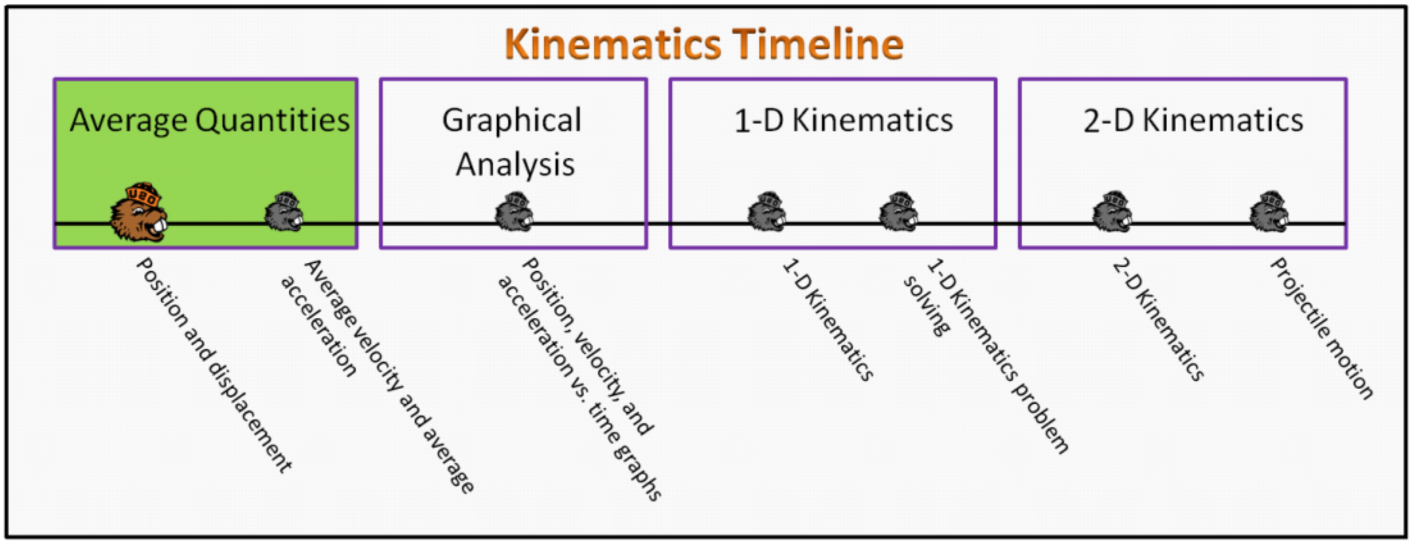
(AQ) Ryan's Space

Monday, January 22, 2018 5:44 PM

Average Quantities
Foundation Stage (AQ.2)

lecture 1
Position and Displacement



Textbook Chapters

- **BoxSand** :: KC videos (Average Quantities)
- **Giancoli** (Physics Principles with Applications 7th) :: 2-1
- **Knight** (College Physics : A strategic approach 3rd) :: 1.2 ; 2.1
- **Knight** (Physics for Scientists and Engineers 4th) :: 1.3

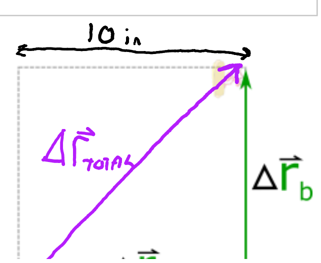
Warm up

AQ.2-1

Description: From a written description, write the mathematical representation of change in position vectors. Use vector algebra to find a resultant vector and the magnitude of the resultant vector.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: Consider a square, of side lengths **L** = 10 inches**,** drawn in ink on a piece of flat paper. Ann the ant initially starts at the bottom left corner as seen from above, then begins to walk along the edges to the opposite corner (up per right corner as seen from above).



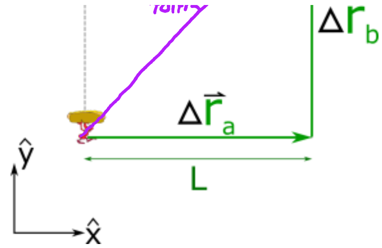
(a) Write the first change in position vector ($\Delta\vec{r}_a$) for Ann the ant from when she starts at the bottom left corner and walks horizontally to the bottom right corner.

$$\boxed{\Delta\vec{r}_a = \langle 10, 0 \rangle \text{ in}}$$

(b) Write the second change in position vector ($\Delta\vec{r}_b$) for Ann the ant from when

$\boxed{\Delta \vec{r}_a = \langle 10, 0 \rangle \text{ in}}$

(b) Write the second change in position vector ($\Delta\vec{r}_b$) for Ann the ant from when she moves from the bottom right corner to the top right corner.



$$\boxed{\Delta \vec{r}_b = \langle 0, 10 \rangle \text{ in}}$$

(c) What is Ann's total change in position from the initial starting location to the final location.

$$\Delta \vec{r}_a + \Delta \vec{r}_b = \langle 10, 0 \rangle \text{ in} + \langle 0, 10 \text{ in} \rangle$$

$$\boxed{\Delta \vec{r}_a + \Delta \vec{r}_b = \langle 10, 10 \rangle \text{ in}} \equiv \Delta \vec{r}_{TOTAL}$$

(d) How far did Ann the ant walk to get from one corner to the other.

$$\text{TOTAL DISTANCE} = |\Delta \vec{r}_a| + |\Delta \vec{r}_b|$$

$$= 10 \text{ in} + 10 \text{ in}$$

$$\boxed{\text{TOTAL DISTANCE} = 20 \text{ in}}$$

(e) How far from the starting point is Ann when she reaches the upper right corner?

$$|\Delta \vec{r}_{TOTAL}| = \sqrt{10^2 + 10^2} \text{ in} = \boxed{14.1 \text{ in}}$$

Selected Learning Objectives

1. Recognize that a vector consists of a magnitude and direction, it is multiple pieces of information, while a scalar is just a number (one piece of information).
2. Identify the difference between a vector and it's magnitude, specifically speed and velocity or distance and displacement.
3. Draw a vector and identify in the drawing the magnitude, direction, and Cartesian components.
4. Mathematically define vectors in terms of Cartesian (x, y) components or magnitude and direction and go between these representations.
5. Recognize that a vector can be translated left, right, or up and down, so long as the magnitude and direction remain unchanged.
6. Add (and subtract) vectors both pictorially with the head to tail method or parallelogram method and recognize that the resultant is equivalent in both.
7. Add (and subtract) vectors using a vector operation diagram.
8. Create vector equations from the graphical representation and vice versa.
9. Add (and subtract) vectors algebraically in component form.
10. Demonstrate that all of the vector rules are still applicable in one dimension.
11. Demonstrate the correct mathematical use of vector algebra rules.
12. Define the characteristics of position, displacement, average velocity, velocity, and average acceleration.
13. Use vectors to determine distances as opposed to using geometry, e.g. finding the displacement between two positions and then the magnitude of that vector is the distance between them.
14. Show that the position of an object depends on the origin but the change in position, the velocity, and the acceleration do not.
15. Understand the directional relationship between two vectors when the vectors are related to each other by a scalar.
16. Demonstrate that the direction of the displacement is the same as the direction of the average velocity.
17. Demonstrate that the direction of the change in velocity is the same as the direction of the average acceleration.
18. Identify kinematic terminology: Displacement is the same as the change in position.
19. Identify kinematic terminology: Distance is the same as the magnitude of the displacement.
20. Identify kinematic terminology: Speed is the same as the magnitude of velocity.
21. Identify kinematic terminology: Time interval is the same as the change in time.
22. Identify kinematic terminology: The magnitude of the acceleration is called the magnitude of acceleration.
23. Show that the total distance traveled is not the same the magnitude of the displacement.
24. Go between the physical and mathematical representation.
25. Calculate quantities involving the average velocity.
26. Calculate quantities involving the average acceleration.

27. Calculate quantities involving the position.
28. Calculate quantities involving the change in position.
29. Show when the instantaneous velocity is equal to the average velocity and that it points in the direction of motion.
30. Show that the change in any vector points from initial to the final vector when they are placed tail to tail.
31. Show that in 1 dimension, when an object is speeding up, the acceleration points in the same direction as the velocity but when it is slowing down they point in opposite directions.
32. Extrapolate the rules for 2D vectors into a 3D application.

Key Terms

- Origin
- Position
- Change-in-position (displacement)
- Distance travelled
- Magnitude of displacement

Key Equations

$$\overbrace{\Delta\vec{r}}^{\text{Change in position (displacement)}} = \overbrace{\vec{r}_f}^{\text{Final position}} - \overbrace{\vec{r}_i}^{\text{Initial position}}$$

In words: The change in position is equal to the final position minus the initial position.

Key Concepts

- The origin is the location where positions are referenced from. The x and y components at the origin are zero by definition.
- Position is a vector, and all positions start from the origin and point to a location of interest.
- A change in position, often referred to as displacement, is a vector subtraction between the initial and final position vectors and points from the head of the initial position vector to the head of the final position vector.
- Positions are dependent on the location of the origin, displacements are not dependent on the origin's location.
- The total distance travelled is not necessarily the same as the magnitude of the displacement.
- Vectors typically simplify an analysis compared to using a geometrical approach.

Act I: Position and Displacement

Questions

AQ.2-2:

Description: Estimate two position vectors in 3D using a given origin, then calculate the displacement vector. (13 minutes)

Learning Objectives: [14, 28, 32]

Problem Statement:

(a) Estimate the position vector to the location in the room defined as point **A**. Set the origin at the front-bottom-right side of the room (as seen by the students). Consider left to right as the positive x direction, bottom to top as positive y direction, and front to back as the positive z direction.
Answer in whole numbers of meters with the following format < x, y, z > m. An example might look like: < -4, 1, 7> m.

(b) Estimate the position vector to the location in the room defined as point **B**.

(c) Using the class average for position vectors **A** and **B**, find the displacement vector from point **A** to **B**. Answer in whole numbers of meters with following format: < Δx, Δy, Δz > m. An example might look like: < 4, -1, 7> m.

New Origin

(d) Estimate the position vector to the locations in the room defined as point **A** and point **B**. This time set the origin at the front-top-left side of the room (as seen by the students). Consider left to right as the positive x direction, bottom to top as positive y direction, and front to back as the positive z direction.
Answer in whole numbers of meters with the following format: $< x_A, y_A, z_A >$ m , $< x_B, y_B, z_B >$ m.
An example might look like: < 4, 1, 7 > m, < -2, 3, 6> m.

(e) Using the class average for position vectors **A** and **B** from the new origin, find the displacement vector from point **A** to **B**. Answer in whole numbers of meters with the following format: < Δx, Δy, Δz > m. An example might look like: < 4, 1, -7> m.

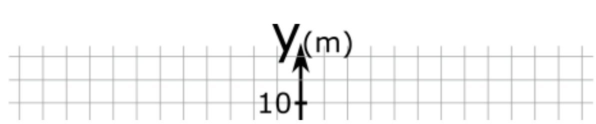
AQ.2-3:

Description: Sketch initial and final position vectors from given mathematical representations. Use a sketch of initial and final position vectors to sketch the change in position. (5 minutes)

Learning Objectives: [7, 18, 24, 27, 28]

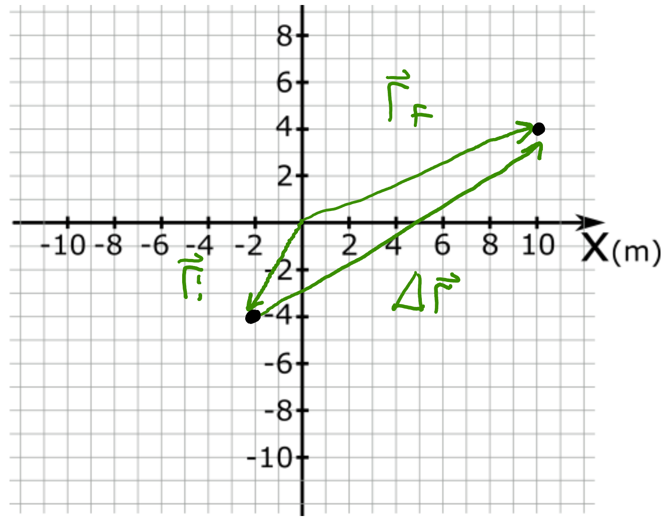
Problem Statement: Cromen the crocodile is initially spotted at a position < -2, -4 > meters from a stationary airboat. After diving back under the water, Cromen resurfaces at a position < 10, 4 > meters from the airboat.

(a) Sketch the initial position vector.



(b) Sketch the final position vector.

(c) Sketch the change in position vector.

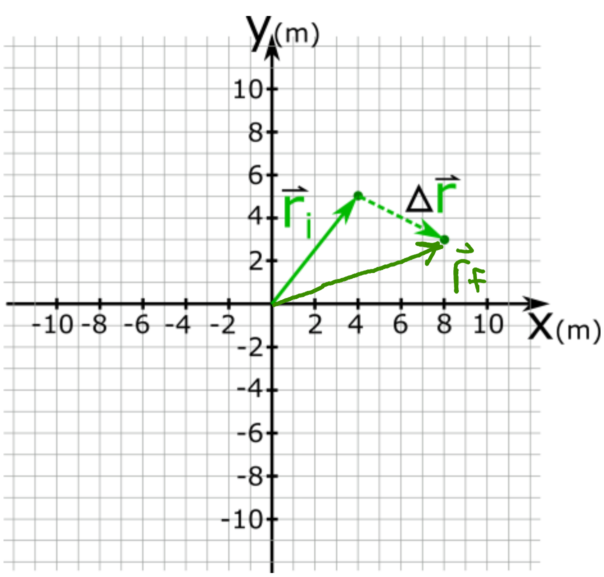


AQ.2-4:

Description: Sketch a physical representation that includes initial position, final position, and the change in position vectors and their relationship to each other. (6 minutes)

Learning Objectives: [7, 27, 28]

Problem Statement: Sketch on the physical representation below the final position vector. Note: the grid spacing between each box is 1 meter.



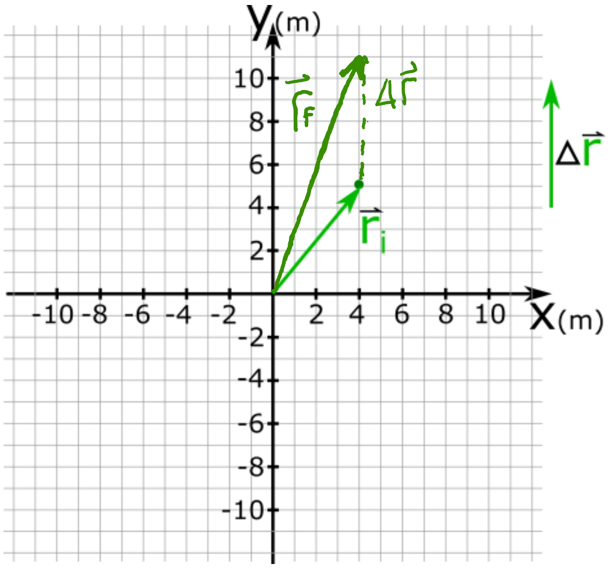
$\Delta\vec{r}$ POINTS FROM INITIAL TO FINAL

AQ.2-5:

Description: Sketch a physical representation that includes initial position, final position, and the change in position vectors and their relationship to each other (6 minutes)

Learning Objectives: [5, 7, 27, 28]

Problem Statement: Sketch on the physical representation below the final position vector. Note: the grid spacing between each box is 1 meter.



AQ.2-6:

Description: From a given path, calculate the distance traveled and the displacement. (6 minutes)

Learning Objectives: [1, 2, 19, 23]

Problem Statement:

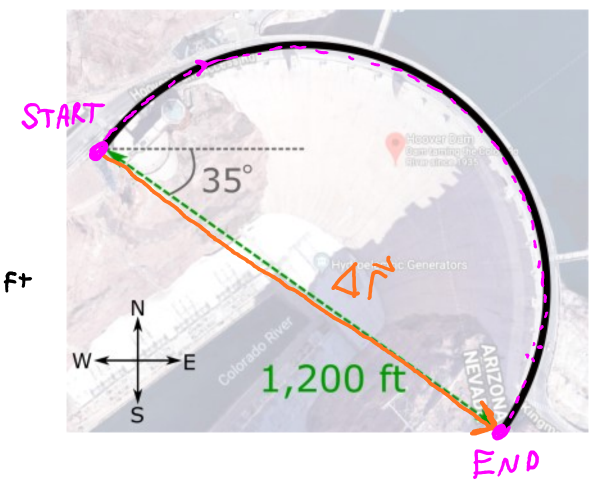
(a) Benny the beaver starts on the western most point of the hoover dam (approximated as a semi-circle as seen from above) and wobbles across the top from one end to the other. What is the total distance that Benny traveled?

(1) 600 ft
(2) 1,200 ft
(3) 1,880 ft
(4) 3,770 ft
(5) None of the above.

$$\text{DIST} = \frac{1}{2}C = \frac{1}{2}2\pi r = \pi r$$

$$\boxed{\text{DIST} = \frac{1}{2}\pi d} \simeq 1880 \text{ ft}$$

$d = 1200$ ft



(b) Which of the following is Benny's displacement? IS A VECTOR!

(1) 600 ft
(2) 1,000 ft
(3) 1,200 ft
(4) 1,900 ft
(5) None of the above

AQ.2-7:

Description: Given a displacement vectors in the mathematical model, sketch a physical representation and use vector algebra to find distances. (6 minutes)

Learning Objectives: [7, 8, 9, 11, 13, 24]

Problem Statement: Bernice the beaver walks through the following displacements sequentially: < 0, -4 > bbl, < 6, 5 > bbl, < -3, 3 > bbl, where bbl is the unit "baseball-bat-length"

(a) Draw a physical representation of Bernice's displacements.

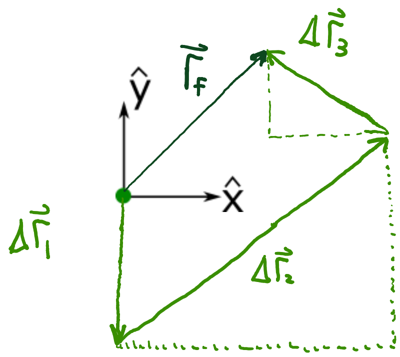
b)

$\vec{r}_f = \Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3$

$\vec{r}_f = \langle 0,-4\rangle \text{bbl} + \langle 6,5\rangle \text{bbl} + \langle -3,3\rangle \text{bbl}$

$\vec{r}_f = \langle 3,4\rangle \text{bbl}$

$|\vec{r}_f| = \sqrt{3^2+4^2}\ \text{bbl} = \boxed{5\ \text{bbl}}$



(b) How far away is Bernice from her original starting position?

AQ.2-8:

Description: Apply the rules of vector addition and subtraction to construct a vector equation. (4 minutes + 2 minutes + 4 minutes + 3 minutes + 5 minutes)

Learning Objectives: [4, 7, 8, 9, 11, 24, 27, 28]

Problem Statement: Upon walking from being hit in the head with a shovel you find yourself in the woods, next to an unfilled grave. You start running from this nightmare in a direction 51.34 degrees from the negative y-direction towards the negative x-direction. Your acute direction senses surprise you but this is no time to contemplate a new super-power. You run along that line for 6.403 miles until you run into a police officer located at a location < -2, -3 > miles from the police station. What are the coordinates of the grave you were presumably destined for?

(a) What location is the best choice to place an origin?

(1) Grave
(2) Police Officer
(3) Police Station

CAN CHOOSE ANY, BUT SINCE PO COORDINATES ARE GIVEN WITH RESPECT TO POLICE STATION, PS IS A GOOD CHOICE

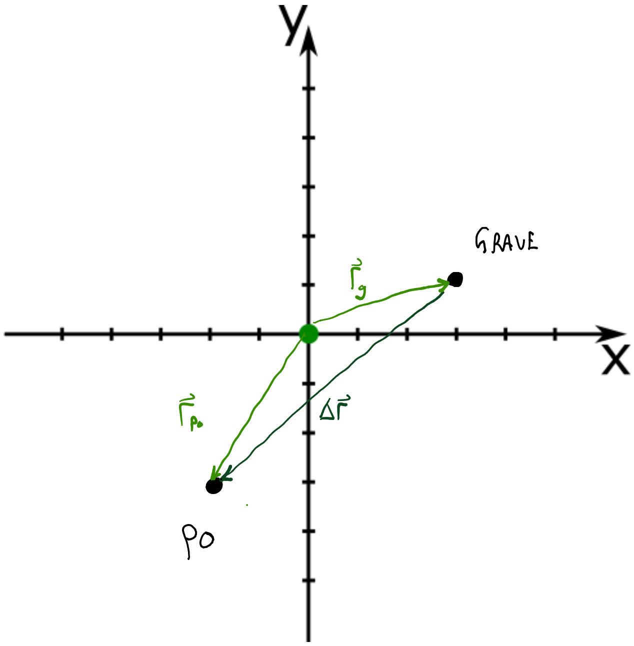
(b) Using the origin you picked from part (a), what location would a final position vector point to?

(1) Grave
(2) Police Officer
(3) Police Station

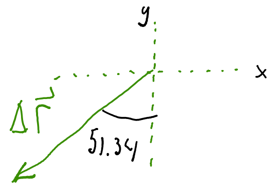
W.R.T. PS ... WAKE UP @ GRAVE, RUN TO PO

INITIAL FINAL

(c) Sketch a physical representation including the three locations, the origin, initial position, final position, and displacement vector.



(d) Find the components of the displacement vector.



$$\Delta y = -6.403\cos(51.34) = \boxed{-4.00\text{ mi}}$$

$$\Delta x = -6.403\sin(51.34) \simeq \boxed{-5.00\text{ mi}}$$

(e) With the origin from part (a), find the coordinates of the grave.

(1) < 3, 1 > miles
(2) < -3, -1 > miles
(3) < 2, 2 > miles
(4) < -2, -2 > miles
(5) < 7, 7 > miles
(6) < -7, -7 > miles

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$

$$\Delta\vec{r} = \vec{r}_{po} - \vec{r}_g$$

$$\vec{r}_g = \vec{r}_{po} - \Delta\vec{r}$$

$$= \langle -2,-3\rangle - \langle -5,-4\rangle$$

$$\vec{r}_g = \langle 3,1\rangle \text{ mi}$$

Conceptual questions for discussion

1. Describe the motion of an dog whose magnitude of displacement is equal to the total distance it travelled.
2. A person initially at the start line on a circular track. The person then runs around the track one time and returns to the same location where they started. Which of the following quantities are zero?
   i. Total distance traveled
   ii. Displacement from the start to the end
   iii. Initial position
   iv. Final position
3. Two pelicans go through the same change in position. Do both pelicans necessarily start and stop at the same locations?
4. Two people can be at different locations in space but also have the same initial position. How can this be true?

Hints

AQ.1-1: No hints.

AQ.1-2: No hints.

AQ.1-3: The tail of position vectors always start at the origin.

AQ.1-4: Displacement points from the initial position to the final position.

AQ.1-5: Displacement points from the initial position to the final position.

AQ.1-6: The circumference of a full circle is 2 π r.

AQ.1-7: Construct "tick" marks to help you scale your sketch.

AQ.1-8: **(a)** Are any vectors, with units of miles, given in reference from a location?
(c) Sketch a the direction of the initial leg of the trip from the grave to the police officer; how long do you think this is re lative to the distance between the police officer and the police station?
(e) Explicitly write out the general mathematical form of what displacement is equal to in terms of initial and final positions. Now match the displacement, initial position, and final position vectors with the components you found for each location: grave, police officer, police station.