IM.2.L1.sols: Impulse and Momentum

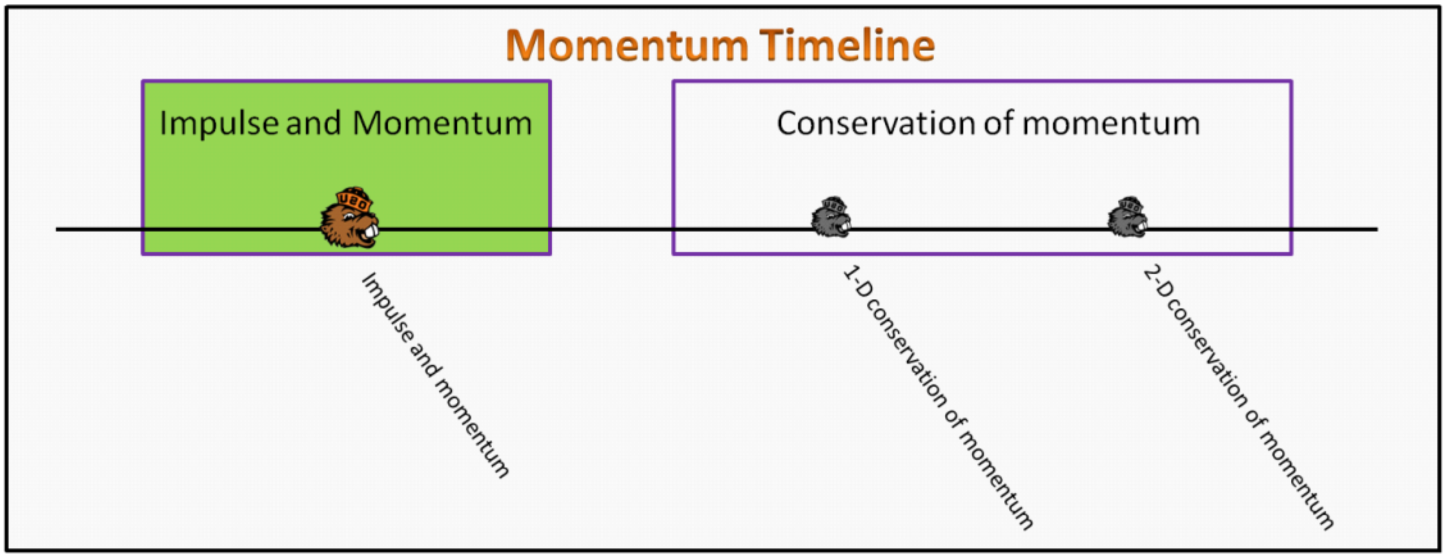
Monday, January 22, 2018 5:44 PM

Impulse and Momentum

Foundation Stage (IM.2.L1)

lecture 1

Impulse and Momentum



Textbook Chapters

- **BoxSand** :: KC videos (Impulse)
- **Giancoli** (Physics Principles with Applications 7th) :: 7-1 ; 7-3 ;
- **Knight** (College Physics : A strategic approach 3rd) :: 9.1 ; 9.2 ; 9.3
- **Knight** (Physics for Scientists and Engineers 4th) :: 11.1

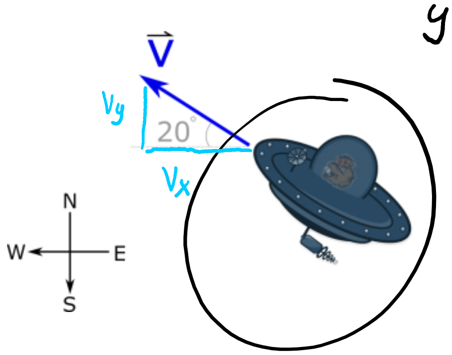
Warm up

IM.2.L1-1:

Description: Calculate momentum of a single object.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: Consider a 100-kg-spaceship floating along with a constant velocity of 400 m/s in the galactic northwest direction as shown in the image below. Momentum ($\vec{p}$) is defined as mass times velocity. What is the momentum of the spaceship. Use a standard coordinate system.

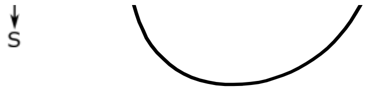


$\hat{y}$ $\hat{x}$

$\vec{p} = m\vec{v}$

$\vec{p} = m\langle v_x, v_y\rangle$

$= m\langle -|\vec{v}|\cos 20^\circ, |\vec{v}|\sin 20^\circ\rangle$



$$= 100\,\text{kg} \langle -400\,\tfrac{m}{s} \cos 20°,\ 400\,\tfrac{m}{s} \sin 20° \rangle$$

$$= 100\,\text{kg} \langle -376\,\tfrac{m}{s},\ 137\,\tfrac{m}{s} \rangle$$

$$\boxed{\vec{p} = \langle -37600,\ 13700 \rangle\ \tfrac{kg\,m}{s}}$$

Selected Learning Objectives

1. Determine the magnitude of the momentum for various objects.
2. Conclude that momentum is a vector because it is the product of a vector and a scalar.
3. Determine the momentum of a system including multiple objects.
4. Demonstrate it is harder to stop an object with larger momentum.
5. Recognize that the impulse-momentum theorem can be derived from Newton's 2nd and 3rd laws.
6. Define impulse as the change in momentum or equivalently as the average net force multiplied by the time elapsed during the interaction.
7. Determine the change in momentum using the appropriate vector operation diagram.
8. Determine the change in momentum using the mathematic representation.
9. Show that impulse is equal to the area under a net force as a function of time curve.
10. Identify that for a system with two objects where the net external force is zero, the impulse on one is equal and opposite of other because of Newton's 3rd law, and thus the net impulse of the system is zero.
11. Recognize that impulse, change in momentum, net force, acceleration, and change in velocity are all vectors that point in the same direction.
12. Use Newton's 3rd law to show that the contribution of the impulse from one object is negative the impulse from another object during the interaction between the two.

Key Terms

- Momentum
- Change in momentum
- Impulse
- Impulse-Momentum theorem
- Collision

Key Equations

Momeuntum | Mass | Velocity

$$\vec{p} = m\,\vec{v}$$

In words: The momentum of an object is equal to the mass of the object multipled by the velocity of the object.

Change in momeuntum | Mass | Change in velocity

$$\Delta\vec{p} = m\,\Delta\vec{v}$$

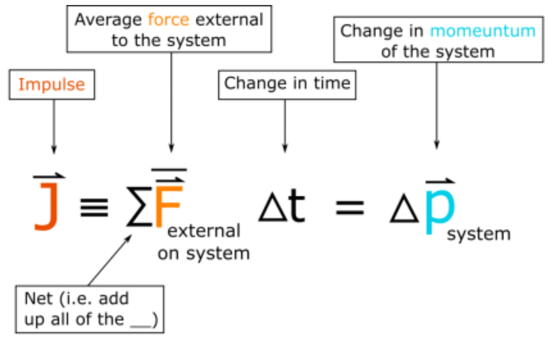
In words: The change momentum of an object is equal to the constant mass of the object multipled by the change in velocity of the object.

Average force external to the system | Impulse | Change in time | Net (i.e. add up all of the __)

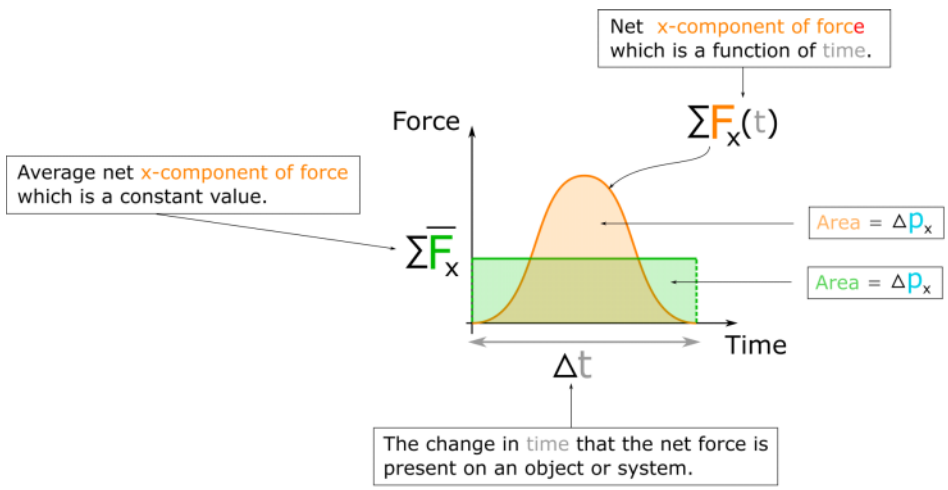
$$\vec{J} \equiv \sum \vec{\overline{F}}_{\substack{\text{external}\\ \text{on system}}}\ \Delta t$$

In words: Impulse is defined as the average net force external to the system multiplied by the change in time that the average net force is present.

Impulse-Momentum Theorem



In words: Impulse, which is defined as the average net force external to the system multipled by the change in time that the average net force is present, is equal to the change in momentum of the system .



In words: The area under a x-component of net force graph vs time is equal to the change in the x-component of momentum. There also exists an average x-component of net force which is a constant value such that the area under this average x-component of net force is equal to the same change in x-component of momentum.

Key Concepts

- The change in momentum is proportional to the net external force, which is proportional to acceleration, which is proportiona l to change in velocity. Therefore, all the above vector quantities point in the same direction.
- The change in time seen in the definition of impulse is the time interval that the external forces are acting on the system.
- The momentum of a system with more than one object is the summation of all of the individual momentum of each object within t he system.
- The momentum-impulse theorem has net force in its definition, thus a force analysis may also be needed when analyzing a system with impulse.

Act I: Momentum and change in momentum

Questions

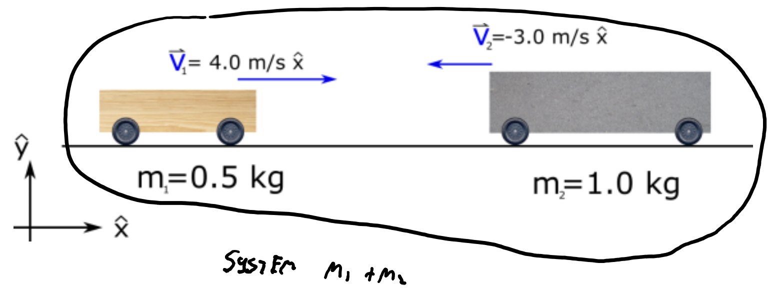
IM.2.L1-2:

Description: Find the momentum of a 1-D system consisting of multiple objects. (4 minutes)

Learning Objectives: [3]

Problem Statement: Two carts are moving along a horizontal track as shown below. Consider a system consisting of both the carts. What is the total momentum of the system?

(1) 1 kg·m/s $\hat{x}$
(2) -1 kg·m/s $\hat{x}$
(3) 5 kg·m/s $\hat{x}$
(4) -5 kg·m/s $\hat{x}$



$$\vec{P}_{sys} = \vec{P}_1 + \vec{P}_2$$

$$\vec{P}_{sys} = m_1 \vec{V}_1 + m_2 \vec{V}_2$$

$$= m_1 \langle V_{1x}, V_{1y} \rangle + m_2 \langle V_{2x}, V_{2y} \rangle$$

$$= 0.5\,kg \langle 4\,m/s, 0 \rangle + 1\,kg \langle -3\,m/s, 0 \rangle$$

$$= \langle 2, 0 \rangle \frac{kg\,m}{s} + \langle -3, 0 \rangle \frac{kg\,m}{s}$$

$$\boxed{\vec{P}_{sys} = \langle -1, 0 \rangle \frac{kg\,m}{s}}$$

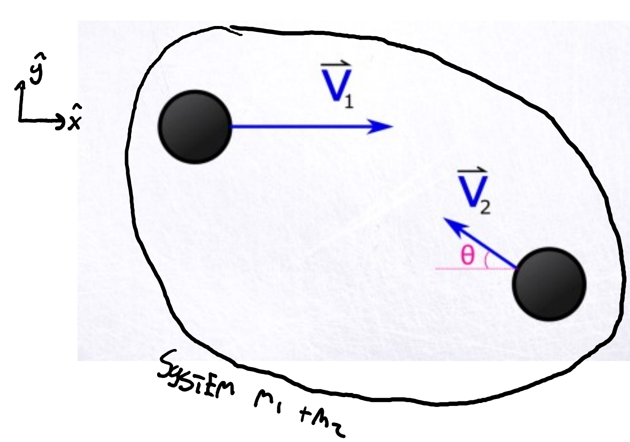
IM.2.L1-3:

Description: Find the momentum of a 2-D system consisting of multiple objects. (5 minutes)

Learning Objectives: [3]

Problem Statement: Two hockey pucks are sliding across an ice rink as shown in the figure below. Both pucks have a mass of 0.165 kg. The speed of puck 1 is 10 m/s and the speed of puck 2 is half the speed of puck 1. Puck 2 is moving at an angle of 30 degrees with respect to the horizontal. Consider a system consisting of both of the hockey pucks. What is the total momentum of the system in a standard coordinate system?

(1) < 0.936 , 0.413 > kg·m/s
(2) < 2.36 , 0.413 > kg·m/s
(3) < 1.24 , 0.714 > kg·m/s
(4) < 2.0625 , 0.714 > kg·m/s



$$|\vec{V}_2| = \tfrac{1}{2} |\vec{V}_1|$$
$$= 5\,m/s$$

$$\vec{P}_{sys} = \vec{P}_1 + \vec{P}_2$$

$$= m_1 \vec{V}_1 + m_2 \vec{V}_2$$

$$= m_1 \langle |\vec{V}_1|, 0 \rangle + m_2 \langle -|\vec{V}_2| \cos\theta, |\vec{V}_2| \sin\theta \rangle$$

$$= 0.165\,kg \langle 10\,m/s, 0 \rangle + 0.165\,kg \langle -5\cos 30, 5\sin 30 \rangle$$

$$= 0.165\ \text{kg} \langle 10\ \text{m/s}, 0 \rangle + 0.165\ \text{kg} \langle -5\cos 30, 5\sin 30 \rangle$$

$$= \langle 1.65, 0 \rangle \frac{\text{kg m}}{\text{s}} + \langle -0.71447, 0.4125 \rangle \frac{\text{kg m}}{\text{s}}$$

$$\boxed{\vec{P}_{sys} = \langle 0.936, 0.413 \rangle \frac{\text{kg m}}{\text{s}}}$$

IM.2.L1-4:

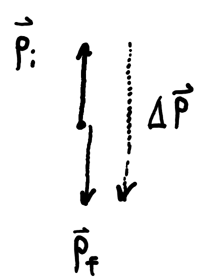
Description: Determine direction of change in momentum. (3 minutes)

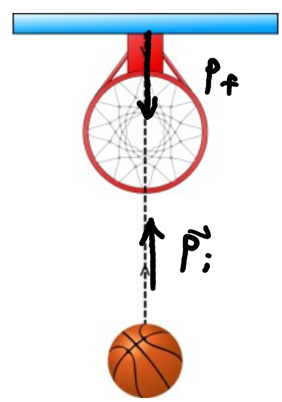
Learning Objectives: [7,11]

Problem Statement: Three basketball players are shooting hoops and bouncing each shot off the backboard. What is the direction of the change in momentum vector, from the moment before the ball hits the backboard to the moment after it leaves the backboard? The dashed line shows the trajectory of the ball from this top view. (Ignore gravity)

(1) Upwards
(2) Downwards
(3) Left
(4) Right
(5) Cannot determine without knowing the initial and final speed of the ball.

VECTOR OPERATION





* NOTE: $\Delta\vec{p} \propto \Sigma\vec{F} \propto \vec{a} \propto \Delta\vec{v}$

ALL THE ABOVE QUANTITIES POINT IN SAME DIRECTION

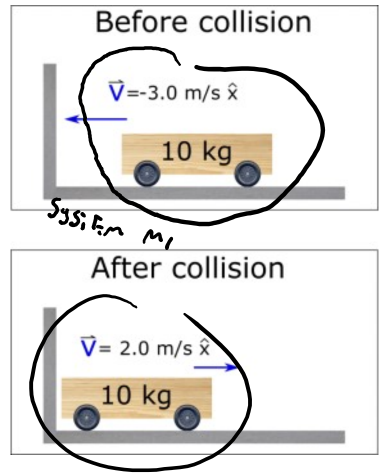
IM.2.L1-5:

Description: Calculate the change in momentum for a 1-D system with one object. (4 minutes)

Learning Objectives: [8,11]

Problem Statement: A cart is initially traveling to the left with a speed of 2 m/s before colliding with a wall. After the collision, the cart is now travelling to the right with a speed of 1 m/s. What is the change in momentum of the cart?

(1) 50 kg·m/s $\hat{x}$
(2) -50 kg·m/s $\hat{x}$
(3) 10 kg·m/s $\hat{x}$
(4) -10 kg·m/s $\hat{x}$



$$\Delta \vec{P}_1 = \vec{P}_f - \vec{P}_i$$

$$= m_1 \vec{V}_{1f} - m_1 \vec{V}_{1i}$$

$$= m_1 (\underbrace{\vec{V}_{1f} - \vec{V}_{1i}}_{\Delta \vec{V}})$$

$$= 10\,kg \left(\langle 2\,m/s, 0 \rangle - \langle -3\,m/s, 0 \rangle \right)$$

$$= 10\,kg \, \langle 5, 0 \rangle \, m/s$$

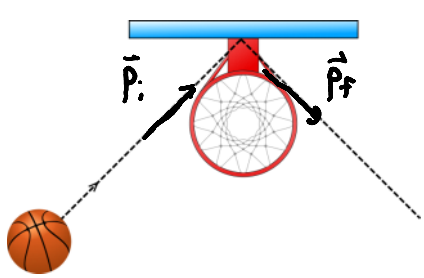
$$\boxed{\Delta \vec{P}_1 = \langle 50, 0 \rangle \, \frac{kg\,m}{s}}$$

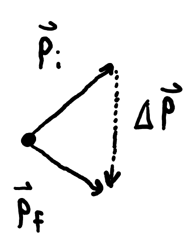
IM.2.L1-6:

Description: Sketch the change in momentum vector. (4 minutes)

Learning Objectives: [7,11]

Problem Statement: Three basketball players are shooting hoops and bouncing each shot off the backboard. Sketch the change in momentum vector of the basketball assuming its speed is the same before and after the collision with the backboard. (Ignore gravity)



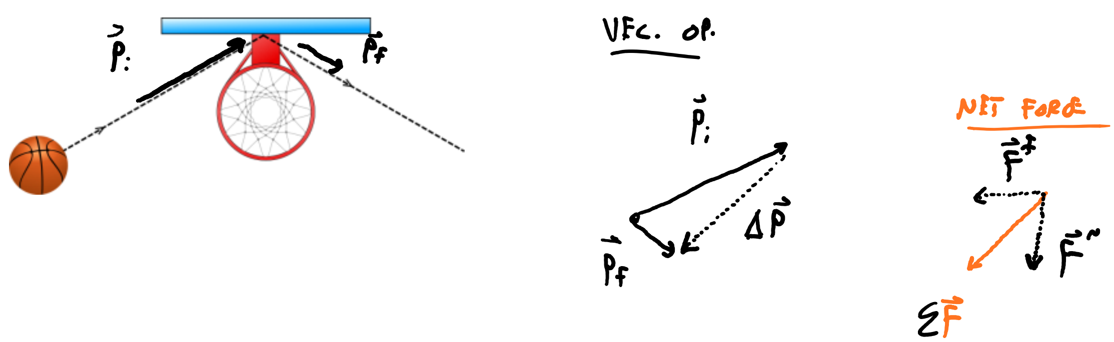


IM.2.L1-7:

Description: Sketch the change in momentum vector. (4 minutes)

Learning Objectives: [7,11]

Problem Statement: Three basketball players are shooting hoops and bouncing each shot off the backboard. Sketch the change in momentum vector of the basketball assuming its speed is reduced during the bounce off the backboard. (Ignore gravity)



Act II: Momentum and Impulse

IM.2.L1-8:

Description: Proportional reasoning with impulse momentum theorem and Newton's 2nd law. (8 minutes)

Learning Objectives: [6, 8]

Problem Statement: The diagram depicts two pucks on a frictionless table. Puck 2 is four times as massive as puck 1. Starting from rest, the pucks are pushed across the table by two equal forces. The forces act on both of them for 6.0 seconds. Rank the final momentum of the two pucks.

(1) $\vec{p}_{1f} > \vec{p}_{2f}$

(2) $\vec{p}_{1f} < \vec{p}_{2f}$

(3) $\vec{p}_{1f} = \vec{p}_{2f}$

(4) Need values of the force to rank their final momentum.

FORCE + TIME ..... MOMENTUM - IMPULSE

$$\sum \vec{F}_{Ext} \Delta t = \Delta \vec{P}$$

$$\sum \vec{F}_1 = \sum \vec{F}_2$$

AND

$$\Delta t_1 = \Delta t_2$$

THUS

$$\Delta \vec{P}_1 = \Delta \vec{P}_2$$

$$\vec{P}_{1f} - \cancel{\vec{P}_{1i}}^{\,0} = \vec{P}_{2f} - \cancel{\vec{P}_{2i}}^{\,0}$$ BOTH AT REST

$$\vec{P}_{1f} = \vec{P}_{2f}$$

IM.2.L1-9:

Description: Proportional reasoning with impulse momentum theorem and Newton's 2nd law. (8 minutes)

Learning Objectives: [6,8]

Problem Statement: The diagram depicts two pucks on a table with friction. Puck 2 is four times as massive as puck 1. Starting from rest, the pucks are pushed across the table by two equal forces. The forces act on both of them for 6.0 seconds. Rank the change in momentum of the two pucks.

(1) $\Delta\vec{p}_1 > \Delta\vec{p}_2$

(2) $\Delta\vec{p}_1 < \Delta\vec{p}_2$

(3) $\Delta\vec{p}_1 = \Delta\vec{p}_2$

(4) Need values of the force to rank their change in momentum.

$$\sum \vec{F}_{Ext} \Delta t = \Delta \vec{P}$$

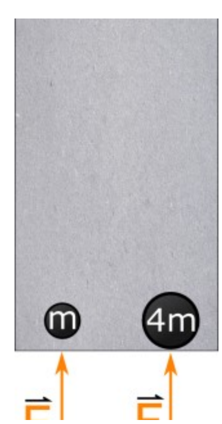
w/ $m_1 < m_2$

THEN $|\vec{F}_1^{N}| < |\vec{F}_2^{N}|$

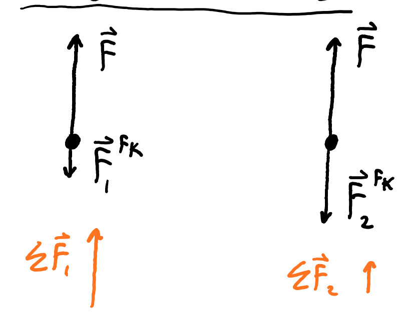
AND $|\vec{F}_1^{fk}| < |\vec{F}_2^{fk}|$

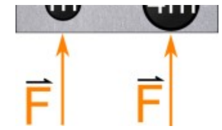
TOP DOWN VIEW FBDS

$\uparrow \vec{F}$ $\uparrow \vec{F}$



TOP DOWN VIEW FBDS





$|\Sigma \vec F_1| > |\Sigma \vec F_2|$

AND w/ $\Delta t_1 = \Delta t_2$

$\Delta \vec P_1 > \Delta \vec P_2$

IM.2.L1-10:

Description: Proportional reasoning with impulse momentum theorem, Newton's 2nd law, and kinematics. (8 minutes)

Learning Objectives: [6,8]

Problem Statement: The diagram depicts two pucks on a frictionless table. Puck 2 is four times as massive as puck 1. Starting from rest, the pucks are pushed across the table by two equal forces. The forces act on both of them all the way to the finish line. Rank the change in momentum of the two pucks.

(1) $\Delta\vec{p}_1 > \Delta\vec{p}_2$

(2) $\Delta\vec{p}_1 < \Delta\vec{p}_2$

(3) $\Delta\vec{p}_1 = \Delta\vec{p}_2$

(4) Need values of the force to rank their change in momentum.

$\Sigma \vec F_{Ext} \Delta t = \Delta \vec P$ $\qquad$ $\Sigma \vec F_{Ext} = m \vec a$

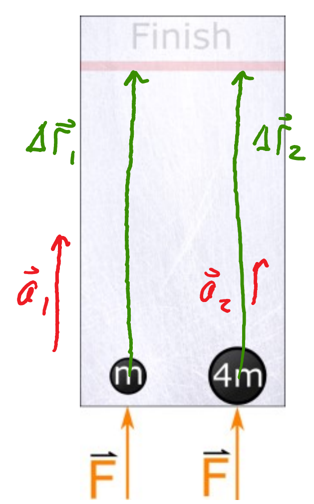
w/ $\Sigma \vec F_1 = \Sigma \vec F_2$

AND $m_1 < m_2$

THEN $\vec a_1 > \vec a_2$

ALSO $\Delta \vec r_1 = \Delta \vec r_2$

SO.. $\Delta t_1 < \Delta t_2$



(2) $\Delta\vec{p}_1 < \Delta\vec{p}_2$

(3) $\Delta\vec{p}_1 = \Delta\vec{p}_2$

(4) Need values of the force to rank their change in momentum.

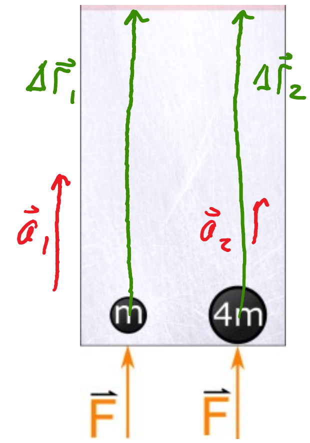
w/ $\Sigma\vec{F}_1 = \Sigma\vec{F}_2$

AND $m_1 < m_2$

THEN $\vec{a}_1 > \vec{a}_2$

ALSO $\Delta\vec{r}_1 = \Delta\vec{r}_2$

SO.. $\Delta t_1 < \Delta t_2$



FINALLY ... $\Delta\vec{p}_1 < \Delta\vec{p}_2$

IM.2.L1-11:

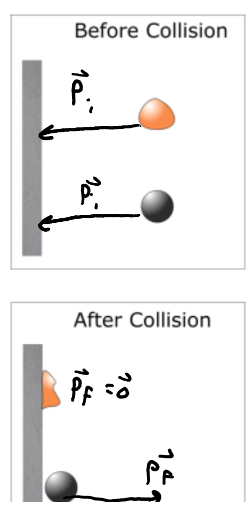
Description: Conceptual question ranking the impulse of two objects based off of their initial and final momentum. (4 minutes)

Learning Objectives: [6, 7]

Problem Statement: A 9.50 g rubber ball and a 9.50 g clay ball are thrown at a wall with equal speeds. The rubber ball bounces off the wall, the clay ball sticks. Which ball exerts a larger impulse on the wall?

(1) They exert equal impulses because they have equal momenta.
(2) The clay ball exerts a larger impulse because it sticks.
(3) Neither exerts an impulse on the wall because the wall doesn't move.
(4) The rubber ball exerts a larger impulse because it bounces.

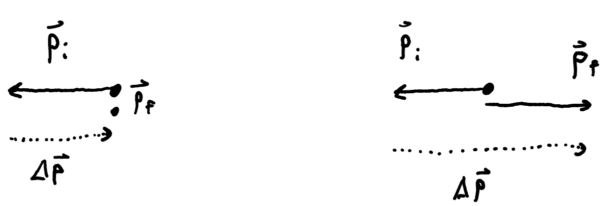
$\vec{J} = \Sigma\vec{F}\Delta t = \Delta\vec{p}$



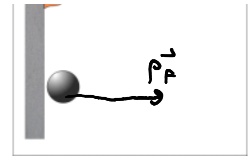
VEC. OPS

CLAY

RUBBER



$\Delta\vec{P}$ $\Delta\vec{P}$



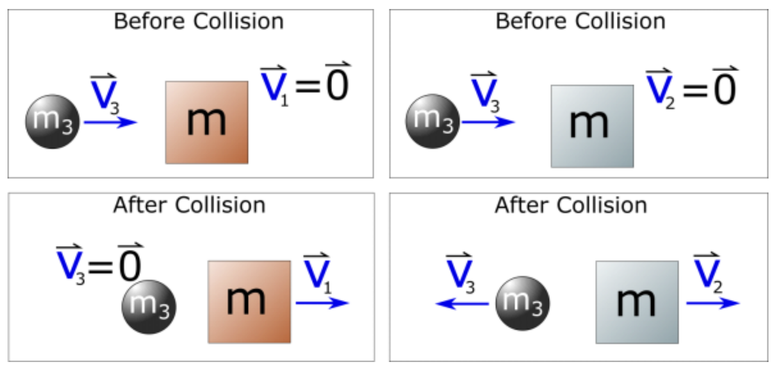
IM.2.L1-12:

Description: Conceptual question ranking the final speeds of two objects based off of their initial and final momentum. (5 minutes)

Learning Objectives: [6,7,12]

Problem Statement: Objects 1 and 2 are made of different materials, with different "springiness", but they have the same mass and are initially at rest. Ball 3 is then thrown at both objects with the same initial speed. Ball 3 remains at rest after colliding with ball 1. Ball 3 bounces back to the left after colliding with ball 2. Rank the final speed of balls 1 and 2 after they collide with ball 3.

(1) $v_1 = v_2$
(2) $v_1 > v_2$
(3) $v_1 < v_2$
(4) Nothing can be said about their relative speeds until we know the initial speed of ball 3 and the mass of each object.



$\Delta\vec{P}_{3A} < \Delta\vec{P}_{3B}$

W/ 3RD LAW

THEN

$\Delta\vec{P}_1 < \Delta\vec{P}_2$

BOTH $\vec{V}_i = \vec{0}$

AND $M_1 = M_2$

$\boxed{\text{SO } \vec{V}_{1f} < \vec{V}_{2f}}$

3RD LAW CASE A

$\Sigma\vec{F}_{31} = -\Sigma\vec{F}_{13}$

W/ $\Sigma\vec{F}\Delta t = \Delta\vec{P}$

THEN $\Delta\vec{P}_3 = -\Delta\vec{P}_1$

3RD LAW CASE B

$\Delta\vec{P}_3 = -\Delta\vec{P}_2$

Act III: Momentum and Impulse graphical analysis

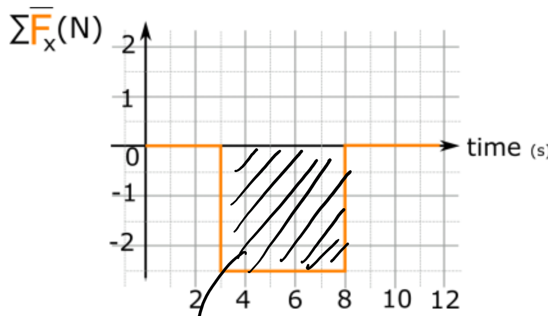
IM.2.L1-13:

Description: Find the impulse given a net force vs time graph. (4 minutes)

Learning Objectives: [9]

Problem Statement: A net force is applied to a 5 kg ball as shown in the graph below. What is the impulse delivered to the 5 kg ball?

(1) 15 N·s $\hat{x}$
(2) -15 N·s $\hat{x}$
(3) 24 N·s $\hat{x}$
(4) -24 N·s $\hat{x}$



AREA = IMPULSE

$-3\,N(8-3)s$ = IMPULSE

-15 Ns

IM.2.L1-14:

Description: Calculate final velocity given a net force vs time graph and initial velocity. (4 minutes + 3 minutes)

Learning Objectives: [9]

Problem Statement: The plot below shows the average net force acting horizontally on a 0.16 kg billiard ball vs time the cue stick strikes it.

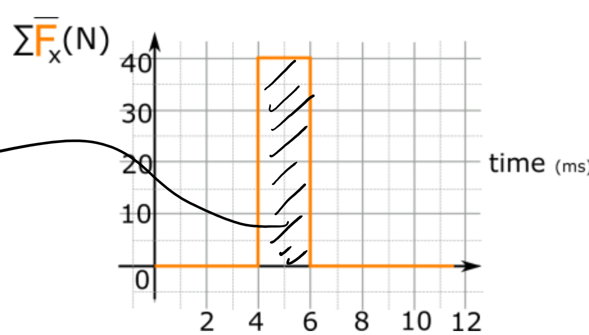
(a) What is the final velocity of the ball assuming it was initially at rest before the collision?

(1) 80.0 m/s $\hat{x}$
(2) 0.08 m/s $\hat{x}$
(3) 0.50 m/s $\hat{x}$
(4) 6.60 m/s $\hat{x}$

AREA = ΔP_x

$40\,N(6\times10^{-3} - 4\times10^{-3})s$

0.08 NS = ΔP_x

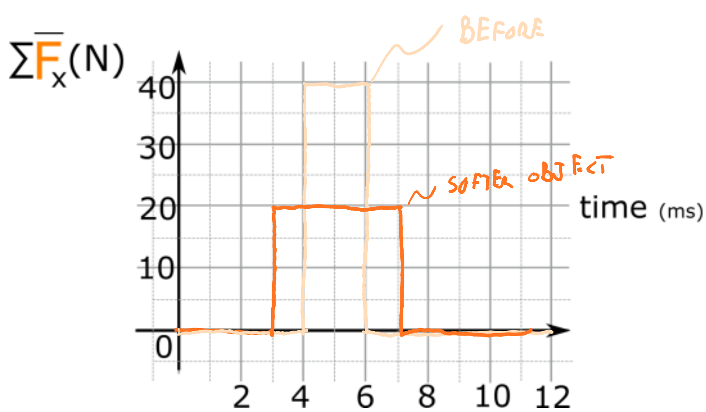


$$0.08\ \text{NS} = P_{fx} - \cancel{P_{ix}}^{\,0}$$

$$0.08\ \text{NS} = m V_{fx}$$

$$V_{fx} = \frac{0.08\ \text{NS}}{.16\ \text{kg}} \approx \boxed{0.5\ \text{m/s}}$$

(b) Sketch the average net horizontal force if the billiard ball was made out of a softer material and went through the same change in momentum.



SAME ΔP_x = SAME AREA

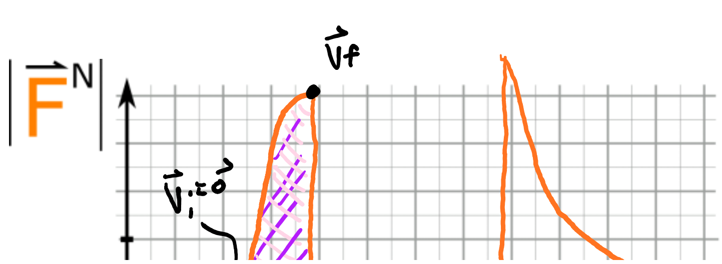
SOFTER IMPLIES $\Delta t \uparrow$

IM.2.L1-15:

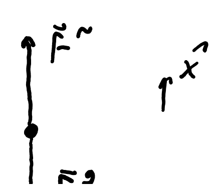
Description: Construct a FBD and apply a force analysis involving a scenario where maximum static friction can point up or down an incline depending on other factors. (6 minutes + 3 minutes)

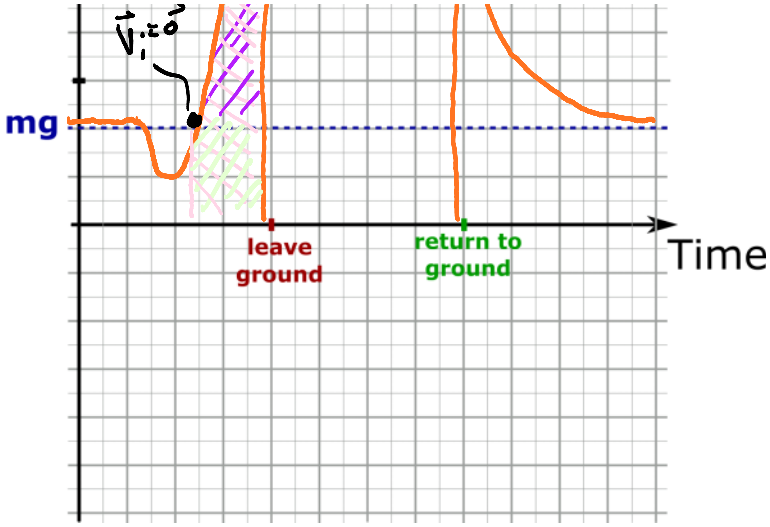
Learning Objectives: [9,11,12]

Problem Statement: In an effort to understand the graphical nature of impulse, I am going to jump into the air and we are going to study the normal force acting on me during this motion. Sketch a plot of what you think the normal force acting on me as a function of time will look like.



STAGE 1: JUMPING





$$\Sigma\vec{F} = \vec{F}^N - \vec{F}^g$$

$$\Sigma\vec{F}\Delta t = \Delta\vec{p}$$

$$\vec{F}^N\Delta t - \vec{F}^g\Delta t = m\vec{v}_f - m\cancel{\vec{v}_i}^{\,0}$$

$$\underbrace{\text{AREA} - \text{AREA}}_{} = m\vec{v}_f$$

$$\text{AREA} = mV_{fx}$$

STAGE 2 : LEAVE GROUND → MAX HEIGHT

KINEMATICS

$\uparrow \hat{x}$

$$V_{fx} = \frac{\text{AREA}}{m}$$

$$\cancel{V_f^2}^{\,0} = V_i^2 + 2a_x\Delta x$$

$$0 = \frac{\text{AREA}^2}{m^2} - 2g\Delta x$$

$$\boxed{\Delta x = \frac{\text{AREA}^2}{2g\,m^2}}$$

Conceptual questions for discussion

1. Consider a car going around a circle at a constant speed. Is the momentum of the car also constant?
2. If the one or more external force acting on a system is not constant (i.e. a function of time), can you use $\Sigma\vec{F}_{ext}\,\Delta t = \Delta\vec{p}$ to find the change in momentum?
3. Consider jumping off a tall table of constant height. It is possible to reduce the impulse on you when you land? Provide ex amples that support your answer.
4. Consider an apple and a large spaceship moving towards each other in space far from any other gravitational objects. When th ey collide, which object goes through a larger change in momentum?

Hints

IM.2.L1-1: No hints.

IM.2.L1-2: Vector components can be positive or negative scalars.

IM.2.L1-3: Vector components can be positive or negative scalars.

IM.2.L1-4: A change in a vector quantity always points from the initial vector tip to the final vector tip when both vectors are tail-to-tail.

IM.2.L1-5: A change in a quantity is always the final value minus the initial value.

IM.2.L1-6: A change in a vector quantity always points from the initial vector tip to the final vector tip when both vectors are tail-to-tail.

IM.2.L1-7: A change in a vector quantity always points from the initial vector tip to the final vector tip when both vectors are tail-to-tail.

IM.2.L1-8: Start with the momentum impulse theorem; which quantities are the same for each puck? Construct a proportional reasoning statement once you determined which quantities are constant.

IM.2.L1-9: Start with the momentum impulse theorem; which quantities are the same for each puck? Construct a proportional reasoning statement once you determined which quantities are constant.

IM.2.L1-10: Start with the momentum impulse theorem; which quantities are the same for each puck? Construct a proportional reasoning statement once you determined which quantities are constant.

IM.2.L1-11: Sketch a vector operation for the change in momentum. How is the change in momentum related to impulse?

IM.2.L1-12: No hints.

IM.2.L1-13: No hints.

IM.2.L1-14: No hints.

IM.2.L1-15: No hints.