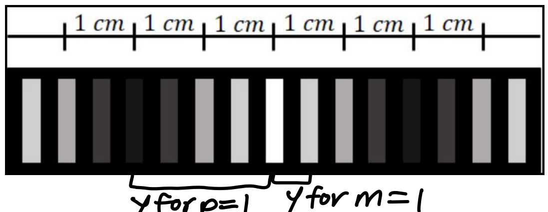
Question 1

Below is an image of the fringe patterns produced by two identical slits and light of wavelength, $\lambda = 600$ nm. Notice that there is both a single slit interference pattern and a double slit interference pattern. The pattern is produced on a screen 1.0 meters from the slits. Use the provided scale.



(a) Determine the separation between the slits. Explain your reasoning.
(b) Determine the width of one individual slit. Explain your reasoning.
(c) Use related quantities sensemaking to compare the spatial interference patterns observed and their respective mathematical models. Do this by answering the following prompts

- Comparing your answers to parts (a) and (b), was the slit separation of the double slit larger, or was the width of the single slit larger?
- Would you predict that this relationship is always the case? Explain your prediction using the mathematical models. (hint: a diagram may also be helpful here!)

a) Known:

$\lambda = 600\,nm = 6\times10^{-7}\,m$

$L = 1\,m$

$m = 1$

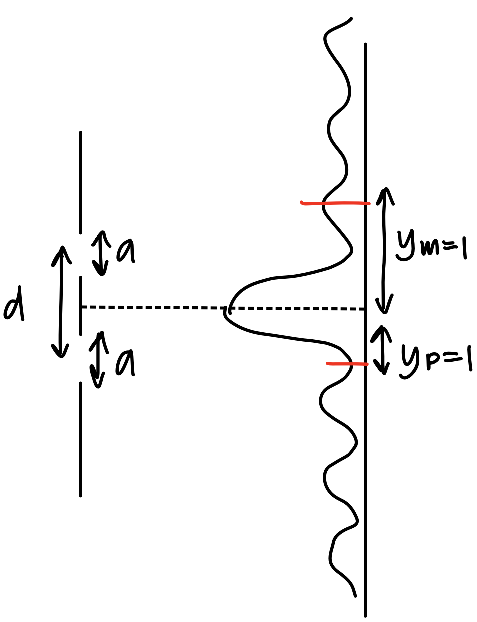
$y_{m=1} = 0.5\,cm = 0.005\,m$

$y_{p=1} = 2\,cm = 0.02\,m$

equations:

double slit:
$d\sin\theta_m = m\lambda$
$y_m = L\tan\theta_m$

single slit:
$a\sin\theta_p = p\lambda$
$y_p = L\tan\theta_p$



want to find d:

$d\sin\theta_m = m\lambda$ need to find θ first
? ? ✓ ✓

$y_m = L\tan\theta_m \Rightarrow \theta_1 = \tan^{-1}\left(\frac{y_1}{L}\right)$
✓ ✓

$\theta_1 = \tan^{-1}\left(\frac{0.005\,m}{1\,m}\right) = 0.29°$

$d\sin\theta_m = m\lambda \Rightarrow d = \frac{m\lambda}{\sin\theta_m}$

$d = \frac{1(6\times10^{-7}m)}{\sin(0.29°)}$

$d = 1.2\times10^{-4}\,m$

b) want to find a:

$a \sin\theta_p = p\lambda$ need to find θ_p first
? ? ✓✓

$y_p = L\tan\theta_p \Rightarrow \theta_1 = \tan^{-1}\left(\frac{y_1}{L}\right)$
✓ ✓ ?

$$\theta_1 = \tan^{-1}\left(\frac{0.02\,m}{1\,m}\right) = 1.15°$$

$$a\sin\theta_p = p\lambda \Rightarrow a = \frac{p\lambda}{\sin\theta_p}$$

$$a = \frac{1(6\times10^{-7}m)}{\sin(1.15°)}$$

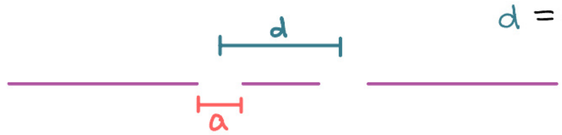
$$a = 3.0\times10^{-5}m$$

c) $d = 1.2\times10^{-4}$
$a = 3.0\times10^{-5}$ $\Big] \rightarrow d > a$

The slit separation of the double slit was larger than the width of the single slit.

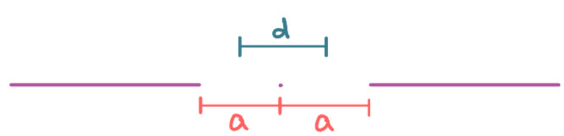
Why is d always larger than a?

d is the measure of the distance between 2 slits, measured from the center of one slit to the center of the adjacent slit.
a is the width of a single slit.



$d = \frac{1}{2}a + \frac{1}{2}a +$ distance btwn slits

To understand why $d > a$, use Special cases sensemaking to consider the case where d is at is minimum length it can be: make the distance between adjacent slits $\rightarrow 0$



Even when the distance between slits is almost zero, d still contains the distances from the center of each slit.

$$d = \frac{1}{2}a + \frac{1}{2}a + \cancelto{0}{\text{distance btwn slits}}$$

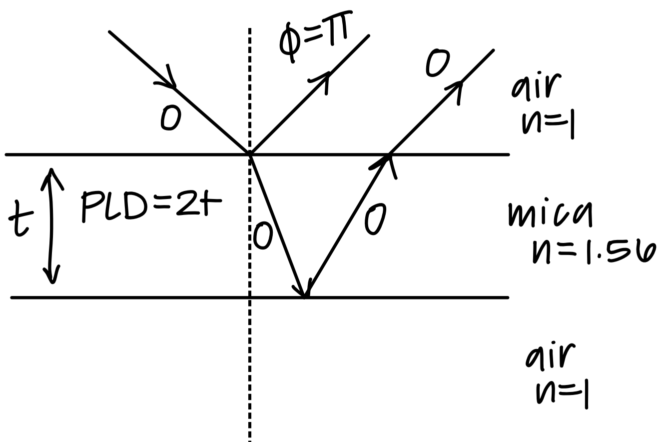
As the distance btwn slits approaches 0, d approaches the value of a

Thus, the smallest distance d can be is still greater than a, so d is always greater than a.

Question 2

Muscovite (n = 1.56), or better known as mica, is a phyllosilicate mineral of aluminum and potassium. New industrial uses include being an insulator, usually for small electrical components. Fabrication of devices with mica often require high precision in the determination of the mica thickness. If a mica sheet is suspended in air and reflected light shows gaps in the visible spectrum at 450, 525, and 630 nm, what is the thickness of the mica sheet?

Hint for a quality solution: please do not just show calculations for one thickness. Instead, show the entirety of the process you used to find the thickness. How do you rule out the many thicknesses that might work for an individual wavelength?



$\Delta\phi = \pi$

Knowns & equations

$n_{air} = 1$

$n_{mica} = 1.56$

$\lambda = 450, 525, 630$ nm

$PLD = 2t = (m+\frac{1}{2})\lambda \rightarrow$ constructive] b/c $\Delta\phi=\pi$

$PLD = 2t = m\lambda \rightarrow$ destructive

$n_1\lambda_1 = n_2\lambda_2$

There are 2 main ways to go about this problem. The first step for both are figuring out the wavelengths of light in the mica using $n_1\lambda_1 = n_2\lambda_2$.

n_1 = n of air = 1

λ_1 = wavelength in air = 450, 525, or 630 nm

n_2 = n of mica = 1.56

λ_2 = our unknowns

if $\lambda_1 = 450$ nm: $1(450\text{nm}) = 1.56\lambda_2 \Rightarrow \lambda_2 = 288.5$ nm

if $\lambda_1 = 525$ nm: $1(525\text{ nm}) = 1.56\lambda_2 \Rightarrow \lambda_2 = 336.5$ nm

if $\lambda_1 = 630$ nm: $1(630\text{ nm}) = 1.56\lambda_2 \Rightarrow \lambda_2 = 403.8$ nm

The thickness will be the same for all wavelengths since PLD=2t. We can make a table to see when t will be the same for our unknown m values of each wavelength, OR find a ratio of m values.

Table method: finding the common thickness

$PLD = 2t = m\lambda_2$

$t = \frac{m\lambda_2}{2}$ ← found above; unique for each λ_1

m	t_{450}	t_{525}	t_{630}
1	144.2	168.3	201.9
2	288.5	336.5	403.8
3	432.7	504.8	605.8
4	576.9	673.1	807.7
5	721.2	841.3	1009.6
6	865.4	1009.6	1211.5
7	1009.6	1177.9	1413.5
8	1153.8	1346.2	1615.4
9	1298.1	1514.4	1817.3

(t shown in nanometers)

The thickness is the same for all 3 wavelengths when m=5 for 630 nm, m=6 for 525 nm, and m=7 for 450 nm. This is the only thickness that would create the correct pattern of reflected light with gaps in the visible spectrum at these wavelengths.

ratio method: finding the specific m values first

$$t = \frac{m\lambda_2}{2}$$

$$t = \frac{m_1(288.5m)}{2} = \frac{m_2(336.5m)}{2} = \frac{m_3(403.8m)}{2}$$

* since all thicknesses are the same *

$\frac{m_1}{m_2} = \frac{336.5}{288.5}$ | $\frac{m_2}{m_3} = \frac{403.8}{336.5}$ | $\frac{m_1}{m_3} = \frac{403.8}{288.5}$

$\frac{m_1}{m_2} = 1.1\overline{666}$ | $\frac{m_2}{m_3} = 1.2$ | $\frac{m_1}{m_3} = 1.4$

$\frac{m_1}{m_2} = \frac{7}{6}$ | $\frac{m_2}{m_3} = \frac{6}{5}$ | $\frac{m_1}{m_3} = \frac{7}{5}$

* should be able to convert to integer fractions in calculator *

$m_1 : m_2$ — 7 : 6 | $m_2 : m_3$ — 6 : 5 | $m_1 : m_3$ — 7 : 5

$$\boxed{m_1 = 7, \quad m_2 = 6, \quad m_3 = 5}$$

$$t = \frac{7(288.5m)}{2} = \frac{6(336.5m)}{2} = \frac{5(403.8m)}{2}$$

$t = 1010 \text{ nm} = 1.01 \times 10^{-6} \text{ m}$