Question 1

In 1911, Ernest Rutherford and his assistants Geiger and Marsden conducted an experiment in which they scattered alpha particles (nuclei of helium atoms) from thin sheets of gold. An alpha particle, having charge +2*e* and mass 6.64 x 10^{-27} kg, is a product of certain radioactive decays. The results of the experiment led Rutherford to the idea that most of the atom's mass is in a very small nucleus, with electrons in orbit around it. This is the nuclear, or planetary, model of an atom. Assume an alpha particle, initially very far from a stationary gold nucleus, is fired with a velocity of 2.00 x 10^7 m/s directly toward the nucleus (charge +79*e*).

(a) What is the smallest distance between the alpha particle and the nucleus before the alpha particle reverses direction? Assume the gold nucleus remains stationary.

(b) How might your answer change if the gold nucleus was allowed to move? Explain your answer qualitatively! There is no need to perform any calculations. (Hint: think a little about energy and/or momentum conservation!)

(c) Use dimensionality sensemaking to show that the units or dimensions of your solution to part (a) work out as they should. Remember to make a prediction for the units/dimensions (probably based on what is asked for in the question prompt), then explain the units/dimensions of your solution and compare them to your prediction.

a) known:

alpha particle:

mass $= 6.64 \times 10^{-27}$ kg

$q_1 = +2e = 3.204 \times 10^{-19}$ C

$V_i = 2 \times 10^7$ m/s

$V_f = 0$ m/s

nucleus:

$q_2 = +79e = 1.27 \times 10^{-17}$ C

equations:

$KE = \frac{1}{2}mv^2$

$U^E = q_0 V = \frac{kq_1q_2}{|\Delta r|}$

$V = \frac{kq}{|\Delta r|}$

(α = alpha particle, n = nucleus)

$$KE_{i\alpha} + \cancel{KE_{in}}^{0} + \cancel{U^E_i}^{0} = \cancel{KE_{f\alpha}}^{0} + \cancel{KE_{fn}}^{0} + U^E_f$$

$$KE_i = U^E_f$$

$$\frac{1}{2}mv_i^2 = \frac{kq_1q_2}{|\Delta r|}$$

$$\Delta r = \frac{kq_1q_2}{\frac{1}{2}mv_i^2} = \frac{9\times10^9 \frac{Nm^2}{C^2}(3.204\times10^{-19}C)(1.27\times10^{-17}C)}{\frac{1}{2}(6.64\times10^{-27}kg)(2\times10^7 m/s)^2}$$

$\Delta r = 2.75\times10^{-14}\ m$

b) If the gold nucleus can move, it will want to get away from the alpha particle (like charges repel!). Some of the initial kinetic energy of the alpha particle will be transferred to the nucleus as kinetic energy. This means the final electric potential energy of the alpha particle will have decreased. So, smaller U^E = larger Δr!

(α = alpha particle, n = nucleus)

$$KE_{i\alpha} + \cancel{KE_{in}}^{0} + \cancel{U_i^E}^{0} = \cancel{KE_{f\alpha}}^{0} + KE_{fn} + U_f^E$$

$$\underbrace{KE_{i\alpha}}_{\text{same}} = KE_{fn}\uparrow + U_f^E\downarrow$$

$$U_f^E\downarrow = \frac{kq_1q_2}{|\Delta r|\uparrow}$$

q_1, k, & q_2 stay the same so Δr has to increase for U_f^E to be smaller

c) prediction: units will be meters since part a asks us to find a distance.

solve: $\Delta r = \frac{kq_1q_2}{\frac{1}{2}mv_i^2}$

$$\Delta r = \frac{\frac{Nm^2}{\cancel{C^2}}(\cancel{C})(\cancel{C})}{kg(m/s)^2} = \frac{(\cancel{kg}\cdot m/\cancel{s^2})\cancel{m^2}}{\cancel{kg}(\cancel{m^2}/\cancel{s^2})}$$

$\Delta r = m$ ← matches our prediction!