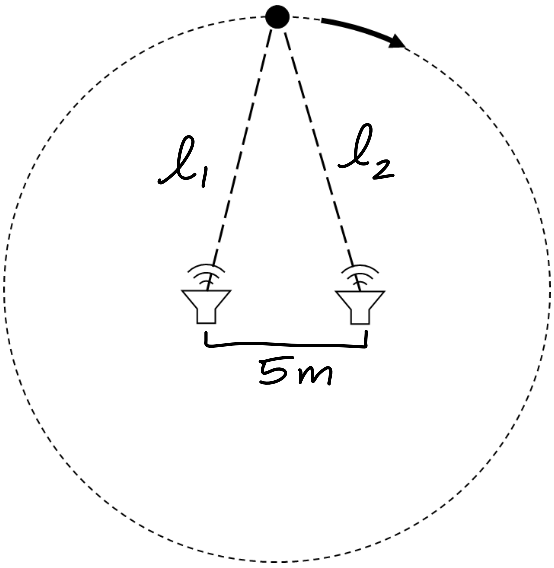
Question 1.



Two spherical speakers are placed 5 meters apart. They emit a constant tone of 686 Hz. You are standing on the dot in the picture, equidistant from each speaker. The speed of sound in air is 343 m/s.

(a) What is the wavelength of the sound emitted by the speakers?

(b) Is your starting position a point of constructive or destructive interference? Explain.

(c) How many spots of constructive interference do you experience if you walk in a complete circle around the speakers?

(d) Use covariational sensemaking to evaluate this situation by answering the following prompts:
  - Make a prediction for how the number of constructive interference spots would change if the frequency were to increase.
  - Explain your prediction using one or more algebraic relationships you used to solve part (c).
  - Compare your prediction with the number of constructive interference spots observed if the frequency of the speakers is tripled.

known:

$f = 686$ Hz

$v = 343$ m/s

$\ell_1 = \ell_2$

equations:

constructive:

$PLD = |\ell_1 - \ell_2| = m\lambda$

destructive:

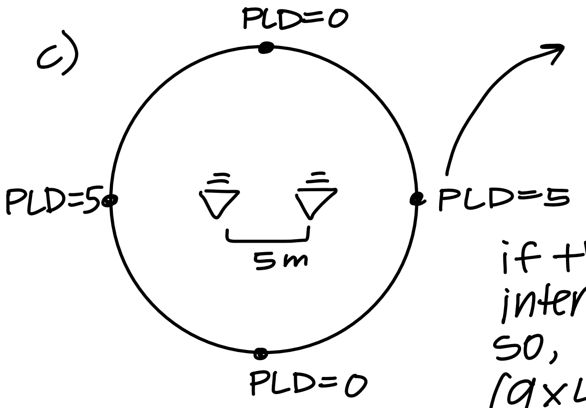
$PLD = (m + \frac{1}{2})\lambda$

a) $v = f\lambda$

$$\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{686 \text{ Hz}} = 0.5 \text{ m}$$

b) At our starting point, $\ell_1 = \ell_2$, so the path length difference is 0. This corresponds with $m=0$ in our constructive interference equation. If it was $m=0$ in the destructive model, the PLD would not be zero.

c)



looking at this point on the circle, the PLD = 5m

$PLD = m\lambda$

$5m = m(0.5m)$

$m = 10$

if this is $m=10$, there are 9 points of constructive interference between the starting point & here.

so, if we do this for each segment:

$(9 \times 4) + 4$ "corners" = 40 spots of constructive interference

d) step 1: make your prediction!

step 2: algebraic relationships

$$\downarrow \lambda = \frac{v^{\text{ same}}}{f \uparrow} \qquad \underset{\text{same}}{PLD} = \underset{\uparrow}{m}\,\underset{\downarrow}{\lambda}$$

seems we'd expect higher m, so more constructive spots

step 3: solve

$$\lambda = \frac{343 \text{ m/s}}{3(686 \text{ Hz})} = 0.1667$$

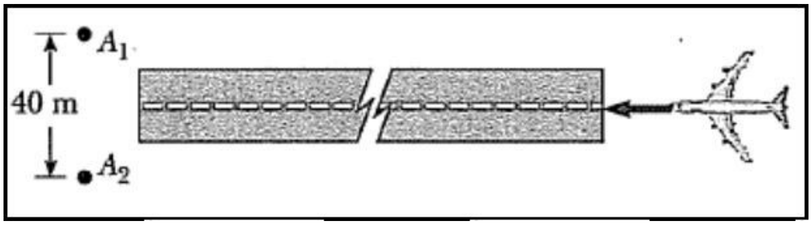
$$5 = m(0.1667 \text{m})$$

$m = 30 \rightarrow$ so 29 constructive spots in each segment

$(29 \times 4) + 4$ "corners" = 120 constructive interference spots

Question 2.

Young's double-slit experiment underlies the instrument landing system used to guide aircraft to safe landings when the visibility is poor. Although real systems are more complicated than the example described here, they operate on the same principles. A pilot is trying to align her plane with a runway as suggest in the figure. Two radio antennas A_1 and A_2, separated by 40.0 m, are positioned adjacent to the runway. The antennas broadcast single frequency, 30.0 MHz, coherent radio waves.



(a) Find the wavelength of the waves. The pilot "locks onto" the strong signal radiated along an interference maximum and steers the plane to keep the received signal strong. If she detects the central maximum, the plane will have the right heading to land when it reaches the runway.

(b) Suppose instead that the plane is flying along the first side maximum, one maxima from the central. How far to the side of the runway centerline is the plane when it is 2.00 km from the antennas?

(c) It is possible to tell the pilot she is on the wrong maximum by sending out two signals from each antenna and equipping the aircraft with a two-channel receiver. The ratio of the two frequencies must not be the ratio of small integers (such as 3/4). Explain how this two-frequency system would work, and why it would not necessarily work if the frequencies were related by an integer ratio. In your explanation, make sure to draw a diagram (physical representation) of a system which uses two distinct frequencies (it's probably most helpful to draw this for two frequencies that ARE related by a ratio of small integers).

known:

$f = 30\text{ MHz} = 30\times10^6\text{ Hz}$

$d = 40\text{ m}$

$V = 3\times10^8\text{ m/s}$

↳ speed of light b/c radio waves!

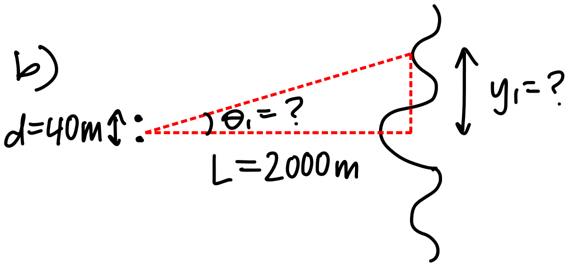
a) $V = f\lambda$

$$\lambda = \frac{V}{f} = \frac{3\times10^8\text{ m/s}}{30\times10^6\text{ Hz}}$$

$\lambda = 10\text{ m}$

first side maximum: constructive @ $m=1$

b)

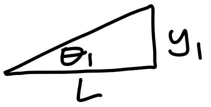


$m\lambda = d\sin\theta$

$(1)\lambda = d\sin\theta_1$

$$\theta_1 = \sin^{-1}\left(\frac{\lambda}{d}\right) = \sin^{-1}\left(\frac{10\text{ m}}{40\text{ m}}\right)$$

$\theta_1 = 14.5°$



$\rightarrow \tan\theta_1 = \frac{y_1}{L} \rightarrow y_1 = L\tan\theta_1 = 2000\tan(14.5°)$

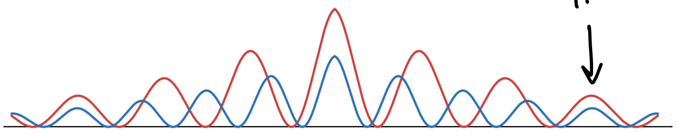
$y_1 = 516.4\text{ m}$

c) Check out this link: Desmos.com/calculator/tjbaswaj6e

ignore the crazy equations, but use the R toggle to examine the ratios between the 2 frequencies.

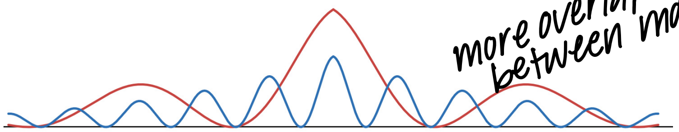
If the frequencies are related by integer ratios, especially small ones ($3/4$ aka 0.75), maximas that are not the central ones will align. Try it on the Desmos link! This may make it hard for the pilot to distinguish the side maximas from the center. If our ratio is not one of small integers, say something like $1/\pi \sim 0.32$, the reading would be more clear about where the central ones are located.

lining up here too!



→ ratio is $3/4$ (0.75)

more overlap between maxima



→ ratio is $1/\pi$ (0.32)