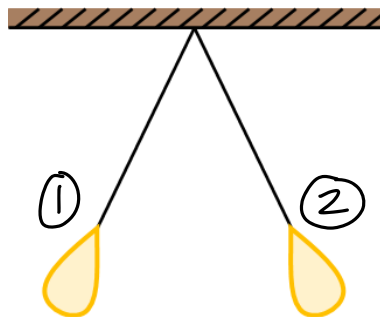


Question 1

Two balloons filled with air are tied to very light 0.5-m-long string. The loose ends of the strings are taped to the same position on a horizontal bar. When both are equally rubbed with the same material they both develop a static charge and repel each other making an angle of 30° with respect to the vertical. If you need a quantity that is not provided, please find a source and *cite it*.

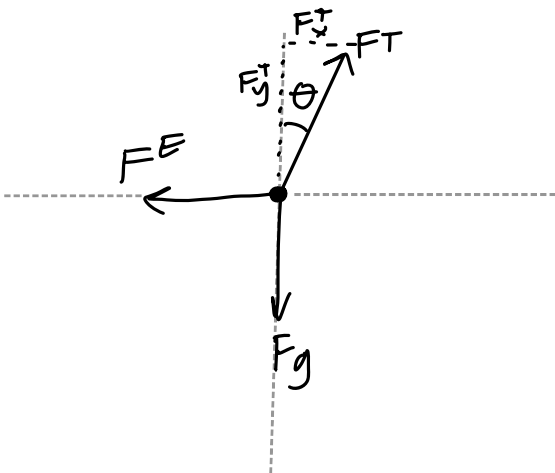
- (a) With this experiment alone can you tell what type of charge is on the balloon? Explain.
- (b) Is this a stable or unstable equilibrium?
- (c) *Estimate* the number of excess elementary charges (likely either an excess or deficit of electrons!) on each balloon. Clearly state any assumptions made in your estimation.
- (d) Use special case sensemaking to estimate the number of charges needed for the balloons to each make an angle of 90° with the vertical. Explain using the relationships (equations) between angle and charge you found in part (c). A useful tactic is to show what happens to the charge as the angle approaches 90° . Make sure to include a prediction and qualitative explanation, then compare that with the result of the mathematical relationship.
- Note: this might also be considered covariational sensemaking, or both methods in conjunction, since we are reasoning using an equation/relationship between quantities.*
- (e) If a small charged sphere is placed in the middle of the two balloons and the angle between them doubles, *estimate* the number of excess elementary charges on the sphere, again being sure to state any assumptions used in your calculation.



a) We can't tell. Because they were rubbed equally with the same material, and they are made of the same material, they will have the same charge. So, they are either both positively or both negatively charged, which leads to the balloons repelling one another.

b) Stable equilibrium. If we pushed one of the balloons it would return back to the same position.

c) FBD (balloon 1)



KNOWN:

mass of balloon $\approx 10\text{ g} = 0.01\text{ kg}$

$\theta = 30^\circ$

$\Delta r = 0.5\text{ m}$

$q_1 = q_2$

$g = 9.8\text{ m/s}^2$

$k = 9 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$

* found online, answers may vary based on mass you use! *

$$\sum F_y = F_y^T - F_y^g = m a_y^0$$

$$F_y^g = F_y^T$$

$$mg = F_y^T$$

$$0.01 \text{ kg}(9.8 \text{ m/s}^2) = F_y^T$$

$$F_y^T = 0.098 \text{ N}$$

$$\sum F_x = F_x^T - F^E = m a_x^0$$

$$F_x^T = F^E$$

$$F_y^T \tan \theta = F^E$$

$$0.098 \text{ N} \tan(30^\circ) = F^E$$

$$F^E = 0.057 \text{ N}$$

$$F^E = \frac{k |q_1| |q_2|}{|\Delta r|^2} = \frac{k q^2}{\Delta r^2} \implies q = \sqrt{\frac{F^E (\Delta r^2)}{k}}$$

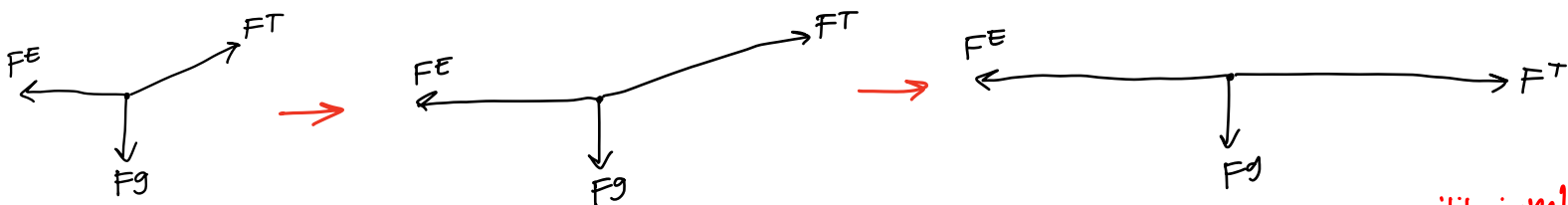
$$q = \sqrt{\frac{0.057 \text{ N} (0.5 \text{ m})^2}{9 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}}}} = 1.25 \times 10^{-6} \text{ C}$$

$$\frac{q}{e} = \frac{1.25 \times 10^{-6} \text{ C}}{1.602 \times 10^{-19} \text{ C}} = 7.83 \times 10^{12}$$

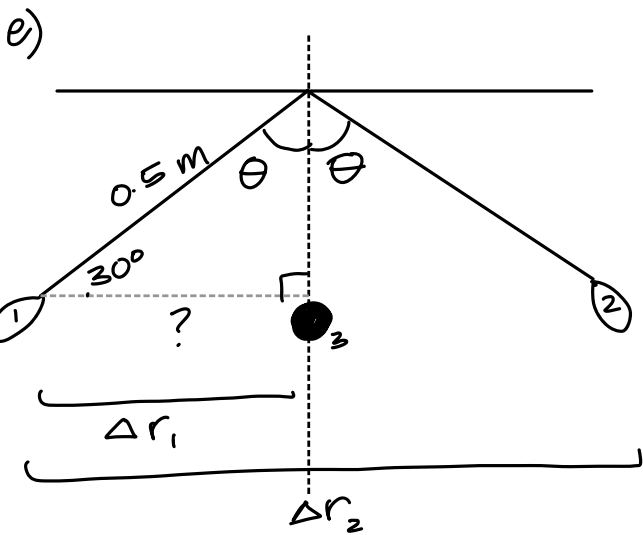
There are about 7.83×10^{12} excess electrons on each balloon.

d) It won't be possible for both balloons to make a 90° angle with the vertical. F_g will always stay the same. F^E would be at a maximum at this point. F^T will be increasing to counteract F^E . However, if both balloons are totally flat along the ceiling, F^T and F^E would be completely in the x direction. With nothing to counteract F_g , they would no longer be in equilibrium so it is not possible.

Try the math — there will most likely be a domain error as you approach 90° .



not equilibrium!



$$\theta = 60^\circ$$

$$\cos 30^\circ = \frac{?}{0.5} \Rightarrow ? = 0.5 \cos 30^\circ = 0.433 \text{ m}$$

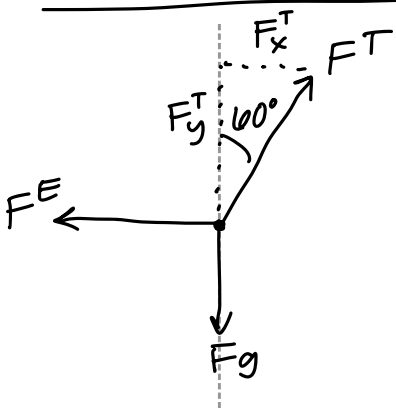
$$\Delta r_1 = 0.433 \text{ m}$$

$$\Delta r_2 = 0.866 \text{ m}$$

$$q_1 = q_2$$

$$q_3 = ?$$

FBD (balloon 1)



$$\sum F_y = F_y^T - F_g = m a_y^0$$

$$F_g = F_y^T$$

$$m g = F_y^T$$

$$0.01 \text{ kg} (9.8 \text{ m/s}^2) = F_y^T$$

$$F_y^T = 0.098 \text{ N}$$

$$\sum F_x = F_x^T - F^E = m a_x^0$$

$$F_x^T = F^E$$

$$F_y^T \tan \theta = F^E$$

$$0.098 \text{ N} \tan(60^\circ) = F^E$$

$$F^E = 0.17 \text{ N}$$

$$F^E = \frac{k q_1^2}{|\Delta r_2|^2} + \frac{k q_1 q_3}{|\Delta r_1|^2} \Rightarrow q_3 = \frac{\left(F^E - \frac{k q_1^2}{\Delta r_2^2} \right) \Delta r_1^2}{k q_1}$$

$$q_3 = \frac{\left(0.17 \text{ N} - \frac{9 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C} (1.25 \times 10^{-6} \text{ C})^2}{(0.866 \text{ m})^2} \right) (0.433 \text{ m})^2}{9 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C} (1.25 \times 10^{-6} \text{ C})}$$

$$q_3 = 2.51 \times 10^{-6} \text{ C}$$

$$\frac{q_3}{e} = \frac{2.51 \times 10^{-6} \text{ C}}{1.602 \times 10^{-19} \text{ C}} = 1.57 \times 10^{13}$$

There are about 1.57×10^{13} excess
charges on the sphere

* answers may vary again
based on balloon mass used! *