text{ N}$$

$$(c) \quad P_1 = P_2 + \rho_f g d$$

Symbolic:

Without Height Difference:

$$P_1 = P_2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$P_2 = 10^6 *$$

$$P_1 = 10^6$$

* based on calculation of these values

With Height Difference:

$$P_1 = P_2 + \rho_f g d, \rho_f g d = 10^1 *$$

$$P_1 = 10^6 + 10^1$$

When comparing the value for P_1 with and without the height difference, we get an answer of 10^6 vs. $10^6 + 10^1$. Because 10^6 is 5 orders of magnitude larger than 10^1 , we can assume the contribution of pressure from the height difference is negligible and it is not necessary to add when solving for the force in part (b).

Numerical:

$$\frac{F_1}{A_1} + \rho_f g d = \frac{F_2}{A_2}$$

→ this accounted for the height difference

$$\text{without it: } F_1 = \frac{F_2 (A_1)}{A_2} = \frac{F_2 (\pi r_1^2)}{\pi r_2^2} = \frac{2222 \cdot 6 \text{ N} (\pi (3.175 \times 10^{-3} \text{ m})^2)}{\pi (1.905 \times 10^{-2} \text{ m})^2}$$

$F_1 = 61.7$ → you still get 61.7! The order of magnitude w/ or w/o $\rho_f g d$ is the same in this case