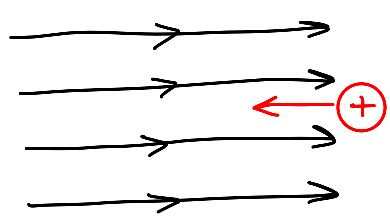
Question 1

A proton is fired directly into a uniform electric field. The field is able to momentarily reduce the charge's momentum to zero.

(a) What must be the orientation of the electric field relative to the initial momentum of the proton?
(b) What type of charge configuration would generate a uniform electric field?
(c) If the proton is initially traveling at **5% the speed of light**, what field strength will stop the charge within **1.00 cm** of entering the field region?
(d) How much work does the electric field do in stopping the proton? Answer this by finding the change in kinetic energy of the proton. Remember that work and kinetic energy are related by **W = ΔKE**.
(e) Use the self-consistency sensemaking technique to test your answers to parts (c) and (d) by answering the following prompts.
 (i) Using your answer to part (c) and the distance the proton travels into the field, find the work done on the proton using the relationship $W = \vec{F} \cdot \Delta\vec{r}$
 (ii) Compare your answers to parts (i) and (d) and show that they are in agreement.

a) The orientation must be opposite and parallel to the initial momentum of the proton. The proton will move in the same direction as the electric field, so if it slows down to zero and doesn't change direction along the x AND y axis, it must be opposite and parallel.



b) Uniformly charged infinite sheet

c) Knowns:

$V_i = 0.05(3\times10^8 m/s) = 1.5\times10^7 m/s$

$V_f = 0\ m/s$

$\Delta X = 1cm = 0.01\ m$

$m_{proton} = 1.67\times10^{-27}\ kg$

$q = 1.602\times10^{-19}\ C$

solving with kinematics:

$V_f^2 = V_i^2 + 2a\Delta X$

$a = \frac{V_f^2 - V_i^2}{2\Delta X} = \frac{0-(1.5\times10^7)^2}{2(0.01m)}$

$a = 1.125\times10^{16} m/s^2$

$\vec{F}_E = m\vec{a} = q|\vec{E}|$

$|\vec{E}| = \frac{m|\vec{a}|}{q} = \frac{1.67\times10^{-27}kg(1.125\times10^{16}m/s^2)}{1.602\times10^{-19}C}$

$|\vec{E}| = 1.17\times10^8 \frac{N}{C}$

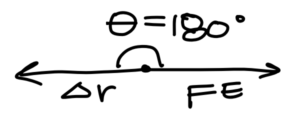
could also solve using an energy analysis!

$$E_i + W = E_f$$

$$KE_i + W = KE_f$$

$$\frac{1}{2}mv_i^2 + F^E \Delta x \cos\theta = \frac{1}{2}m\overset{0}{\cancel{v_f^2}}$$

$$F^E = \frac{-\frac{1}{2}mv_i^2}{\Delta x \cos\theta}$$



$$F^E = \frac{-\frac{1}{2}(1.67\times10^{-27}\,kg)(1.5\times10^{7}\,m/s)^2}{(0.01\,m)\underbrace{\cos(180°)}_{-1}}$$

$$F^E = 1.88\times10^{-11}\,N$$

$$|\vec{E}| = \frac{F^E}{q} = \frac{1.88\times10^{-11}\,N}{1.602\times10^{-19}\,C}$$

$$|\vec{E}| = 1.17\times10^{8}\,\frac{N}{C}$$

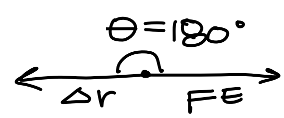
d) $W = \Delta KE = KE_f - KE_i = \frac{1}{2}m\overset{0}{\cancel{v_f^2}} - \frac{1}{2}mv_i^2$

$$W = -\frac{1}{2}mv_i^2 = -\frac{1}{2}(1.67\times10^{-27}\,kg)(1.5\times10^{7}\,m/s)^2$$

$$W = -1.88\times10^{-13}\,J$$

e)

i) $\vec{W} = \vec{F}\cdot\Delta\vec{r} = F^E \Delta x \cos\theta$



$$W = 1.88\times10^{-11}\,N\,(0.01\,m)\cos(180°)$$

$$W = -1.88\times10^{-13}\,J$$

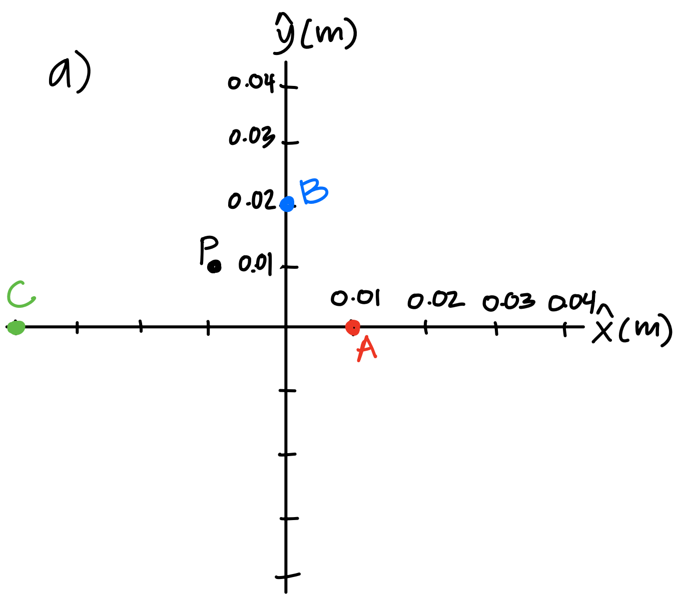
ii) The answers from part d and i are equal!

Question 2

Three **+2 μC** point charges are fixed in space in the following locations: the first at **<1, 0, 0> cm**, second at **<0, 2, 0> cm**, and a third at **<-4, 0, 0> cm**.

(a) Find the net electric field vector at position **<-1, 1, 0> cm**. (hint: draw a diagram!)
(b) At this location, what is the force vector on a **-3 μC** test charge?
(c) Use the sign sense-making technique to check that your answers to parts (b) and (a) are consistent by answering the following prompts.
  (i) Make a prediction for the relationship between the direction of an electric field vector and the direction of the force on a negatively charged particle placed in that electric field. Explain what this means for the signs of each component (X, Y, and Z) of the force and electric field.
  (ii) Compare your prediction with the answers you found in parts (a) and (b) and show that they are consistent.

a)



☆ convert to meters! ☆

Knowns & equations:

$\Delta\vec{r}_{AP} = \langle -0.02, -0.01, 0\rangle$ m

$\Delta\vec{r}_{BP} = \langle -0.01, -0.01, 0\rangle$ m

$\Delta\vec{r}_{CP} = \langle 0.03, 0.01, 0\rangle$ m

☆ $\Delta\vec{r}$ goes from charge to point of interest ☆

$$\vec{E}_q(\vec{r}_p) = \frac{kq}{|\Delta\vec{r}|^2}\Delta\hat{r}$$

$$k = 9\times10^9 \frac{N\cdot m^2}{C^2}$$

$$q_A = q_B = q_C = 2\times10^{-6}\ C$$

A: $|\Delta\vec{r}_{AP}| = \sqrt{(-0.02)^2 + (-0.01)^2 + 0^2}$ m

$|\Delta\vec{r}_{AP}| = \sqrt{5\times10^{-4}}$ m

$$\Delta\hat{r}_{AP} = \left\langle \frac{\Delta x_{AP}}{\Delta r_{AP}}, \frac{\Delta y_{AP}}{\Delta r_{AP}}, 0 \right\rangle$$

$$\Delta\hat{r}_{AP} = \left\langle \frac{-0.02}{\sqrt{5\times10^{-4}}}, \frac{-0.01}{\sqrt{5\times10^{-4}}}, 0 \right\rangle$$

$$\vec{E}_q(A) = \frac{9\times10^9 \frac{N\cdot m^2}{C^2}(2\times10^{-6} C)}{(\sqrt{5\times10^{-4}}\ m)^2} \left\langle \frac{-0.02}{\sqrt{5\times10^{-4}}}, \frac{-0.01}{\sqrt{5\times10^{-4}}}, 0 \right\rangle$$

$$\vec{E}_q(A) = \langle -3.2\times10^7, -1.6\times10^7, 0\rangle \frac{N}{C}$$

B: $|\Delta\vec{r}_{BP}| = \sqrt{(-0.01)^2 + (-0.01)^2 + 0^2}\ m$

$|\Delta\vec{r}_{BP}| = \sqrt{2\times10^{-4}}\ m$

$$\Delta\hat{r}_{BP} = \left\langle \frac{\Delta x}{\Delta r}, \frac{\Delta y}{\Delta r}, 0 \right\rangle$$

$$\Delta\hat{r}_{BP} = \left\langle \frac{-0.01}{\sqrt{2\times10^{-4}}}, \frac{-0.01}{\sqrt{2\times10^{-4}}}, 0 \right\rangle$$

$$\vec{E}_q(B) = \frac{9\times10^9 \frac{N\cdot m^2}{C^2}(2\times10^{-6}C)}{(\sqrt{2\times10^{-4}}m)^2} \left\langle \frac{-0.01}{\sqrt{2\times10^{-4}}}, \frac{-0.01}{\sqrt{\ \times10^{-4}}}, 0 \right\rangle$$

$$\vec{E}_q(B) = \langle -6.4\times10^7, -6.4\times10^7, 0 \rangle \frac{N}{C}$$

C: $|\Delta\vec{r}_{CP}| = \sqrt{(0.03)^2 + (0.01)^2 + 0^2}\ m$

$|\Delta\vec{r}_{CP}| = \sqrt{1\times10^{-3}}\ m$

$$\Delta\hat{r}_{CP} = \left\langle \frac{\Delta x}{\Delta r}, \frac{\Delta y}{\Delta r}, 0 \right\rangle$$

$$\Delta\hat{r}_{CP} = \left\langle \frac{0.03}{\sqrt{1\times10^{-3}}}, \frac{0.01}{\sqrt{1\times10^{-3}}}, 0 \right\rangle$$

$$\vec{E}_q(C) = \frac{9\times10^9 \frac{N\cdot m^2}{C^2}(2\times10^{-6}C)}{(\sqrt{1\times10^{-3}}m)^2} \left\langle \frac{0.03}{\sqrt{1\times10^{-3}}}, \frac{0.01}{\sqrt{1\times10^{-3}}}, 0 \right\rangle$$

$$\vec{E}_q(C) = \langle 1.7\times10^7, 5.7\times10^6, 0 \rangle \frac{N}{C}$$

$$\Sigma\vec{E}_q = \vec{E}_q(A) + \vec{E}_q(B) + \vec{E}_q(C)$$

$$\Sigma\vec{E}_q = \langle -3.2\times10^7, -1.6\times10^7, 0 \rangle \frac{N}{C} + \langle -6.4\times10^7, -6.4\times10^7, 0 \rangle \frac{N}{C} + \langle 1.7\times10^7, 5.7\times10^6, 0 \rangle \frac{N}{C}$$

$$\Sigma\vec{E}_q = \langle -7.9\times10^7, -4.2\times10^7, 0 \rangle \frac{N}{C}$$

b) $\vec{F}^E = q|\vec{E}|$

$\vec{F}^E = -3\times10^{-6}\,C \langle -7.9\times10^7, -4.2\times10^7, 0\rangle \frac{N}{C}$

$\vec{F}^E = \langle 237, 126, 0\rangle\ N$

c) i) $\vec{F}^E = q\vec{E}$

$\downarrow$

if q is negative $\rightarrow \vec{F}^E = -q\vec{E}$

prediction: for each component, $\vec{F}^E$ and $\vec{E}$ will need to have opposite signs for the equation to be true.

ii) our answer to part a: $\Sigma\vec{E}_q = \langle -7.9\times10^7, -4.2\times10^7, 0\rangle \frac{N}{C}$

$\vec{E}_q = \langle -, -, 0\rangle$

our answer to part b: $\vec{F}^E = \langle 237, 126, 0\rangle\ N$

$\vec{F}^E = \langle +, +, 0\rangle$

$\vec{E}_q$ and $\vec{F}^E$ have opposite signs; our prediction is true.