CE.2.L1.sols: Potential Energy and Conservation of Energy

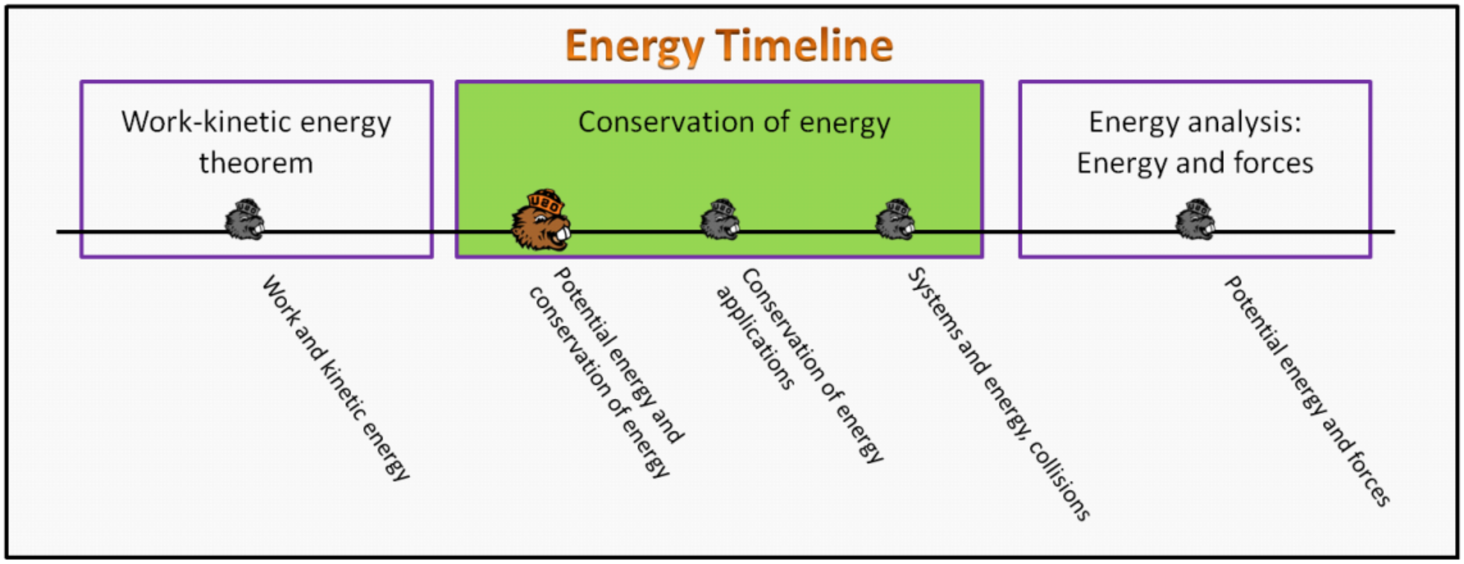
Monday, January 22, 2018 5:44 PM

Conservation of Energy

Foundation Stage (CE.2.L1)

lecture 1

Potential energy and conservation of energy



Textbook Chapters

- **BoxSand** :: KC videos (potential energy ; conservation of energy)
- **Giancoli** (Physics Principles with Applications 7th) :: 6-4 ; 6-6 ; 6-7
- **Knight** (College Physics : A strategic approach 3rd) :: 10.4
- **Knight** (Physics for Scientists and Engineers 4th) :: 10.1 ; 10.2 ; 10.3 ; 10.4 ; 10.5

Warm up

CE.2.L1-1:

Description: Construct equation for work due to gravity for a ball falling vertically downwards given mass and vertical distance.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

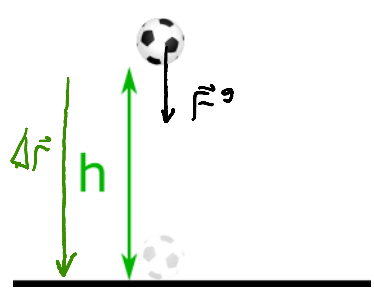
Problem Statement: A ball of mass m is dropped from a vertical height h above the level ground near the surface of the earth. Consider the ball as your system, what is the work from gravity on the ball as it falls from h to the moment before it reaches the gro und?

(1) g
(2) -g
(3) h
(4) -h
(5) mg
(6) -mg
(7) mgh
(8) -mgh

$$W^g = \vec{F}^g \cdot \Delta\vec{r}$$

$$= |\vec{F}^g||\Delta\vec{r}|\,\overset{+1}{\cancel{\cos\theta_g}} \quad \text{b/c } \theta_g = 0$$

$$\boxed{W^g = mgh}$$



Selected Learning Objectives

1. Define a system and identify internal and external objects, forces, and work.
2. Show that conservative forces yield work that is independent of the path taken.
3. Identify conservative and non-conservative forces and work.
4. (UPMF) Use the independence of path for conservative work to create a function of position to account for conservative work. We call that function a potential energy function for all internal conservative work.
5. Construct the final form of the work-energy theorem using the concept of potential energy.
6. Show that energy is conserved for systems where the net external work is zero.
7. Show that application of an energy analysis involves bookkeeping an initial and final state of the system.
8. Use the gravitational potential energy function, for near Earth objects, in an energy analysis.
9. Use the spring potential energy function for Hooke's law springs in an energy analysis.
10. Construct the graphical representation depicting the kinetic, potential, thermal, and total energy as a function of position.
11. (UPMF) When non-conservative work is internal to a system, there is a form of energy associated with that work and it is not called potential energy. E.g. work done by friction converting macroscopic kinetic energy to microscopic kinetic energy (aka thermal energy)
12. Identify other forms of energy such as thermal, chemical potential, electric potential, sound, light, … etc.
13. Identify energy transformations within a system and energy transfers into/out of a system.
14. Apply a conservation of energy equation for a system from its initial to final state.
15. Apply conservation of energy to system with multiple internal objects.
16. Define elastic collisions and apply a conservation of energy and momentum analysis to the collision.
17. Define inelastic collisions and apply a conservation of energy and momentum analysis to the collision.
18. Derive the relationship between kinetic energy and momentum.

Key Terms

- External work
- Internal work
- Conservative force
- Gravitational potential energy
- Hooke's law spring (i.e. ideal spring)
- Spring potential energy
- Energy diagram
- Work - energy theorem
- Conservation of energy

Key Equations

Gravitational potential energy

Type of potential energy: Gravitational

Vertical displacement from a chosen horizontal reference line

Potential Energy

$$U^g = m\,g\,y$$

mass

magnitude of free-fall acceleration

In words: The gravitational potential energy near the surface of the earth is equal to mass of the object moving near the surface of the earth times the magnitude of free-fall acceleration times the vertical displacement from a chosen horizontal reference line.

Change in gravitational potential energy

Type of potential energy: Gravitational

Vertical displacement from a chosen horizontal reference line

Potential Energy

$$\Delta U^g = m\,g\,\Delta y$$

mass

magnitude of free-fall acceleration

In words: The change in gravitational potential energy near the surface of the earth is equal to mass of the object moving near the surface of the earth times the magnitude of free-fall acceleration times the change in the vertical displacement from a chosen horizontal reference line.

Spring potential energy

Type of potential energy: Spring

Displacement from spring's equilibrium

Potential Energy

Spring constant

$$U^s = \frac{1}{2} k x^2$$

In words: The spring potential energy is equal to one half of the spring constant times the squared displacement from the spring's equilibrium.

Change in spring potential energy

Type of potential energy: Spring

Displacement from spring's equilibrium

Potential Energy

Spring constant

$$\Delta U^s = \frac{1}{2} k \Delta x^2$$

In words: The change in spring potential energy is equal to one half of the spring constant times the change in the squared displacement from the spring's equilibrium.

Work-Energy theorem

Initial Kinetic Energy | Initial gravitational potential energy | Initial spring potential energy | Work | Final Kinetic Energy | Final gravitational potential energy | Final spring potential energy

$$\underbrace{KE_i + U_i^g + U_i^s}_{E_i} + \sum W_{\substack{\text{external}\\ \text{to system}}} = \underbrace{KE_f + U_f^g + U_f^s}_{E_f}$$

Net (i.e. add up all of the __)

$$E_i + \sum W_{\substack{\text{external}\\ \text{to system}}} = E_f$$

In words: The sum of the initial kinetic energy, initial gravitational potential energy, and initial spring potentilal energy (which is defined as initial energy) plus the net external work is equal to the sum of the final kinetic energy, final gravitational potential energy, and final spring potential energy (which is defined as final energy).

…OR…

Work-Energy theorem

Work | Final Kinetic Energy | Final gravitational potential energy | Final spring potential energy

$$\sum W_{\substack{\text{external}\\ \text{to system}}} = \underbrace{\Delta KE + \Delta U^g + \Delta U^s}_{\Delta E}$$

Net (i.e. add up all of the __)

$$\sum W_{\substack{\text{external}\\ \text{to system}}} = \Delta E$$

In words: The net external work is equal to the sum of change in kinetic energy, gravitational potential energy, and spring potential energy (which is defined as change in energy).

Key Concepts

- After defining a system, external work is the work done by any forces external to the system. Thus external work is a mechanism for how to transfer energy into or out of a system.
- After defining a system, internal work is the work done by any forces internal to the system. Thus internal work is a mechanism for how to transform energy, from one type to another, within a system.
- If the work done by a force on an object as it moves between two locations is independent of the path taken between the two points, then the force is called a conservative force.
- The work due to gravity near the surface of the earth is only a function of the vertical height above or below any arbitrary horizontal reference line. The functional form is: $W^g = -\,m\,g\,\Delta y$. Note that it doesn't matter what path an object takes, the work from gravity near the surface of the earth is still $-\,m\,g\,\Delta y$. Since the worke done by gravity is only a function of vertical displacemnt (i.e. independent of the path taken) the force of gravity is a conservative force.
- When gravity is doing work and also internal to a system, we no longer use the term "work due to gravity", rather we say that there is a gravitational potential energy within our system that is changing.
- A Hooke's law spring (i.e. ideal spring) is modeled with a force function that is linearly proportional to the displacement (stretched or compressed) from the springs equilibrium position.
- The work done by an ideal spring is only a function of the change in displacement from the spring's equilibrium position. Note that the work done by an ideal spring is not dependent on the path it takes to get from one displacement to another. Thus the ideal spring force is a conservative force. When a spring does work and is also internal to a system, we no longer use the term "work due to spring", rather we say that there is a spring potential energy within our system that is changing.
- In general, whenever there is work internal to a system, we no longer call it work, rather we associate the internal work with a change in some form of energy which is associated with that work. For example, when gravity is internal to the system, the system's gravitational potential energy can change; when a spring is internal to a system, the system's spring potential energy can change; when electric forces are internal to a system, the electric potential energy of the system can change; etc…
- When each form of energy in a system, along with the total energy of the system, is plotted as a function of position, the resulting graph is known as an energy diagram.
- The work-energy equation is the most general equation telling us how a system's energy changes from one snapshot to the next. This is analogous to the impulse-momentum theorem which is the most general equation telling us how the momentum of a system changes from one snapshot to another.
- Conservation of energy refers to the scenario when no external work is done on a system, thus the change in energy of the system is then zero. This is analogous to conservation of momentum, where if there is no net external force, then the change in momentum of the system is zero.

Act I: Gravitational potential energy

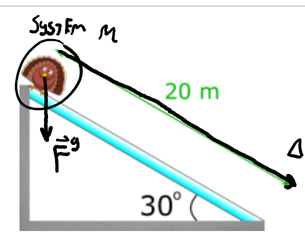
Questions

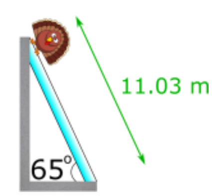
CE.2.L1-2:

Description: Find work from gravity for object sliding down frictionless incline. Find final speed using work kinetic energy theorem for object sliding down frictionless incline.

Learning Objectives: [1, 2, 3, 4]

Problem Statement: Two identical 2-kg turkeys slide down frictionless inclines as shown below. Consider just the turkey as your system for both parts (a) and (b).





(a) How much work does gravity do on the turkey sliding down the 30-degree incline?

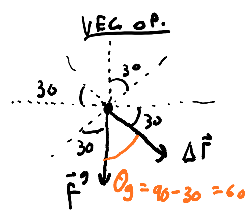
(1) 0 J
(2) 9.8 J
(3) 17.0 J
(4) 196 J
(5) 339 J

$W^g = \vec{F}^g \cdot \Delta\vec{r}$

$= |\vec{F}^g||\Delta\vec{r}|\cos\theta_g$

$= mg(20\,m)\cos(60°)$

$= (2\,kg)(9.8\,m/s^2)(20\,m)\cos(60)$

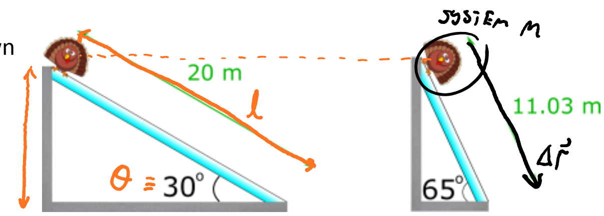


$$= (2\,kg)(9.8\,m/s^2)(20\,m)\cos(60)$$

$$\boxed{W^g = 196\,J}$$

(b) Starting from rest, how fast will the 2-kg turkey be moving as it slides down the 65-degree incline?

(1) 4.21 m/s
(2) 9.56 m/s
(3) 14.0 m/s
(4) 24.0 m/s



$$\sin\theta = \frac{|\Delta y|}{\ell}$$

$$\ell = \frac{|\Delta y|}{\sin\theta}$$

$$\cancel{KE_i}^{\,0} + \Sigma W_{EXT} = KE_F$$

B/L $\theta_N = 90°$

$$\cancel{W^N}^{\,0} + W^g = \tfrac{1}{2} m v_F^2$$

$$mg|\Delta\vec{r}|\cos\theta_g = \tfrac{1}{2} m v_F^2$$

$$(2\,kg)(9.8\,m/s^2)(11.03\,m)\cos(90-65) = \tfrac{1}{2} m v_F^2$$

$$* \quad 196\,J = \tfrac{1}{2} m v_F^2$$

WAIT... SAME W^g AS PART (a)?

$$v_F \approx 14\,m/s$$

$$W^g = |\vec{F}^g||\Delta\vec{r}|\cos\theta_g$$

$$= mg\,\ell\cos(90-\theta)$$

$$= mg\frac{|\Delta y|}{\cancel{\sin\theta}}\cancel{\cos(90-\theta)}$$

....BUT $\cos(90-\theta) = \sin\theta$

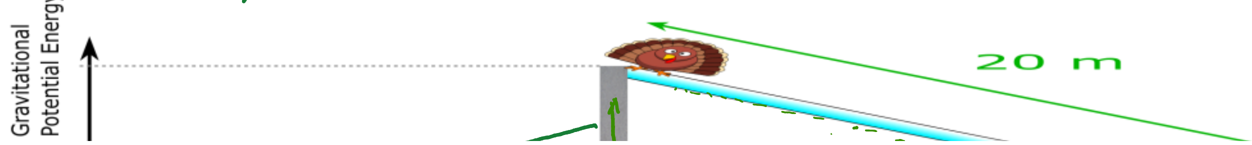
$$W^g = mg|\Delta y|$$

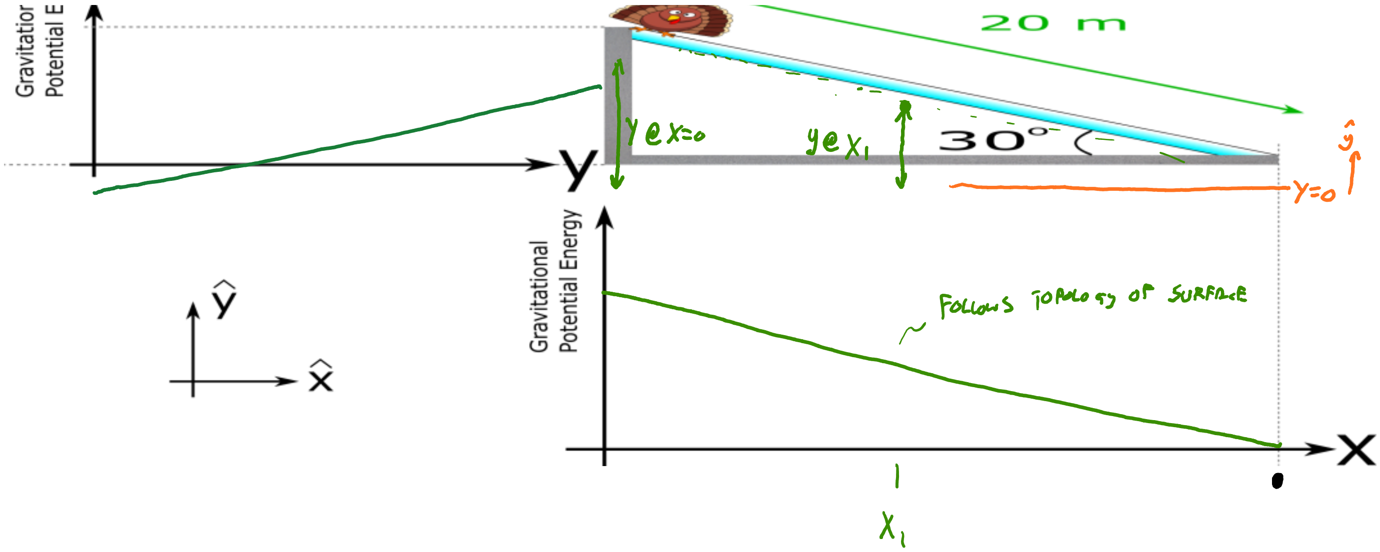
W^g IS ONLY A $\underbrace{\text{FUNCTION OF VERTICAL DISPLACEMENT}}$ (Δy)

SO $\vec{F}^g$ IS A CONSERVATIVE FORCE ..

... I.E. WORK IS INDEPENDENT OF PATH.

$$U^g = mgy$$





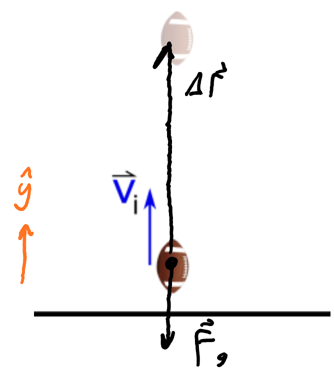
CE.2.L1-3:

Description: Determine sign of work from gravity. (3 minutes)

Learning Objectives: [1, 2, 3]

Problem Statement: Consider throwing a ball vertically upwards. After the ball leaves your hand, when can the work from gravity be equal to - 20 J?

(1) While on the way up. $\theta_g = 180$
(2) While momentarily at rest at its max height.
(3) While on the way down.
(4) During the entire motion while going up and down.



LAST LECTURE

$W^g = \vec{F}^g \cdot \Delta\vec{r}$

$= mg|\Delta\vec{r}|\cos\theta_g$ (-1)

$W^g = -mg|\Delta\vec{r}|$

Don't match.

FROM ABOVE (CE.2.L1-2)

$W^g = mg|\Delta y|$

↑ must be (-) on way up..

$\boxed{W^g = -mg\,\Delta y}$

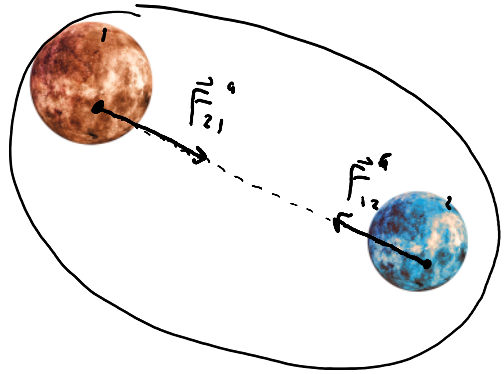
check ... $\underbrace{\underbrace{-mg}_{(-)}\underbrace{(y_f - y_i)}_{(+)}}_{(-)}$ ✓

CE.2.L1-4:

Description: Conceptual question about internal work. (2 minutes + 2 minutes)

Learning Objectives: [1, 3, 4, 6]

Problem Statement: Two large planets are separated by some distance as show in the figure below. Consider a system that contains both planets which are far away from all other objects.



(a) What is the sign of the external work on the system?

(1) 0
(2) Positive
(3) Negative

ALL FORCES ARE INTERNAL.

(b) If the planets are initially at rest and far from all other massive objects, what happens to the kinetic energy of the system when released from rest?

(1) Stays the same
(2) Increases
(3) Decreases

$$\sum KE_i + \cancel{\sum W_{ext}}^{\,0} = \sum KE_f$$

$$\sum KE_i = \sum KE_f$$

⋮

BUT BOTH PLANETS SPEED UP AND KE IS (+) SO... KE OF SYSTEM SHOULD INCREASE!

NEED TO UPDATE WORK - KINETIC ENERGY

$$KE_i + \sum W_{ext} + \sum W_{INT} = KE_f$$

IF CONSERVATIVE FORCES INTERNAL THE THEN WORK $= -\Delta U^c$

e.g. $W^g_{int} = -\Delta U^g$

w/ $U^g = mgy$

So.. $\Delta U^g = mg\Delta y$

FINALLY... WORK-ENERGY THEOREM

$$KE_i + U^g_i + \sum W_{ext} = KE_f + U^g_f$$

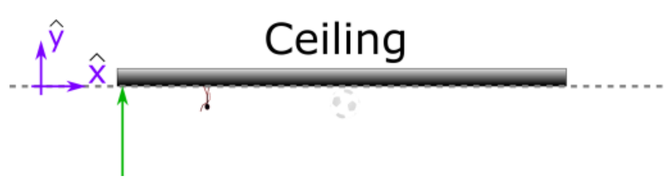
CE.2.L1-5:

Description: Conceptual question regarding origin of gravitational potential energy. (2 minutes + 4 minutes)

Learning Objectives: [2, 3]

Problem Statement: A ball is kicked straight into the air from a height of 0.5 meters above the ground. It flies to a maximum height of 3 meters above the ground.

(a) The change in gravitational potential energy from 0.5 meters to 3 meters is equal to 10.5 J when the origin is set at the floor. What is the change in gravitational potential energy when the origin is placed on the ceiling?



$$\Delta U^g = mg\Delta y$$

change in gravitational potential energy when the origin is placed on the ceiling?

$\Delta U^g = mg\Delta y$

VERTICAL DISPLACEMENT

INDEPENDENT OF ORIGIN

(1) 0 J
(2) -10.5 J
(3) 10.5 J
(4) Need the mass of the ball to determine.

(b) If the mass of the ball is 0.43 kg, calculate the gravitational energy using an origin at the ground with the positive y dire ction pointing downwards.

(1) -10.5 J
(2) 10.5 J
(3) 12.6 J
(4) -12.6 J

$\hat{x}$, $\hat{y}$

$\Delta U^g = mg\Delta y$

$= (0.43\text{ kg})(9.8\text{ m/s}^2)(-3\text{m} - (-0.5\text{ m}))$

$\Delta U^g = -10.5\text{ J}$

But we defined $W^g = -\Delta U^g$

So w/ this coord sys

$W^g = -(-) = (+)$ HUH?

$\Delta\vec{r}$

$\vec{F}^g$

*MUST USE POSITIVE y DIRECTION AS UPWARDS FOR U^g

CE.2.L1-6:

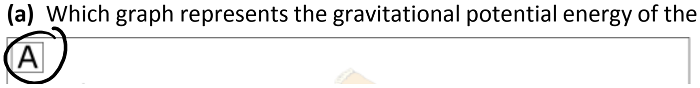
Description: Match the correct energy diagrams with a picture of a system evolving in time. (3 minutes + 4 minutes + 4 minutes)

Learning Objectives: [1, 6, 10]

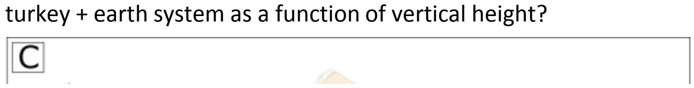
Problem Statement: A slice of pumpkin pie is sliding from right to left across a frictionless surface as shown in the images below. Assume a standard coordinate system for all parts below.

(a) Which graph represents the gravitational potential energy of the turkey + earth system as a function of vertical height?

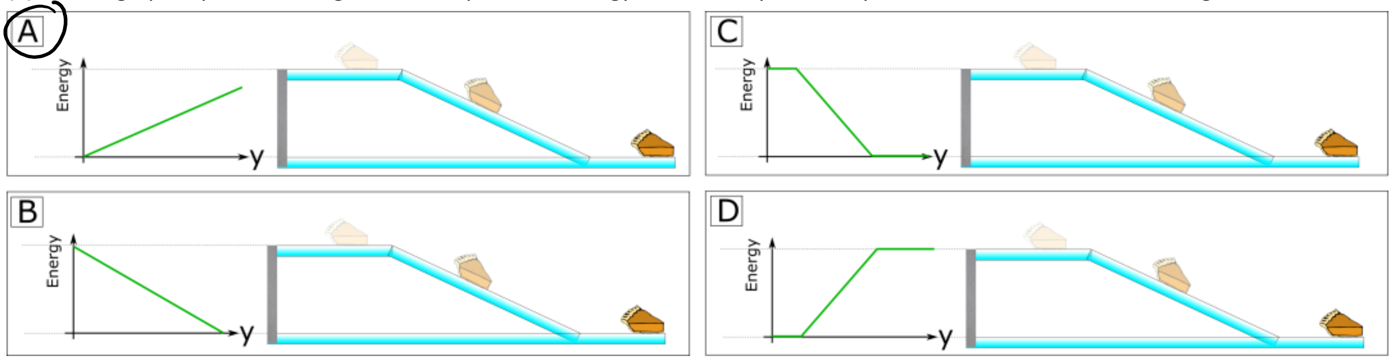
A



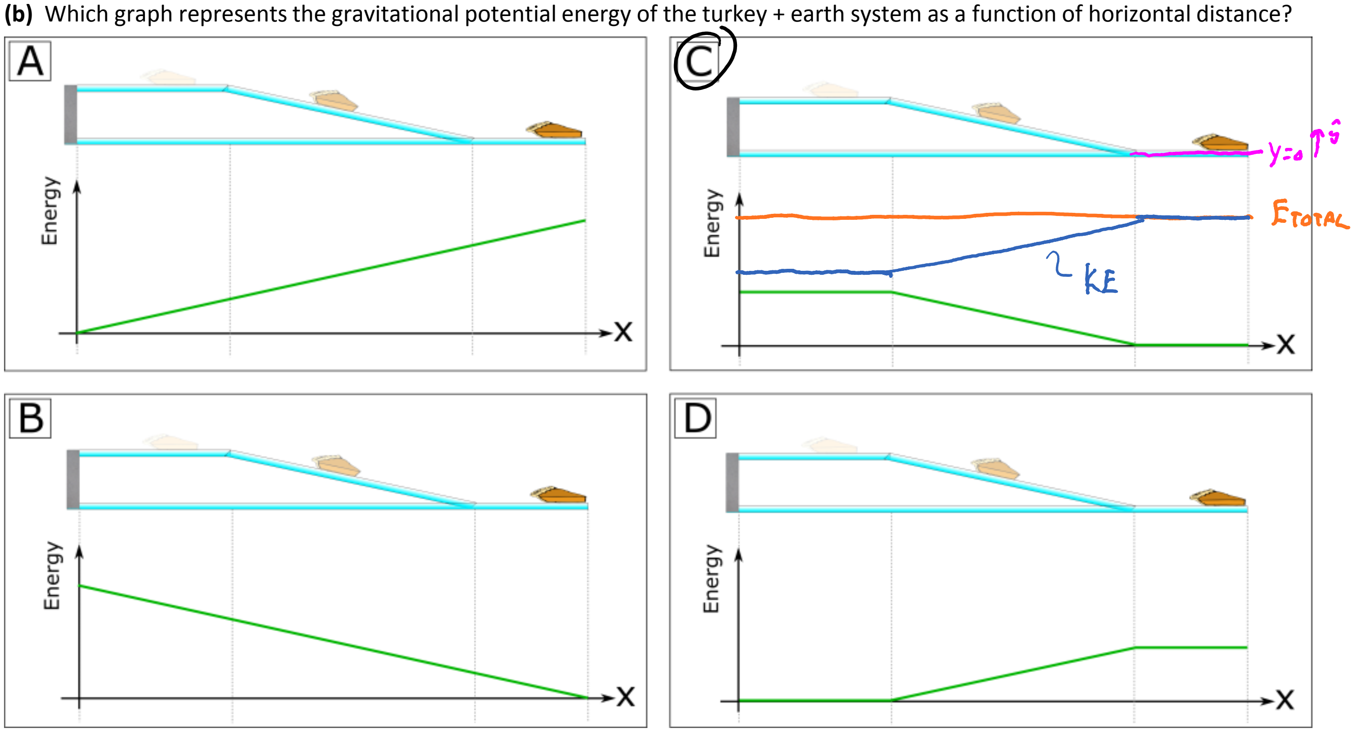
C



(a) Which graph represents the gravitational potential energy of the turkey + earth system as a function of vertical height?



(b) Which graph represents the gravitational potential energy of the turkey + earth system as a function of horizontal distance?



(c) Sketch the kinetic energy of the turkey as a function of horizontal distance on the graph of your answer to part (b).

$$KE_i + U^g_i + \cancel{W_{EXT}}^{0} = KE_f + U^g_f$$

$$\underbrace{KE_i + U^g_i} = \underbrace{KE_f + U^g_f}$$

$$E_{TOTAL\,i} = E_{TOTAL\,f}$$

$$E_{TOTAL} = \underbrace{KE + U^g}_{\text{CONSTANT}}$$

Act II: Spring potential energy

CE.2.L1-7:

Description: Use Hooke's law to determine spring constant from graph. Sketch spring potential energy. Proportional reasoning with spring potential energy. Conceptual question about stretching vs compressing a spring. (4 minutes + 3 minutes + 2 minutes + 2 minutes)

Learning Objectives: [10]

Problem Statement: The force from an ideal spring is given by the equation below, where **k** is a positive constant called the spring constant, and **x** is the displacement of the spring from its equilibrium position. The equation is often referred to as Hooke's law.

$$F_x^s = -k\,x$$

(a) The graph to the right shows the force measured from a spring as it is displaced from its equilibrium location. What is the spring constant of the spring?

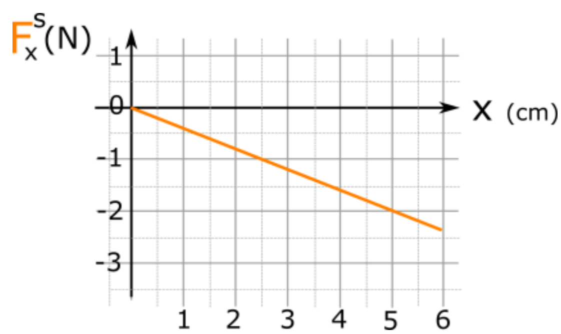
(1) 5 N/m
(2) - 5 N/m
(3) 40 N/m
(4) -40 N/m

$|\text{Slope}| = \left|\frac{-2N}{(5/100)m}\right|$

$= 40\ \frac{N}{m}$

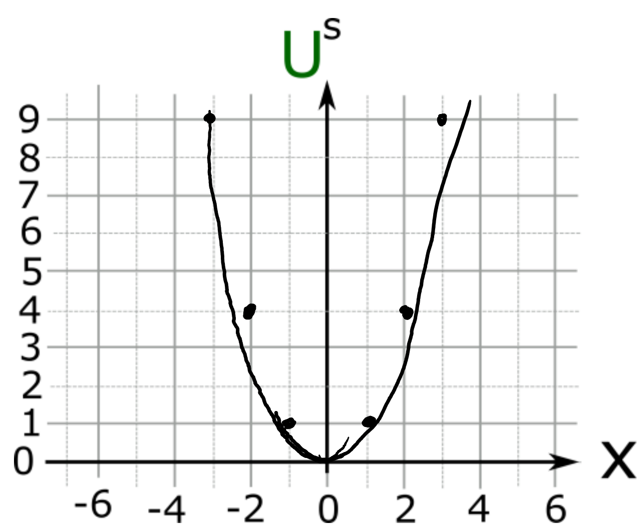
$y = mx + b$

$F_x^s = -kx + 0$



(b) The spring potential energy is given in the equation below where **k** and **x** have the same meaning as they do in Hooke's law.

$$U^s = \frac{1}{2}k\,x^2$$



(c) If the potential energy of a spring increases by a factor of 4, by what factor must the displacement change by?

(1) 1
(2) 2
(3) 3

$U^s \propto x^2$

(d) When a spring is stretched from its equilibrium position by 5 cm, it is noted that the spring potential energy is 20 J. What is the spring potential energy if the same spring is compressed by 5 cm from its equilibrium position?

(1) 10 J

(1) 1
(2) 2
(3) 3
(4) 4
(5) 9

$U^s \propto x^2$

$x \propto \sqrt{U^s}$

if $U^s \rightarrow 4U^s$

$x \rightarrow \sqrt{4}\, x$

What is the spring potential energy if the same spring is compressed by 5 cm from its equilibrium position?

(1) 10 J
(2) -10 J
(3) -20 J
(4) 20 J
(5) Need value spring constant to determine.

UPDATE WORK-ENERGY

$$KE_i + U^g_i + U^s_i + \Sigma W_{EXT} = KE_f + U^g_f + U^s_f$$

Act III: Work-energy theorem

CE.2.L1-8:

Description: Use conservation of energy to find vertical height for object sliding up a frictionless incline. (1 minute + 1 minute + 8 mi nutes)

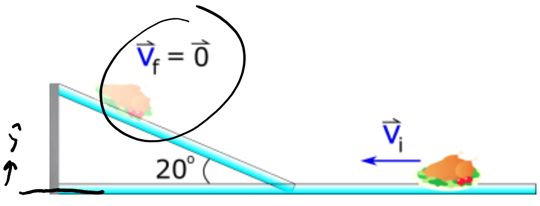
Learning Objectives: [1, 8, 14]

Problem Statement: A 4 kg turkey dinner is sliding across a frictionless horizontal ground with a velocity of 5 m/s to the left. The dinner then encounters a frictionless incline that makes a 20 degree angle with respect to the horizontal. We wish to use an energy analysis to find high above the ground the turkey gets before coming to rest.

(a) Define your system.

(1) Turkey dinner.
(2) Earth
(3) Turkey dinner + Earth

SYSTEM T + E



$y=0$

(b) Is there any external work between the initial snapshot and the final snapshot?

(1) Yes
(2) No

$\Sigma W_{EXT} = \cancel{W}^{0}$

(c) How high above the ground does the Turkey dinner get before coming to a rest?

$$KE_i + \cancel{U^g_i}^{0} + \cancel{\Sigma W_{EXT}}^{0} = \cancel{KE_f}^{0} + U^g_f$$

$$KE_i = U^g_f$$

$$\frac{1}{2}\cancel{m}V_i^2 = \cancel{m}g y_f$$

$$y_f = \frac{V_i^2}{2g} = \frac{(5\ m/s)^2}{2(9.8\ m/s^2)} \simeq \boxed{1.28\ m}$$

Conceptual questions for discussion

1. Consider a ball falling near the surface of the earth: If the earth is not included in your system, is there gravitational potential energy in your system?
2. One of your friends claims that the gravitational potential energy of a book on a shelf is 25J. Another friend claims that the same book has a gravitational potential energy of 50J. Is one of your friends necessarily wrong?
3. Consider a single spring: Describe 2 actions you can do to the spring to get a negative change in spring potential energy. (E.g. stretching the spring from its equilibrium position will result in a positive change in spring potential energy.)

Hints

CE.2.L1-1: No hints.

CE.2.L1-2: Create a vector operation to help find the angle that goes into the cosine function seen in the work definition.

CE.2.L1-3: No hints.

CE.2.L1-4: No hints.

CE.2.L1-5: The Δy seen in the change in gravitational potential energy function is the vertical component of displacement. What do you know about displacements and their relationship to origins?

CE.2.L1-6: Part (a): what is the functional form of gravitational potential energy?

CE.2.L1-7: Part (a): What is the generic equation for a linear line: y = ?

CE.2.L1-8: No hints.