

Week 7 Challenge Homework

Fluid Dynamics and Simple Harmonic Oscillation

Submission Details | Submit a digital copy (PDF, jpg, etc.) to Canvas. Please use the interface to associate each page of your submission with the corresponding question number! It makes grading much easier. Please clearly indicate which question is being solved. If data is needed to complete a problem, be sure to cite the source you've acquired your data from. See the course website for further details.

Group Submissions | You may submit a group collaboration to Canvas. Add each group member to the submission. Each group member should contribute to the work. Clearly indicate which part of the submission is written by each member (color or labels are preferable).

Sensemaking | You will be asked to apply sensemaking in some problems. More information about sensemaking can be found on the Boxsand [Sensemaking](#) page, which is linked on the Canvas homepage. There are many different types of sensemaking. In this course, we will focus on evaluative sensemaking. This is often phrased as checking if your answer is reasonable or not. Usually, the evaluative sensemaking process consists of making a prediction with an explanation, then a comparison. First, make a **prediction** of what you would expect your answer or solution to look like, based on one of the techniques below. Then, provide an **explanation** for the prediction using arguments based in physical reasoning. Finally, make a **comparison** between the answer that you did find and your prediction. Briefly explain why they do or do not match.

Short explanations of the various sensemaking techniques we will use in this course are included below. Please see the Boxsand sensemaking page for more complete details.

- *Sign*: Explain a prediction for the **sign** of your answer or a quantity in your solution. Compare your prediction with the found quantity.
- *Dimensionality*: Explain a prediction for the **units or dimensions** of your answer or a quantity in your solution. Compare your prediction with the found quantity.
- *Related Quantities*: Explain a prediction for the **relationship between two quantities** within your solution. Compare your prediction with the found relationship.
- *Proportionality*, also known as *Covariation*: Usually using a symbolic solution, explain a prediction for the **behavior of one quantity in your solution when another or others are changed**. Compare your prediction with the found quantity.
- *Order of Magnitude*: Explain a prediction for the **order of magnitude** of your answer or a quantity in your solution. Compare your prediction with the found quantity.
- *Graphical Analysis*: Explain a prediction for your answer or a quantity in your solution based on arguments made from **analysis of a graph**. Compare your prediction with the found quantity or relationship.
- *Special Cases*: Explain a prediction for your answer or a quantity in your solution by examining the behavior as another quantity is taken to a limit or **special case**, such as 0, infinity, 0 degrees, or 90 degrees. Compare your prediction with the found relationship.
- *Self-consistency*: Explain or show that your answer is **self-consistent**. Usually this involves using your found answer within an earlier part of your solution and showing that the result is as expected.
- *Known Values*: Explain a prediction of your answer or a quantity in your solution based on a **known value**, such as the speed of light or the density of water. This can involve research and citing a source. Compare your prediction with the found quantity.

Question 1:

You are on a cruise ship to the Bahamas. **The door to your room faces the outside of the ship, and swings inwards to your room.** On a breezy day you notice that after turning the door knob, it is very difficult to pull the door open (legend has it, this happened to the mother of a friend of your instructor's cousin's dog's grandpa's owner). The wind is blowing steadily parallel to the side of the ship. **The wind is not at all directed into the door!**



- What is the origin of the force preventing you from opening the door? Explain why the wind is making it difficult to open the door. Hint: if your door opened swinging outwards, it would be initially easy to open!
- Your door is 1.00 meters wide and 1.93 meters tall. The door handle is 87.5 cm from the door hinges. If you have to exert a force of 45 lbs to start to open the door, how fast is the wind travelling outside your door? You may assume the force discussed in part (a) acts at the center of the door. Give the speed both in m/s and m/hr.
- Use known value sensemaking to evaluate your answer to question (b). Make sure to state a relevant known value (which is a prediction of an acceptable value), compare with your found value, and explain whether or not you think your answer to part (b) is reasonable.

Question 2:

In an engine, a piston oscillates with simple harmonic motion so that its position varies according to the expression $x(t) = 15.0 \cos(70.0 t)$ where x is in centimeters and t is in seconds.

- What is the period of the motion?
- What is the amplitude of the motion?
- At $t = 1$ s, find the position of the piston.
- At $t = 1$ s, find the velocity of the piston.
- At $t = 1$ s, find the acceleration of the piston.
- Sketch a plot of the acceleration as a function of time. Scale the plot properly.
- Apply related quantities sensemaking to your answer by
 - Making a **prediction** for similarities and differences you expect to see between the position and acceleration of an oscillating object
 - Explaining** each prediction
 - Finding an equation that describes the acceleration as a function of time for this piston and **comparing** this equation with the position equation in light of your predictions.

Question 1:

a) The air outside the door is moving at a faster speed than the inside air. Because of this, the pressure outside the door is smaller than inside because of proportional reasoning & Bernoulli's equation. The origin of force is this pressure difference between the inside & outside of the door.

$$\overbrace{P_1 + \frac{1}{2}\rho v_1^2 + \cancel{\rho g d_1}}^{\text{outside}} = \overbrace{P_2 + \frac{1}{2}\rho v_2^2 + \cancel{\rho g d_2}}^{\text{inside}} \quad F = \frac{\Delta P}{A}$$

↓ ↑
↑ ↓

b) KNOWN:

$$\begin{aligned}
 A(\text{door}) &= 1 \text{ m}(1.93 \text{ m}) = 1.93 \text{ m}^2 \\
 r_w(\text{wind}) &= 0.5 \text{ m} \\
 r_h(\text{handle}) &= 87.5 \text{ cm} = 0.875 \text{ m} \\
 F_H &= 45 \text{ lbs} = 200 \text{ N} \\
 P_2 &= 101,325 \text{ Pa} \\
 v_2 &= 0 \text{ m/s} \\
 \rho &= 1.204 \text{ kg/m}^3 \\
 d_1 &= d_2
 \end{aligned}$$

equations

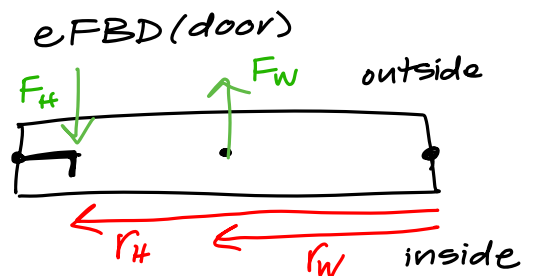
$$\begin{aligned}
 P_1 + \frac{1}{2}\rho v_1^2 + \rho g d_1 &= P_2 + \frac{1}{2}\rho v_2^2 + \rho g d_2 \\
 \tau &= I\alpha \\
 \tau &= |r||F|\sin\theta \\
 P &= \frac{F}{A}
 \end{aligned}$$

$$\sum \tau = I\alpha \neq 0$$

$$\sum \tau = 0 \Rightarrow |\tau_H(\text{handle})| = |\tau_W(\text{wind})|$$

$$|r_H||F_H|\sin\theta_H = |r_W||F_W|\sin\theta_W$$

$$(r_H)(F_H) = (r_W)(A\Delta P)$$



$$\theta_H = 90^\circ$$

$$\theta_W = 90^\circ$$

$$\overbrace{P_1 + \frac{1}{2}\rho v_1^2 + \cancel{\rho g d_1}}^{\text{outside}} = \overbrace{P_2 + \frac{1}{2}\rho v_2^2 + \cancel{\rho g d_2}}^{\text{inside}}$$

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2$$

$$\Delta P = \frac{1}{2}\rho v_1^2 \leftarrow \text{plug into equation from above}$$

$$(r_H)(F_H) = (r_W)(A\Delta P)$$

$$r_H F_H = r_W A \frac{1}{2} \rho V_i^2$$

$$V_i = \sqrt{\frac{2 r_H F_H}{r_W A \rho}}$$

$$V_i = \sqrt{\frac{2(0.875\text{m})(200\text{N})}{0.5\text{m}(1.93\text{m}^2)(1.204\text{kg/m}^3)}}$$

$$V_i = 17.4 \text{ m/s} \approx 38.9 \text{ mph}$$

c) Known value: find a source that provides the average wind at sea. How does your value compare? If it's not close to the average, compare it to the maximum wind speed at sea. When would your value be reasonable?

Question 2:

$$x(t) = 15 \cos(70t)$$

↑ ↑
 x_{\max} ω

a) period = T $\omega = \frac{2\pi}{T} \rightarrow T = \frac{2\pi}{\omega}$

$$T = \frac{2\pi}{70}$$

$$T = 0.0898 \text{ s}$$

b) amplitude = $x_{\max} = 15 \text{ cm}$

c) $x(1) = 15 \cos(70(1))$

$$x(1) = 9.5 \text{ cm}$$

★ calculator mode should be in RADIANS for these calculations★
Because ω is in units of rad/s

d) $v(t) = -v_{\max} \sin(\omega t)$ $v_{\max} = x_{\max} \omega$

$$v(t) = -x_{\max} \omega \sin(\omega t)$$

$$v(1) = -15(70) \sin(70(1))$$

$$v(1) = -813 \text{ cm/s}$$

e) $a(t) = -a_{\max} \cos(\omega t)$ $a_{\max} = v_{\max} \omega = x_{\max} \omega^2$

$a(t) = -x_{\max} \omega^2 \cos(\omega t)$

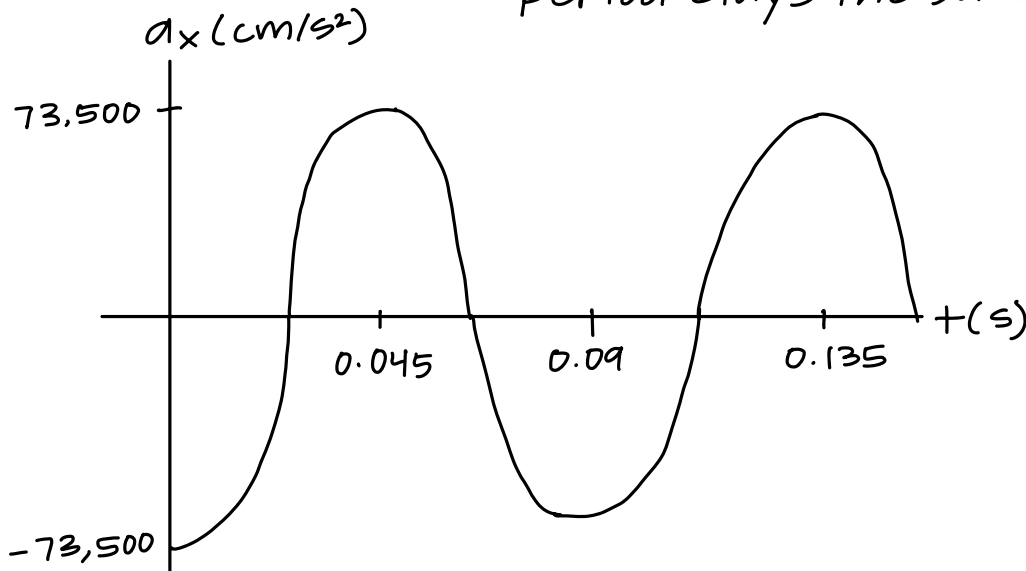
$a(1) = -15(70)^2 \cos(70(1))$

$a(1) = -46,549 \text{ cm/s}^2$

f) $a_{\max} = x_{\max} \omega^2 = 15(70)^2 = 73,500 \text{ cm/s}^2$

$a(0) = -15(70^2) \cos(0) = -73,500 \text{ cm/s}^2$

period stays the same! $\approx 0.09 \text{ s} = T$



g) try comparing equations & graphs!

$x(t) = x_{\max} \cos(\omega t)$ $a(t) = -\overbrace{x_{\max} \omega^2}^{a_{\max}} \cos(\omega t)$

- both have same period & ω
- both are cos functions but $x(t)$ is $+\cos$ and $a(t)$ is $-\cos$
 \hookrightarrow how will their graphs compare w/c of this? what does it mean is actually happening?
- $|a_{\max}| > |x_{\max}|$ but they have different units. can we compare their amplitudes if the units are different? (not really!)
- what about forces? Think about this relationship:
 $\Sigma F_x = -k\Delta x = ma_x \rightarrow$ what does this tell you about position, acceleration, & their graphs?