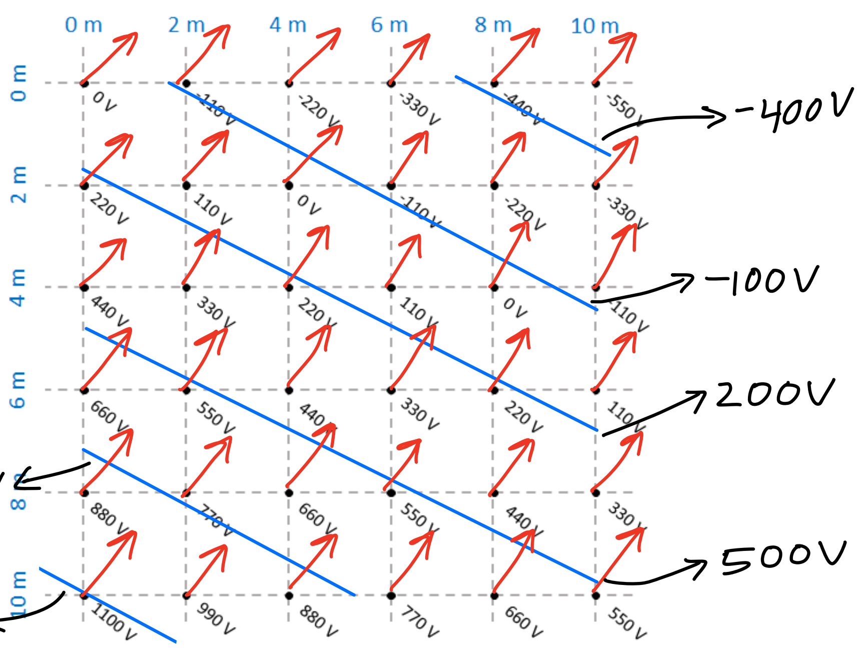
Question 1

The electric potential is mapped out everywhere in a 2D-space for a certain distribution of charge. Please be clear and precise with your solutions to the following questions.

(a) Draw a set of equipotential lines for V = -400V, -100V, 200V, 500V, 800V and 1100V.
(b) Using the vector field representation, sketch the electric field in this region.
(c) Assuming a standard coordinate system, find an algebraic equation for the **scalar** electric potential as a function of x and y.
(d) Again assuming a standard coordinate system, find an algebraic equation for the **vector** electric field as a function of x and y.
(e) Using the *self-consistency* sensemaking technique, predict from your answer to part (d) what you would expect the spacing of the equipotential lines to be (and explain why). Compare this prediction to what you found in part (a).

a) see blue lines on image

b) see red arrows on image (perpendicular to equipotential lines & pointing down electric potential "hill")



c) $V = ?\Delta x + ?\Delta y$ → how does voltage change if you move only in the x or only in the y direction

example: @ 4 meters $\hat{x}$, voltage has changed by −220V

so → $\frac{-220}{4} = -55$

example: @ 6 meters $\hat{y}$, voltage has changed by 660V

so → $\frac{660}{6} = 110$

$$V = -55\Delta x + 110\Delta y$$

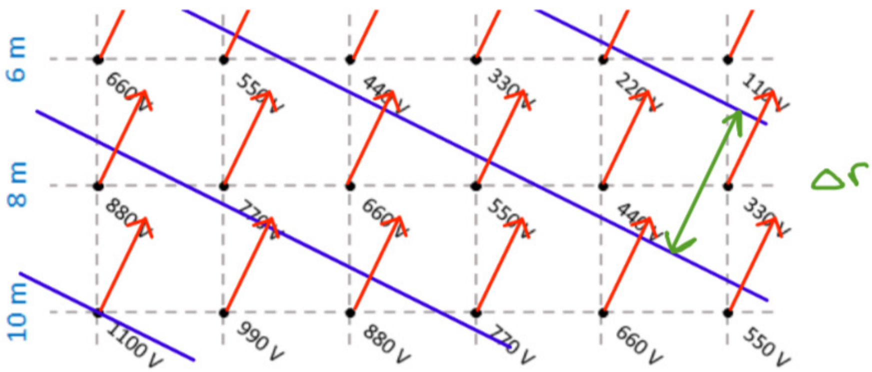
x decreases by 55V each meter
y increases by 110 V each meter

d) $\vec{E} = -\left\langle \frac{\Delta V_x}{\Delta x}, \frac{\Delta V_y}{\Delta y} \right\rangle = -\left\langle -\frac{55 \Delta x}{\Delta x}, \frac{110 \Delta y}{\Delta y} \right\rangle$

$\vec{E} = \langle 55, -110 \rangle \frac{V}{m}$

The voltage depends on change in position but the electric field does not.

e)



Based on the distance between the lines and the dimensions of the grid, the equipotential lines appear to be between 2 & 3 meters apart.

$$|E_r| = \frac{\Delta V_r}{\Delta r} \Rightarrow \Delta r = \frac{\Delta V_r}{|E_r|} = \frac{300\ V}{\sqrt{55^2 + (-110)^2}\ V/m}$$

$\Delta r = 2.44\ m$

Question 2

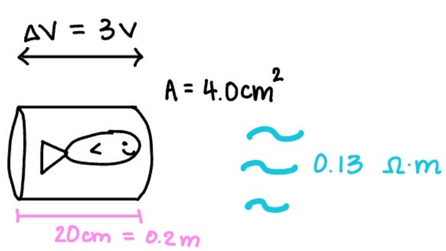
Certain fish, such as the Elephant Fish (Mormyridae), concentrate charges in their head and tail, thereby producing an electric field in the water around them. This field creates a potential difference of a few volts between the head and tail, which in turn causes current to flow in the conducting seawater. As the fish swims, it passes near objects that have a resistivity different from that of seawater, which in turn causes the current to vary. Cells in the skin of the fish are sensitive to this current and can detect changes in it. The changes in the current allow the fish to navigate. (In the next few chapters, we shall investigate how the fish might detect this current.) Since the electric field is weak far from the fish, we shall consider only the field in the vicinity of the fish. If the fish is 20 cm long (certain members of the family can reach 1.5 m), we can model the seawater through which that field passes as a conducting tube around the fish of area 4.0 cm^2 which has a potential difference of 3.0 V between its ends. The resistivity of seawater is 0.13 Ω·m. For this model let's assume that the fish has negligible width so that it does not impact the conductivity of the tube.

(a) How large is the current through the tube of the seawater?

(b) Suppose the fish swims along the side of a vertical rock face such that one quarter of the tube is now taken up by the rock. The rock has a metal content that gives it twice the conductivity of seawater. What is the difference in current that the fish detects? (Hint: How are the rock and the remaining water in the tube connected, in series or in parallel?)

(c) Use known value sensemaking to evaluate your answer to part (b). Make a prediction for whether the current should increase or decrease. Explain your prediction using conceptual arguments. Finally, compare your prediction with your found answer to part (b), does they agree?

(a)



$$4.0cm^2 \cdot \frac{(10^{-2}m)^2}{1cm^2} = 4\times10^{-4}m^2$$

Resistivity $= \rho = 0.13\ \Omega\cdot m$

$$\text{Resistance} = \frac{\rho L}{A} = \frac{(0.13\ \Omega\cdot m)(0.2\ m)}{4\times10^{-4}m^2} = 65\ \Omega$$

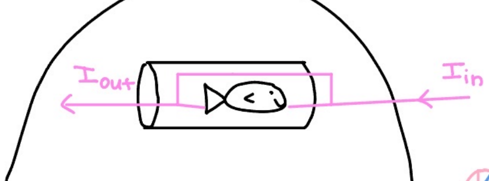
$$I = \frac{\Delta V}{R} = \frac{3V}{65\ \Omega} = 0.046\ A$$

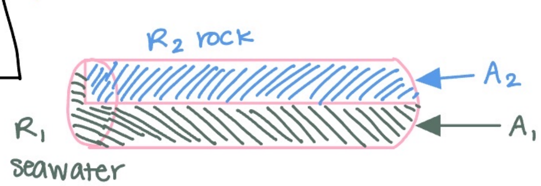
or 46.2 mA

Fish and rock resistors in parallel: current either flows thru seawater into fish or thru seawater into the rock.

$$A = 4cm^2 - \tfrac{1}{4}(4cm^2) = 3cm^2 \cdot \frac{(10^{-2}m)^2}{1cm^2} = 3\times10^{-4}m^2$$

(b)





$$R_1 = \rho_1 \frac{L}{A_1} = 0.13\,\Omega\cdot m \; \frac{0.2\,m}{3\times10^{-4}\,m^2} = 86.7\,\Omega$$

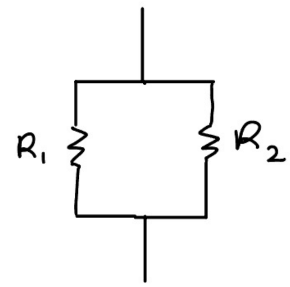
$$R_2 = \rho_2 \frac{L}{A_2} = \frac{\rho_1}{2}\cdot\frac{L}{\frac{1}{3}A_1} = \frac{0.13\,\Omega\cdot m\,(0.2\,m)}{2(\frac{1}{3})(3\times10^{-4}\,m^2)} = 130\,\Omega$$

$$\sigma_2 = 2\sigma_1$$

$$\frac{1}{\rho_2} = 2\cdot\frac{1}{\rho_1}$$

$$\rho_2 = \frac{1}{2\cdot\frac{1}{\rho_1}} = \frac{\rho_1}{2}$$

Because rock & seawater tube are in parallel:



$$R_{eq} = \frac{1}{\frac{1}{86.7\,\Omega} + \frac{1}{130\,\Omega}} = 52\,\Omega$$

$$I_{new} = \frac{V}{R_{eq}} = \frac{3V}{52\,\Omega} = 0.0577\,A \text{ or } 57.7\,mA$$

$$\Delta I = 0.0577\,A - 0.0462\,A = 0.0115\,A \text{ or } 11.5\,mA$$

c) prediction: make one!

known value: conductivity of rock is twice that of seawater,
so resistivity of rock is half that of seawater.

$$\sigma = \frac{1}{\rho} \qquad 2\sigma_1 = \sigma_2 \qquad \frac{1}{2}\rho_1 = \rho_2$$

so, our new overall resistivity is less than before.
(w/ rock vs. only seawater)

$$\underbrace{R_2}_{\downarrow} = \underbrace{\rho_2}_{\downarrow} \frac{L}{A_2} \implies \text{if } R_2 \text{ decreases, } R_{eq} \text{ decreases}$$

decreases, so $R_2 \downarrow$

so, $I = \frac{V}{R_{eq}} \implies$ w/ $R_{eq}\downarrow$, $I \uparrow$

we expect the current to increase

does your prediction match?