Week 9 Challenge Homework

Travelling Waves

Submission Details | Submit a digital copy (PDF, jpg, etc.) to Canvas. Please use the interface to associate each page of your submission with the corresponding question number! It makes grading much easier. Please clearly indicate which question is being solved. If data is needed to complete a problem, be sure to cite the source you've acquired your data from. See the course website for further details.

Group Submissions | You may submit a group collaboration to Canvas. Add each group member to the submission. Each group member should contribute to the work. Clearly indicate which part of the submission is written by each member (color or labels are preferable).

Sensemaking | You will be asked to apply sensemaking in some problems. More information about sensemaking can be found on the Boxsand Sensemaking page, which is linked on the Canvas homepage. There are many different types of sensemaking. In this course, we will focus on evaluative sensemaking. This is often phrased as checking if your answer is reasonable or not. Usually, the evaluative sensemaking process consists of making a prediction with an explanation, then a comparison. First, make a **prediction** of what you would expect your answer or solution to look like, based on one of the techniques below. Then, provide an **explanation** for the prediction using arguments based in physical reasoning. Finally, make a **comparison** between the answer that you did find and your prediction. Briefly explain why they do or do not match.

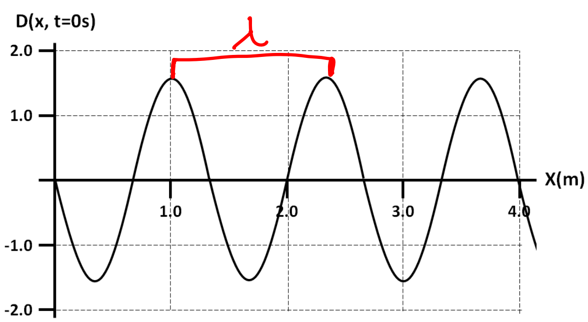
Short explanations of the various sensemaking techniques we will use in this course are included below. Please see the Boxsand sensemaking page for more complete details.

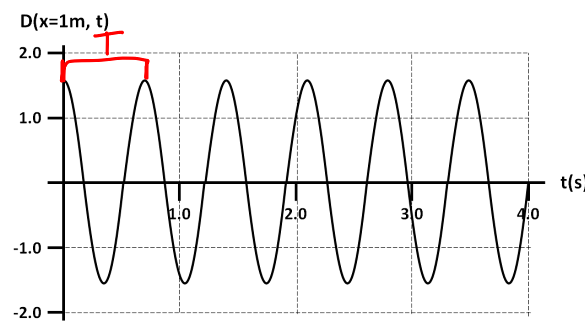
- *Sign:* Explain a prediction for the **sign** of your answer or a quantity in your solution. Compare your prediction with the found quantity.
- *Dimensionality:* Explain a prediction for the **units or dimensions** of your answer or a quantity in your solution. Compare your prediction with the found quantity.
- *Related Quantities:* Explain a prediction for the **relationship between two quantities** within your solution. Compare your prediction with the found relationship.
- *Proportionality,* also known as *Covariation:* Usually using a symbolic solution, explain a prediction for the **behavior of one quantity in your solution when another or others are changed.** Compare your prediction with the found quantity.
- *Order of Magnitude:* Explain a prediction for the **order of magnitude** of your answer or a quantity in your solution. Compare your prediction with the found quantity.
- *Graphical Analysis:* Explain a prediction for your answer or a quantity in your solution based on arguments made from **analysis of a graph**. Compare your prediction with the found quantity or relationship.
- *Special Cases:* Explain a prediction for your answer or a quantity in your solution by examining the behavior as another quantity is taken to a limit or **special case**, such as 0, infinity, 0 degrees, or 90 degrees. Compare your prediction with the found relationship.
- *Self-consistency:* Explain or show that your answer is **self-consistent**. Usually this involves using your found answer within an earlier part of your solution and showing that the result is as expected.
- *Known Values:* Explain a prediction of your answer or a quantity in your solution based on a **known value**, such as the speed of light or the density of water. This can involve research and citing a source. Compare your prediction with the found quantity.

Question 1:

Pictured below are a snapshot and history graph of a travelling wave.

(a) Predict what you expect to happen to the amplitude of a travelling wave if the angular frequency is doubled.
(b) What is the period of the wave.
(c) What is the wavenumber of the wave.
(d) What is the speed of the wave.
(e) Write an equation for the displacement of this wave as function of position (x) and time (t).
(f) Can you tell if the wave is a transverse or longitudinal wave? Explain.
(g) Use Covariation (also known as proportional reasoning) Sensemaking to analyze the equation of motion you found in part (d). Make sure to explain why it agrees with or disagrees with your prediction in part (a).





a) $D(x,t) = \pm D_{max}$ sin or cos $(\pm kx \pm \omega t)$

D_{max}: amplitude, ω: angular frequency

Based on the equation, what is their relationship? Make a prediction.

b) could estimate but we see there are 5.75 oscillations in 4 seconds and the period is the time for one oscillation, so:

$$T = \frac{\text{total time}}{\#\text{oscillations}} = \frac{4\,s}{5.75 \text{ oscillations}} = \frac{16}{23}\,s \approx 0.70\,s$$

★ use graph on the right

c) wavenumber $= k = \frac{2\pi}{\lambda}$

$$\lambda = \frac{\text{total distance}}{\#\text{oscillations}} = \frac{4\,m}{3 \text{ osc.}} = \frac{4}{3}\,m$$

$$k = \frac{2\pi}{4/3\,m} = \frac{3\pi}{2}\,m^{-1} \approx 4.71\,m^{-1}$$

★ use graph on the left

d) $v = \lambda f$

$v = \lambda \frac{1}{T} = 1.33\,m\left(\frac{1}{0.7s}\right) \approx 1.92\,m/s$

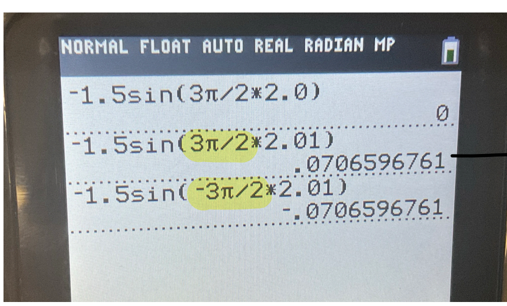
e) $D(x,t) = \pm D_{max} \overset{\sin \text{ or}}{\cos}(kx \pm \omega t)$

- $D_{max} = 1.5\,m$
- $-\sin$ b/c starts at $D(x,t) = 0$ and goes in negative direction
- $k = 4.71\,m^{-1}$
- $\omega = \frac{2\pi}{T} = \frac{2\pi}{16/23s} = \frac{23\pi}{8} s^{-1} \approx 9.03\,s^{-1}$

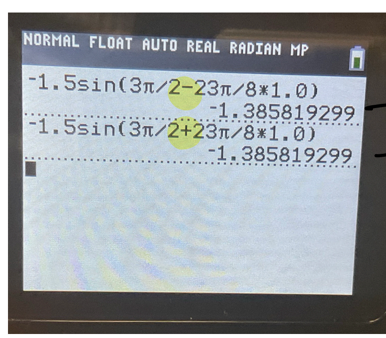
$$D(x,t) = -1.5\sin\left(\frac{3\pi}{2}x \pm \frac{23\pi}{8}t\right)$$

To determine the sign (+ or −) of k, set t equal to zero. Plug in small increments of x, starting somewhere where the result should be zero. For example, at $x=2$, $D(x,t)=0$. At $x=2.01$ we should expect a positive result. If this is what you get, your sign for k is correct. In this case, positive k is correct.

We would use the same strategy for ω, but setting $x=1$ since this is what we see on the history graph. When we start at $t=1$, our result would be the same if ω was + or −, so we can't be sure of its sign.



→ what we expect so +k



→ same answer for + or − ω so we don't know!

f) We can't tell! We know that the wave is moving in x-plane, but the displacement does not distinguish if oscillators are moving in x or y plane.

IF: $D_y(x,t) \rightarrow$ transverse IF: $D_x(x,t) \rightarrow$ longitudinal

g) $D(x,t) = -1.5 \sin\left(\frac{3\pi}{2} x \pm \frac{23\pi t}{8}\right)$

If we change ω, amplitude stays the same. They are independent of one another.