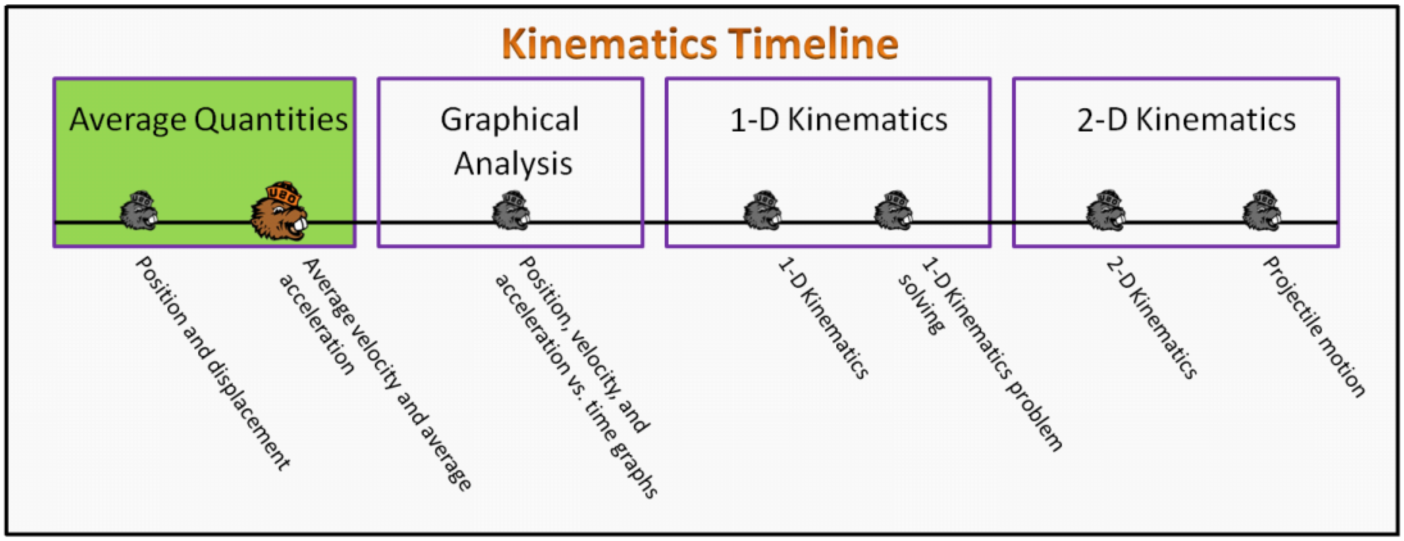
Average Quantities: Average Velocity and Acceleration

Friday, August 10, 2018 10:26 AM

Average Quantities
Foundation Stage (AQ.2)

lecture 2
Average velocity and average acceleration



Textbook Chapters

- **BoxSand** :: KC videos (Average Quantities)
- **Giancoli** (Physics Principles with Applications 7th) :: 2-2 ; 2-3 ; 2-4
- **Knight** (College Physics : A strategic approach 3rd) :: 1.3 ; 2.1 ; 2.2 ; 2.3 ; 2.4
- **Knight** (Physics for Scientists and Engineers 4th) :: 1.4 ; 1.5

Warm up

AQ.2-1

Description: Identify which quantities depend on the location and orientation of the origin.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: Which of the following physical quantities depend on the choice of origin?

(1) Position
(2) Change in position
(3) Average velocity
(4) Instantaneous velocity
(5) Average acceleration
(6) Time

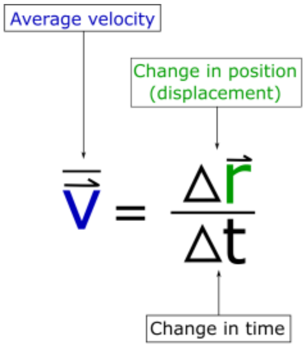
Selected Learning Objectives

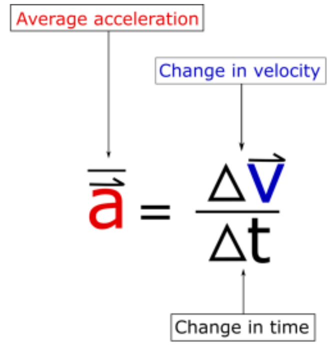
1. Recognize that a vector consists of a magnitude and direction, it is multiple pieces of information, while a scalar is just a number (one piece of information).
2. Identify the difference between a vector and it's magnitude, specifically speed and velocity or distance and displacement.
3. Draw a vector and identify in the drawing the magnitude, direction, and Cartesian components.
4. Mathematically define vectors in terms of Cartesian (x, y) components or magnitude and direction and go between these representations.
5. Recognize that a vector can be translated left, right, or up and down, so long as the magnitude and direction remain unchanged.
6. Add (and subtract) vectors both pictorially with the head to tail method or parallelogram method and recognize that the resultant is equivalent in both.
7. Add (and subtract) vectors using a vector operation diagram.
8. Create vector equations from the graphical representation and vice versa.
9. Add (and subtract) vectors algebraically in component form.
10. Demonstrate that all of the vector rules are still applicable in one dimension.
11. Demonstrate the correct mathematical use of vector algebra rules.
12. Define the characteristics of position, displacement, average velocity, velocity, and average acceleration.
13. Use vectors to determine distances as opposed to using geometry, e.g. finding the displacement between two positions and then the magnitude of that vector is the distance between them.
14. Show that the position of an object depends on the origin but the change in position, the velocity, and the acceleration do not.
15. Understand the directional relationship between two vectors when the vectors are related to each other by a scalar.
16. Demonstrate that the direction of the displacement is the same as the direction of the average velocity.
17. Demonstrate that the direction of the change in velocity is the same as the direction of the average acceleration.
18. Identify kinematic terminology: Displacement is the same as the change in position.
19. Identify kinematic terminology: Distance is the same as the magnitude of the displacement.
20. Identify kinematic terminology: Speed is the same as the magnitude of velocity.
21. Identify kinematic terminology: Time interval is the same as the change in time.
22. Identify kinematic terminology: The magnitude of the acceleration is called the magnitude of acceleration.
23. Show that the total distance traveled is not the same the magnitude of the displacement
24. Go between the physical and mathematical representation.
25. Calculate quantities involving the average velocity
26. Calculate quantities involving the average acceleration
27. Calculate quantities involving the position
28. Calculate quantities involving the change in position
29. Show when the instantaneous velocity is equal to the average velocity and that it points in the direction of motion.
30. Show that the change in any vector points from initial to the final vector when they are placed tail to tail.
31. Show that in 1 dimension, when an object is speeding up, the acceleration points in the same direction as the velocity but when it is slowing down they point in opposite directions.
32. Extrapolate the rules for 2D vectors into a 3D application

Key Terms

- Average velocity
- Instantaneous velocity
- Speed
- Change in velocity
- Average acceleration

Key Equations





In words: The average velocity is equal to the change in position divided by the change in time.

In words: The average acceleration is equal to the change in velocity divided by the change in time.

Key Concepts

- The average velocity points in the same direction as the displacement.
- The instantaneous velocity is tangent to the trajectory.
- The average acceleration points in the same direction as the change in velocity.
- If two vectors are related to each other via a positive scalar, then the two vectors must point in the same direction.
- While keeping the magnitude and direction constant, vectors can be translated left, right, up, or down to help simplify vector algebra in the physical representation.

Act I: Average velocity

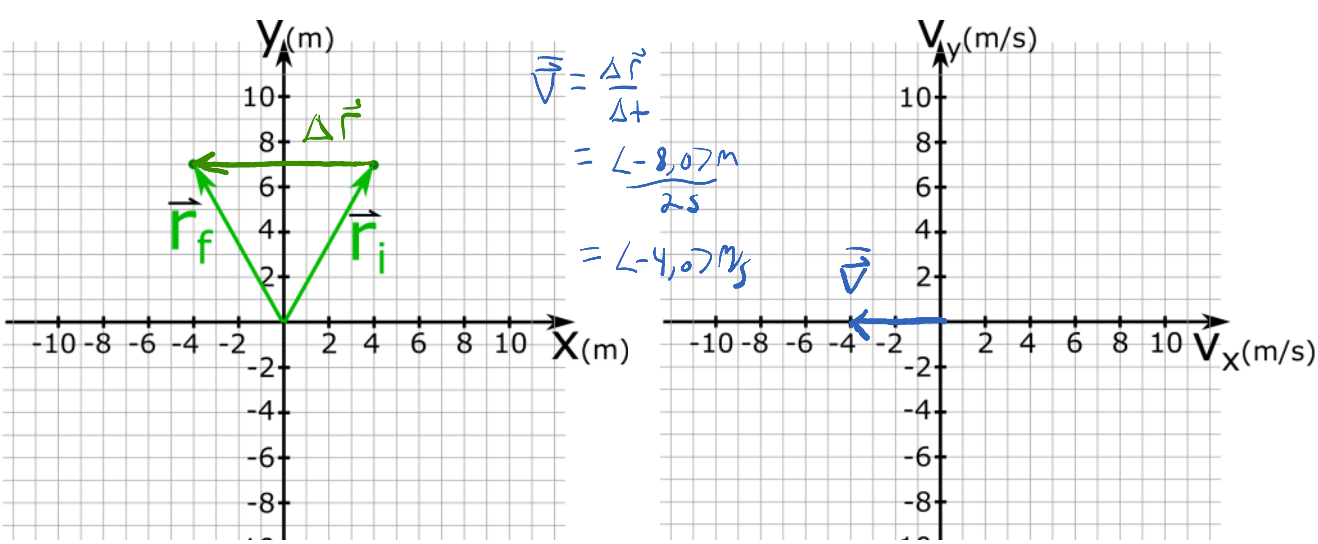
Questions

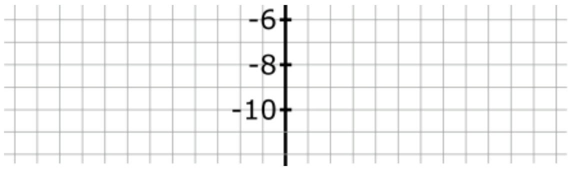
AQ.2-2:

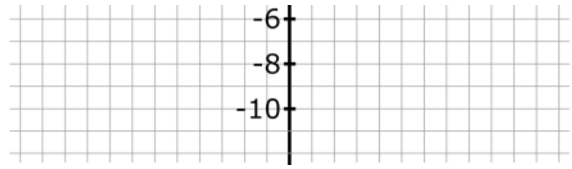
Description: Given position vectors, sketch average velocity. (5 minutes)

Learning Objectives: [4, 5, 14, 17, 19]

Problem Statement: A physical representation of two position vectors on a grid is shown below. If a Dawson the dog took 2 seconds to go from the initial position to the final position, sketch the average velocity vector of the dog on the velocity grid.







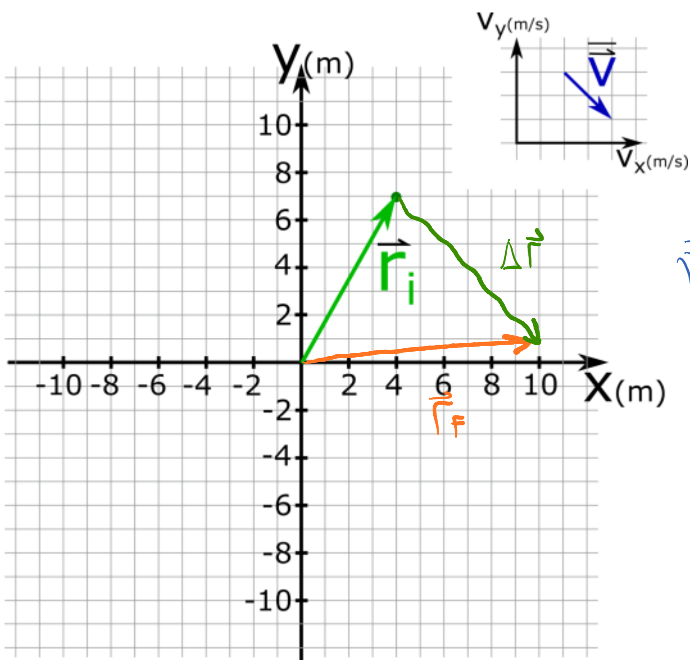
AQ.2-3:

Description: Given initial (or final) position vector and average velocity, find the final (or initial) position vector. (3 minutes + 4 minutes)

Learning Objectives: [4, 5, 14, 17, 19]

Problem Statement: Carol the cat starts at the initial position indicated below. She travels with an average velocity represented below for 3 seconds.

(a) Sketch on the grid below the final position of Carol.



(b) What is the final position vector of Carol?

(1) < -2, 12 > m
(2) - < 10, 1 > m
(3) - < -10, -1 > m
(4) < 4, -10 > m
(5) < 10, -1 > m
(6) < 10, 13 > m

$\vec{\bar{v}} = \frac{\Delta \vec{r}}{\Delta t}$

$\Delta \vec{r} = \Delta t \, \vec{\bar{v}}$

$\vec{r}_f - \vec{r}_i = \Delta t \, \vec{\bar{v}}$

$\vec{r}_f = \vec{r}_i + \Delta t \, \vec{\bar{v}}$

$= \langle 4, 7 \rangle m + 3 sec \langle 2, -2 \rangle m/s$

$= \langle 4, 7 \rangle m + \langle 6, -6 \rangle m/s$

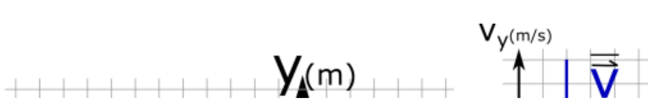
$\boxed{\vec{r}_f = \langle 10, 1 \rangle m}$

AQ.2-4:

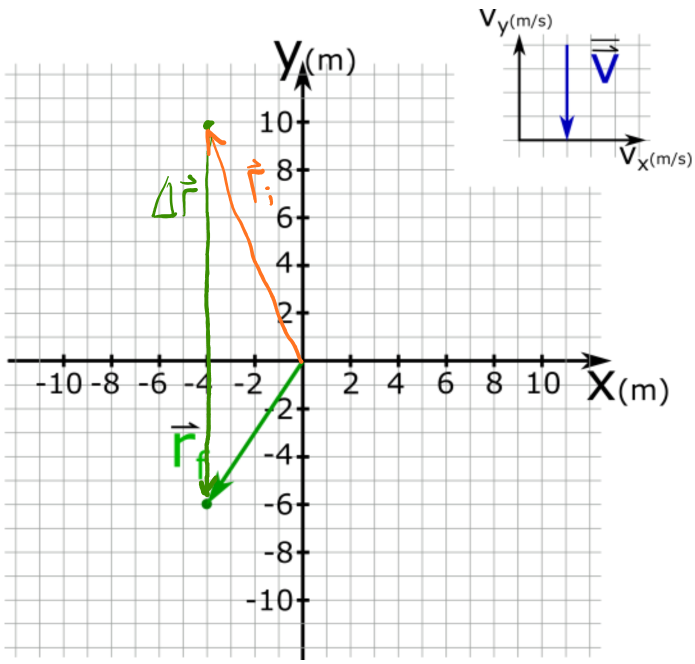
Description: Given initial (or final) position vector and average velocity, find the final (or initial) position vector. (4 minutes)

Learning Objectives: [4, 5, 14, 17, 19]

Problem Statement: Barry the bird has a final position as indicated below. Barry travels with an average velocity represented below for 4 seconds. Sketch the initial position vector of Barry.



$\vec{\bar{v}} = \frac{\Delta \vec{r}}{}$



$$\vec{v} = \frac{\Delta\vec{r}}{\Delta t}$$

$$\Delta\vec{r} = \Delta t\,\vec{v}$$

$$\vec{r}_f - \vec{r}_i = \Delta t\,\vec{v}$$

$$\vec{r}_i = \vec{r}_f - \Delta t\,\vec{v}$$

$$= \langle -4, -6 \rangle m - 4\,sec\,\langle 0, -4 \rangle m/s$$

$$= \langle -4, -6 \rangle m - \langle 0, -16 \rangle m$$

$$\boxed{\vec{r}_i = \langle -4, 10 \rangle m}$$

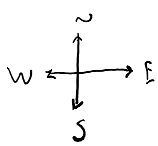
AQ.2-5:

Description: Given displacements vectors in the mathematical representation, calculate the average velocity. (8 minutes)

Learning Objectives: [1, 14, 17]

Problem Statement: A person walks 12 m in a direction 40° east of north for 45 seconds. They then walk a displacement of < 23.6, 4.17 > m for 1 minute. What was their overall average velocity? Assume a standard coordinate system.

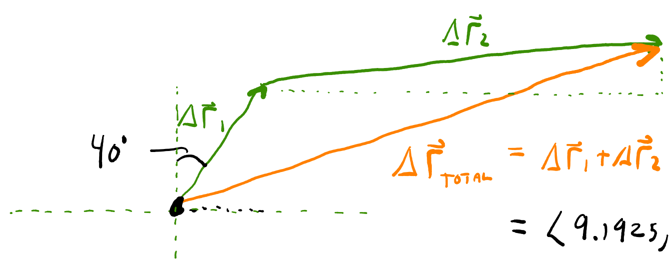
(1) < 32.8 , 11.9 > m/s
(2) < 0.312 , 0.113 > m/s
(3) < 16.4 , 5.94 > m/s
(4) < 15.7 , 6.68 > m/s
(5) 17.1 m/s



$\hat{y}$, $\hat{x}$

$$\Delta x_1 = 12m\cos(40) \approx 9.1925m$$

$$\Delta y_1 = 12m\sin(40) \approx 7.7135m$$



$$\Delta\vec{r}_{TOTAL} = \Delta\vec{r}_1 + \Delta\vec{r}_2$$

$$= \langle 9.1925, 7.7135 \rangle + \langle 23.6, 4.17 \rangle$$

$$\vec{v}_{TOTAL} = \frac{\Delta\vec{r}_{TOTAL}}{\Delta t_{TOTAL}} \qquad \Delta\vec{r}_{TOTAL} = \langle 32.7925, 11.8835 \rangle m$$

$$\Delta t_{TOTAL} = 45 + 60 = 105\,sec$$

$$\boxed{\vec{v} = \langle 0.312, 0.113 \rangle m/s}$$

*NOTE $\vec{v} \neq \vec{v}_1 + \vec{v}_2$

*NOTE $\vec{V} \neq \frac{\vec{V}_1 + \vec{V}_2}{2}$

AQ.2-6:

Description: Sketch an instantaneous velocity vector and average velocity vector given a trajectory. (5 minutes)

Learning Objectives: [1, 5, 18]

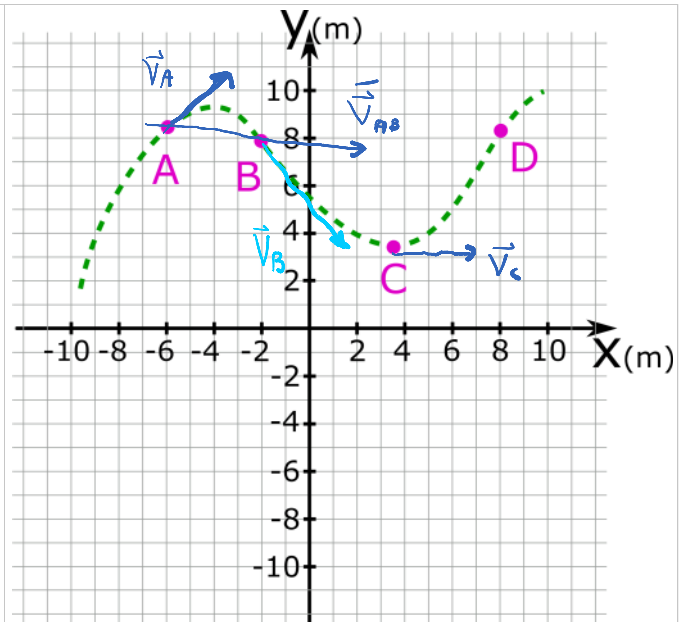
Problem Statement: On the position grid below, the dashed green line represents the trajectory for Camila the camel as she wonders through a desert. Camila passes through location **A** first, then **B**, then **C**, and then **D**.

(a) Sketch the direction of the instantaneous velocity vector at location **A**.

(b) Sketch the direction of the instantaneous velocity vector at location **B**.

(c) Sketch the direction of the average velocity vector between locations **A** and **B**.

(d) Sketch the direction of the instantaneous velocity vector at location **C**.



Act 2: Average acceleration

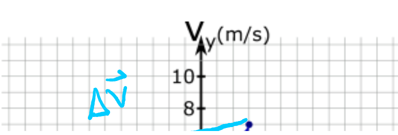
AQ.2-7:

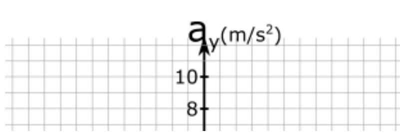
Description: Given velocity vectors, sketch the average acceleration. (3 minutes + 3 minutes)

Learning Objectives: [15, 17, 26]

Problem Statement: A GPS recorded the velocity of a car at two different times which were 2 seconds apart and plotted this information on the velocity grid shown below.

(a) Sketch the average acceleration on the acceleration grid to the right.

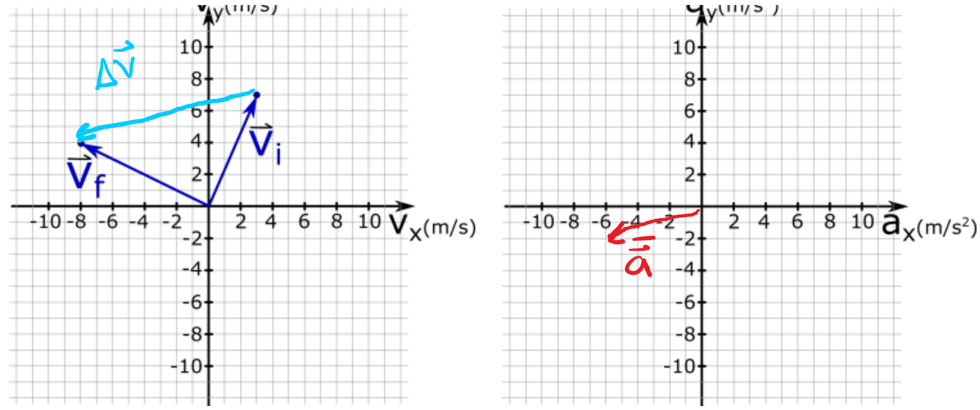




(b) What is the magnitude of the average acceleration?

$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$

$\langle -11, -37 \rangle$ m/s



$\vec{a} = \frac{\Delta v}{\Delta t}$

$= \frac{\langle -11, -3 \rangle \text{ m/s}}{2 \text{ s}}$

$\vec{a} = \langle -5.5, -1.5 \rangle \text{ m/s}^2$

$|\vec{a}| = \sqrt{(5.5)^2 + (1.5)^2} \text{ m/s}^2$

$\approx \boxed{5.70 \text{ m/s}^2}$

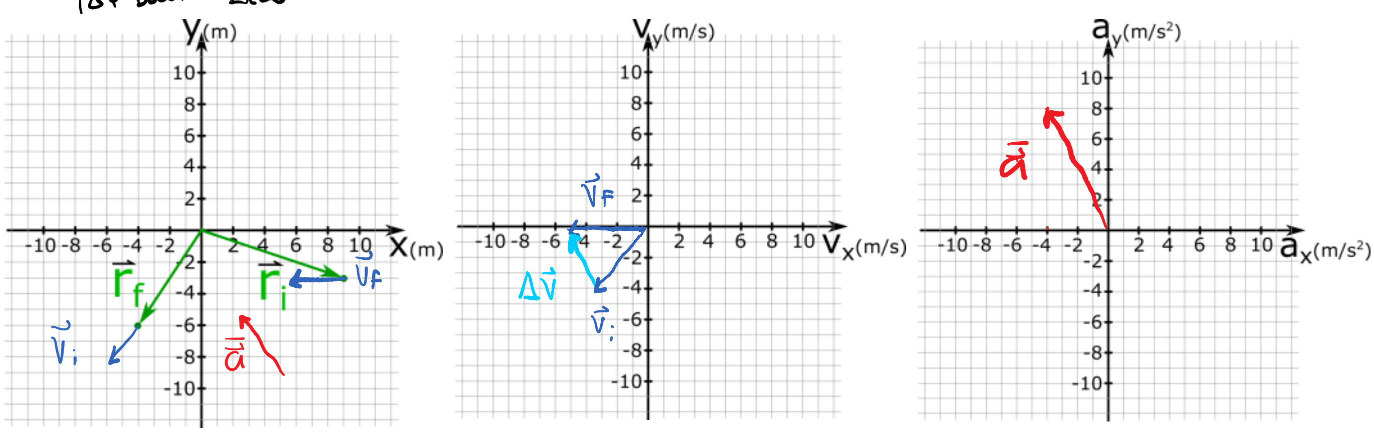
AQ.2-8:

Description: Given position vectors and velocity vectors, sketch the average acceleration vector. (6 minutes)

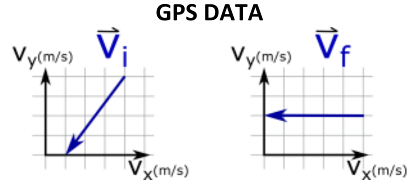
Learning Objectives: [5, 15,17, 26]

Problem Statement: A physical representation of two position vectors on a gird is show below. Hugo the horse took 0.5 seconds to trot from the initial position to the final position. A GPS was used to record Hugo's instantaneous velocity at both the initial position and final position. These velocities are shown on the velocity grids below. Sketch the average acceleration vector on the acceleration grid.

TOP DOWN VIEW



GPS DATA



$\Delta\vec{v} = \langle -2, 4 \rangle \text{ m/s}$

$\vec{a} = \frac{\langle -2, 4 \rangle \text{ m/s}}{0.5 \text{ s}} = \langle -4, 8 \rangle \text{ m/s}^2$

Conceptual questions for discussion

1. If two vectors are related to each other via a scalar, do the two vectors necessarily point in the same direction?
2. A person initially at the start line on a circular track. The person then runs around the track one time and returns to the same

location where they started. Which of the following quantities are zero?

i. Total distance traveled
ii. Displacement from the start to the end
iii. Initial position
iv. Final position
v. Total time.
vi. Average velocity.
vii. Average acceleration.

Hints

AQ.2-1: No hints.

AQ.2-2: Sketch the change in position vector on the position grid first.

AQ.2-3: The average velocity points in the same direction as the displacement.

AQ.2-4: The average velocity points in the same direction as the displacement.

AQ.2-5: Draw a physical representation of the change in position and label the known sides and angles.

AQ.2-6: No hints.

AQ.2-7: Sketch the change in velocity vector on the velocity grid first.

AQ.2-8: If you keep the magnitude and direction of a vector the same, you can move the vector anywhere on the page. Perhaps use this to move the velocity vectors onto the velocity grid.