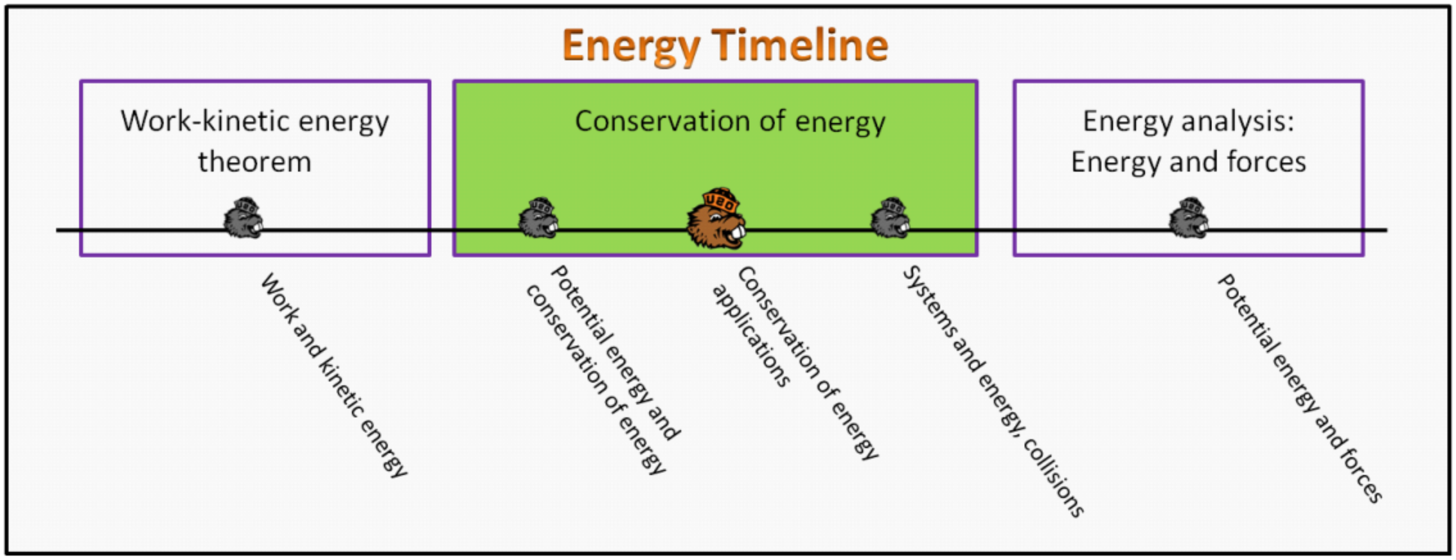
CE.2.L2.sols: Conservation of Energy Application

Monday, January 22, 2018 5:44 PM

Conservation of Energy
Foundation Stage (CE.2.L2)

lecture 2
Conservation of energy applications



Textbook Chapters (* Calculus version)

- **BoxSand** :: KC videos (conservation of energy)
- **Giancoli** (Physics Principles with Applications 7th) :: 6-5 ; 6-6 ; 6-7 ; 6-9
- **Knight** (College Physics : A strategic approach 3rd) :: 10.5 ; 10.6
- ***Knight** (Physics for Scientists and Engineers 4th) :: 10.7 ; 10.8

Warm up

CE.2.L2-1:

Description: Use work-energy theorem to determine final energy of system.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: Consider a system with initial gravitational potential energy of 20 J, initial spring potential energy of 5 J, and initial kinetic energy of 15 J. If the net external work is -10 J, how much final energy is remaining in the system?

$$KE_i + U^g_i + U^s_i + \Sigma W_{EXT} = \underbrace{KE_f + U^g_f + U^s_f}$$

$$15\,J + 20\,J + 5\,J - 10\,J = E_f$$

$$\boxed{30\,J = E_f}$$

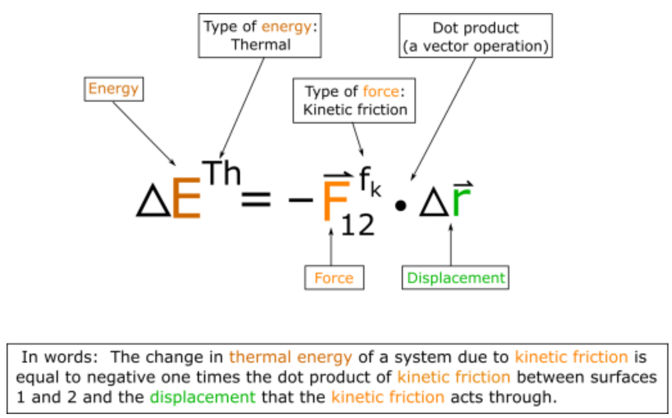
Selected Learning Objectives

1. Define a system and identify internal and external objects, forces, and work.
2. Show that conservative forces yield work that is independent of the path taken.
3. Identify conservative and non-conservative forces and work.
4. (UPMF) Use the independence of path for conservative work to create a function of position to account for conservative work. We call that function a potential energy function for all internal conservative work.
5. Construct the final form of the work-energy theorem using the concept of potential energy.
6. Show that energy is conserved for systems where the net external work is zero.
7. Show that application of an energy analysis involves bookkeeping an initial and final state of the system.
8. Use the gravitational potential energy function, for near Earth objects, in an energy analysis.
9. Use the spring potential energy function for Hooke's law springs in an energy analysis.
10. Construct the graphical representation depicting the kinetic, potential, thermal, and total energy as a function of position.
11. (UPMF) When non-conservative work is internal to a system, there is a form of energy associated with that work and it is not called potential energy. E.g. work done by friction converting macroscopic kinetic energy to microscopic kinetic energy (aka thermal energy)
12. Identify other forms of energy such as thermal, chemical potential, electric potential, sound, light, … etc.
13. Identify energy transformations within a system and energy transfers into/out of a system.
14. Apply a conservation of energy equation for a system from its initial to final state.
15. Apply conservation of energy to system with multiple internal objects.
16. Define elastic collisions and apply a conservation of energy and momentum analysis to the collision.
17. Define inelastic collisions and apply a conservation of energy and momentum analysis to the collision.
18. Derive the relationship between kinetic energy and momentum.

Key Terms

- Thermal energy
- Non-conservative force

Key Equations



Work-Energy theorem

Initial Kinetic Energy | Initial gravitational potential energy | Initial spring potential energy | Initial thermal energy | Work | Final Kinetic Energy | Final gravitational potential energy | Final spring potential energy | Final thermal energy

$$KE_i + U_i^g + U_i^s + E_i^{Th} + \Sigma W_{\substack{\text{external} \\ \text{to system}}} = KE_f + U_f^g + U_f^s + E_f^{Th}$$

Net (i.e. add up all of the __)

$$E_i + \Sigma W_{\substack{\text{external} \\ \text{to system}}} = E_f$$

In words: The sum of the initial kinetic energy, initial gravitational potential energy, initial spring potential energy , and intial thermal energy (which is defined as initial energy) plus the net external work is equal to the sum of the final kinetic energy, final gravitational potential energy, final spring potential energy , and final thermal energy (which is defined as final energy).

...OR...

Work-Energy theorem

Work | Change in Kinetic Energy | Change in potential energy | Change in potential energy | Change in thermal energy

$$\Sigma W_{\substack{\text{external} \\ \text{to system}}} = \Delta KE + \Delta U^g + \Delta U^s + \Delta E^{Th}$$

Net (i.e. add up all of the __)

$$\Sigma W_{\substack{\text{external} \\ \text{to system}}} = \Delta E$$

In words: The net external work is equal to the sum of change in kinetic energy, gravitational potential energy, spring potential energy , and thermal energy (which is defined as change in energy).

Key Concepts

- The work due to kinetic friction from surface 1 on surface 2 plus the work due to kinetic friction from surface 2 on surface 1 depends on the path taken. Thus kinetic friction is a non-conservative force. When kinetic friction is present, the two surfaces in contact sliding relative to each other must be included within the system. This internal work done by kinetic friction on both of the surfaces is then said to change the thermal energy of the system.
- Thermal energy is related to temperature. Generally, if the thermal energy of an object increases, then the temperature of that object increases. (This is not a true statement if a phase change is present, for example water to steam, etc..).

Act I: Conservation of energy

Questions

CE.2.L2-2:

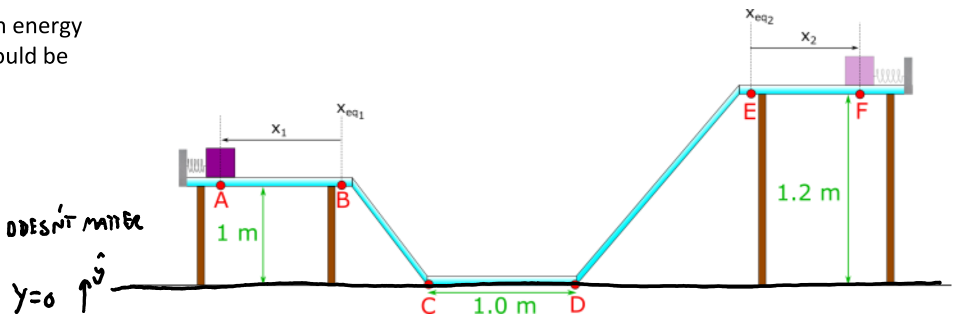
Description: Apply conservation of energy for a mass changing height with springs involved.

Learning Objectives: [1, 3, 5, 6, 7, 8, 9, 10, 14]

Problem Statement: A 2-kg box is compressed 1 meter against a spring whose spring constant is 20 N/m. Once released, the mass slides down a frictionless ramp then back up to another spring with the same spring constant. After compressing the second spring, it comes to rest and is held in place. (Use **g** = 10 m/s²)

(a) We want to analyze this system using an energy analysis. Which of the following objects should be included in our system?

(1) Box
(2) Spring 1
(3) Spring 2
(4) Earth
(5) Tables/surfaces ~ NO FRICTION SO DOESN'T MATTER



(b) Which interval is the work from the normal force the maximum?

(1) **A** to **B**
(2) **B** to **C**
(3) **C** to **D**
(4) **D** to **E**
(5) **E** to **F**
(6) The normal force here does no work for all intervals.

$\vec{F}^N \perp$ TO $\Delta\vec{r}$ FOR ALL INTERVALS

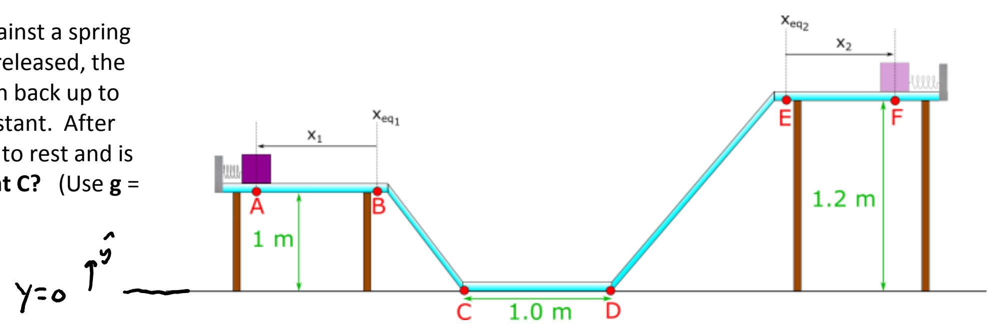
(c) With the system chosen in part (a), is there any net external work between locations **A** and **C** ?

(1) Yes
(2) No

(d) Where would be the best choice for a horizontal **y** = 0 reference axis be?

(1) A
(2) C
(3) E

(e) A 2-kg box is compressed 1 meter against a spring whose spring constant is 20 N/m. Once released, the mass slides down a frictionless ramp then back up to another spring with the same spring constant. After compressing the second spring, it comes to rest and is held in place. **What is the speed at point C?** (Use **g** = 10 m/s²)



$$KE_A^0 + U_A^g + U_A^{s_1} + U_A^{s_2\,0} + \sum W_{EXT}^0 = KE_C + U_C^{g\,0} + U_C^{s_1\,0} + U_C^{s_2\,0}$$

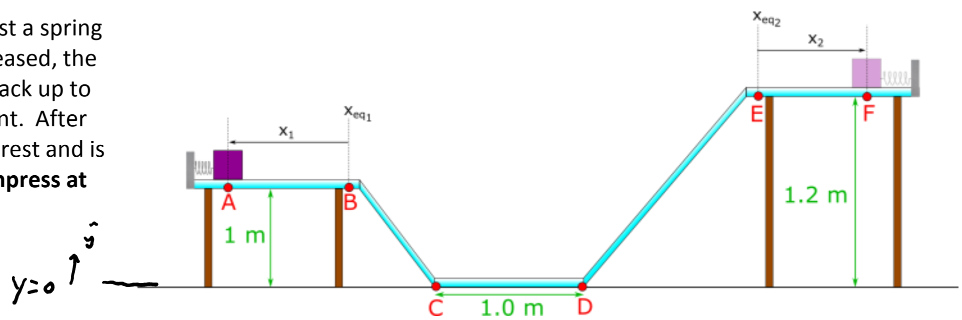
$$U_A^g + U_A^{s_1} = KE_C$$

$$m g y_A + \frac{1}{2} k_1 x_1^2 = \frac{1}{2} m v_C^2$$

$$(2\,kg)(10\,m/s^2)(1\,m) + \frac{1}{2}(20\,N/m)(1\,m)^2 = \frac{1}{2}(2\,kg)\,V_C^2$$

$$\boxed{V_C \approx 5.48\ m/s}$$

(e) A 2-kg box is compressed 1 meter against a spring whose spring constant is 20 N/m. Once released, the mass slides down a frictionless ramp then back up to another spring with the same spring constant. After compressing the second spring, it comes to rest and is held in place. **How far does the spring compress at location F?** (Use **g** = 10 m/s²)



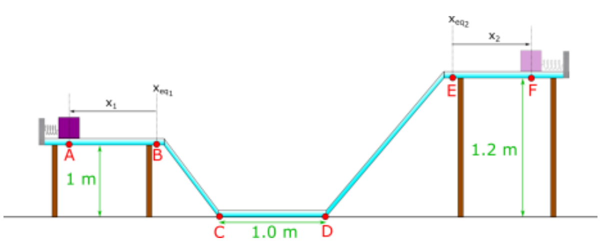
$$KE_A^{\,0} + U_A^g + U_A^{S_1} + U_A^{S_2\,0} + \Sigma W_{Ext}^{\,0} = KE_F^{\,0} + U_F^g + U_F^{S_1\,0} + U_F^{S_2}$$

$$U_A^g + U_A^{S_1} = U_F^g + U_F^{S_2}$$

$$mgy_A + \tfrac{1}{2}k_1x_1^2 = mgy_F + \tfrac{1}{2}k_2x_2^2$$

$$(2\,kg)(10\,m/s^2)(1\,m) + \tfrac{1}{2}(20\,N/m)(1\,m)^2 = (2\,kg)(10\,m/s^2)(1.2) + \tfrac{1}{2}(20\,\tfrac{N}{m})\,x_2^2$$

$$\boxed{x_2 = 0.775\ m}$$



(g) Fill in the value of each form of energy for location **C** in the table. (Use **g** = 10 m/s²)

(h) There is one column of energies that can be filled out without doing any other calculations, which column is it?

	KE(J) ($\tfrac{1}{2}mv^2$)	U^g(J) (mgy)	U^{S_1}(J) ($\tfrac{1}{2}k_1x_1^2$)	U^{S_2}(J) ($\tfrac{1}{2}k_2x_2^2$)	E^{total}(J)
A	0	20	10	0	30
B	10	20	0	0	30

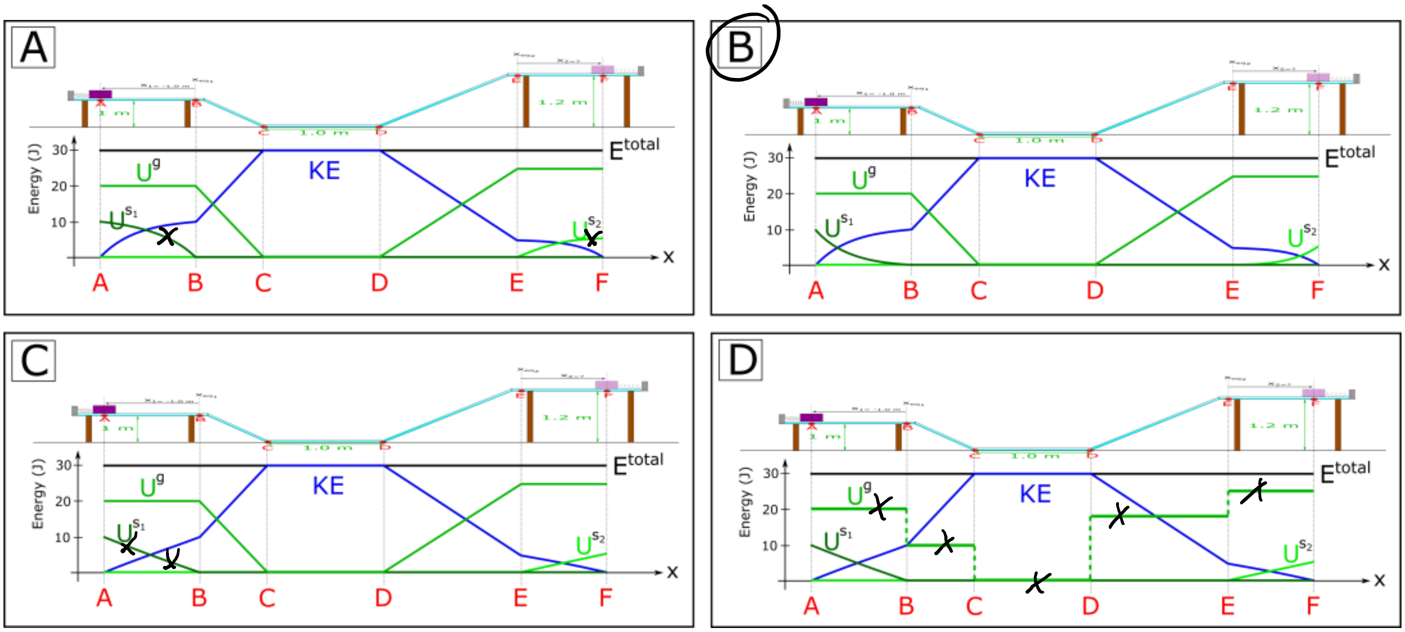
(h) There is one column of energies that can be filled out without doing any other calculations, which column is it?

(1) KE
(2) U^g
(3) U^{s1}
(4) U^{s2}
(5) E^{total}

(i) Complete the rest of the table. (Use **g** = 10 m/s²)

B	10	20	0	0	30
C	30	0	0	0	30
D	30	0	0	0	30
E	6	24	0	0	30
F	0	24	0	6	30

(j) Which of the following energy diagrams correctly describes this scenario? $U^s = \frac{1}{2}kx^2$ $x^2 \rightarrow$ ∪



Act II: Conservation of energy with friction

CE.2.L2-3:

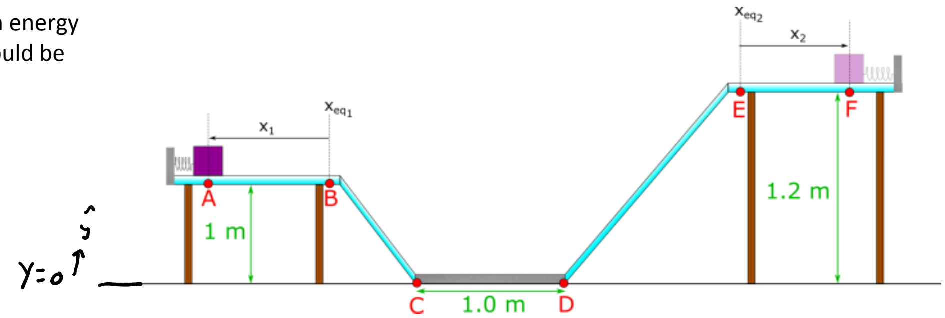
Description: Apply conservation of energy for mass changing height with springs and kinetic friction. (3 minutes)

Learning Objectives: [1, 3, 5, 7, 8, 9, 11, 14]

Problem Statement: When setting up a scenario like the previous problem, you find the bottom freezing element is broken and your box slides along the 1 meter bottom part coming to rest just at the end.

(a) We want to analyze this system using an energy analysis. Which of the following objects should be included in our system?

1. Box
2. Spring 1
3. Spring 2
4. Earth
5. Tables/surfaces



(b) How many joules of energy did friction convert to thermal energy during the friction stage?

$$\cancel{KE_A}^0 + U^g_A + U^{s_1}_A + \cancel{U^{s_2}_A}^0 + E^{th}_A + \cancel{\Sigma W_{ext}}^0 = \cancel{KE_D}^0 + \cancel{U^g_D}^0 + \cancel{U^{s_1}_D}^0 + \cancel{U^{s_2}_D}^0 + E^{th}_D$$

$$U^g_A + U^{s_1}_A + E^{th}_A = E^{th}_D$$

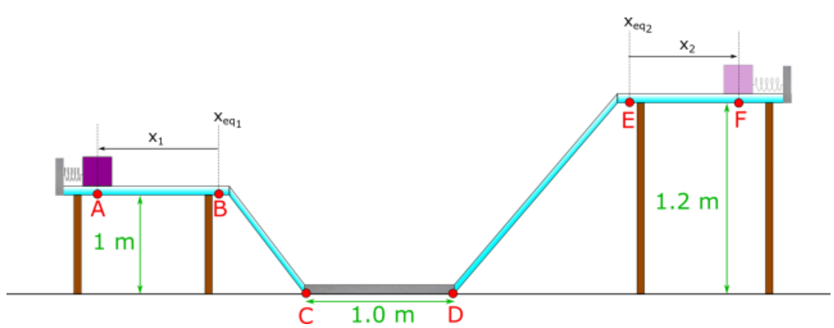
$$U^g_A + U^s_A = E^{th}_D - E^{th}_A$$

$$U^g_A + U^s_A = \Delta E^{th}$$

$$mgy_A + \tfrac{1}{2}k_1 x_1^2 = \Delta E^{th}$$

$$\boxed{30\ J = \Delta E^{th}}$$

(c) A 2-kg box is compressed 1 meter against a spring whose spring constant is 20 N/m. Once released, the mass slides down a frictionless ramp then encounters a 1.0 m long section with no ice. The box comes to a rest at the end of the bottom surface. **What is the coefficient of kinetic friction between the box and the bottom surface?**



$$30\ J = \Delta E^{th}$$

$$30\ J = -\vec{F}^{f_k} \cdot \Delta\vec{r}_{CD}$$

$$= -|\vec{F}^{f_k}||\Delta\vec{r}_{CD}|\cos\theta^{-1} \quad \text{w/ } \theta = 180$$

$$= -|\vec{F}^{f_k}||\Delta\vec{r}_{CD}|(\cos\theta)^{-1} \quad \text{b/c } \theta=180$$

$$= +\mu_k|\vec{F}^{N}||\Delta\vec{r}_{CD}|$$

$$30\,J = \mu_k mg|\Delta\vec{r}_{CD}|$$

$$30\,J = \mu_k (2\,kg)(10\,m/s^2)(1\,m)$$

$$\vdots$$

$$\boxed{\mu_k \approx 1.5}$$

CE.2.L2-4:

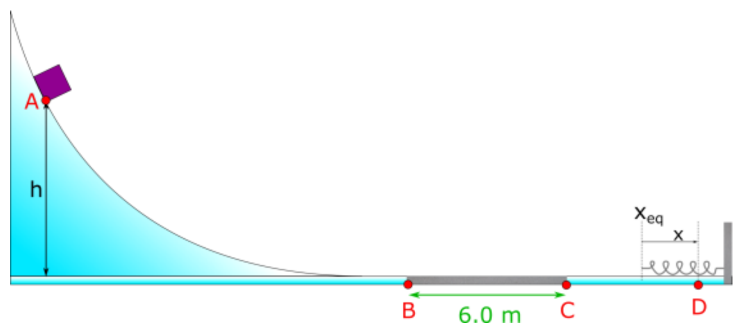
Description: Apply conservation of energy for mass changing height with springs and kinetic friction. (2 minutes + 2 minutes)

Learning Objectives: [1, 3, 5, 7, 8, 9, 10, 11, 14]

Problem Statement: A 10-kg box is released from point **A** in the figure below. The track is frictionless except for the portion between **B** and **C**, which has a length of 6.00 m. The block travels down the track, hits a spring of force constant k = 2250 N/m, and compresses the spring 0.300 m from its equilibrium position before coming to rest momentarily. The coefficient of kinetic friction between the box and the surface between **B** and **C** is 0.328.

(a) Define your system.

(1) Box
(2) Box + earth
(3) Box + earth + spring
(4) Box + earth + spring + surface
(5) Box + spring



(b) Is there any external or internal non-conservative work between locations **A** and **D**?

(1) Yes
(2) No

(c) A 10-kg box is released from point **A** in the figure below. The track is frictionless except for the portion between **B** and **C**, which has a length of 6.00 m. The block travels down the track, hits a spring of force constant k = 2250 N/m, and compresses the spring 0.300 m from its equilibrium position before coming to rest momentarily. The coefficient of kinetic friction between the box and the surface between **B** and **C** is 0.328. **How high above the level ground did the box start?**

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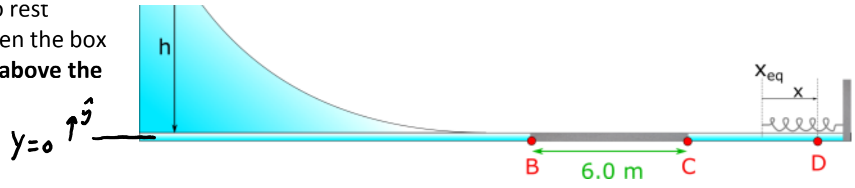


$$\cancel{KE_A} + U^g_A + \cancel{U^s_A} + E^{th}_A + \cancel{\Sigma W_{EXT}} = \cancel{KE_D} + \cancel{U^g_D} + U^s_D + E^{th}_D$$

$$U^g_A + E^{th}_A = U^s_D + E^{th}_D$$

$$U^g_A = U^s_D + \Delta E^{th}$$

$$mgy_A = \tfrac{1}{2}kx^2 - |\vec{F}^{fk}||\Delta\vec{r}_{BC}|\cancel{\cos\theta}^{\,-1}$$

$$mgy_A = \tfrac{1}{2}kx^2 + \mu_k mg|\Delta\vec{r}_{BC}|$$

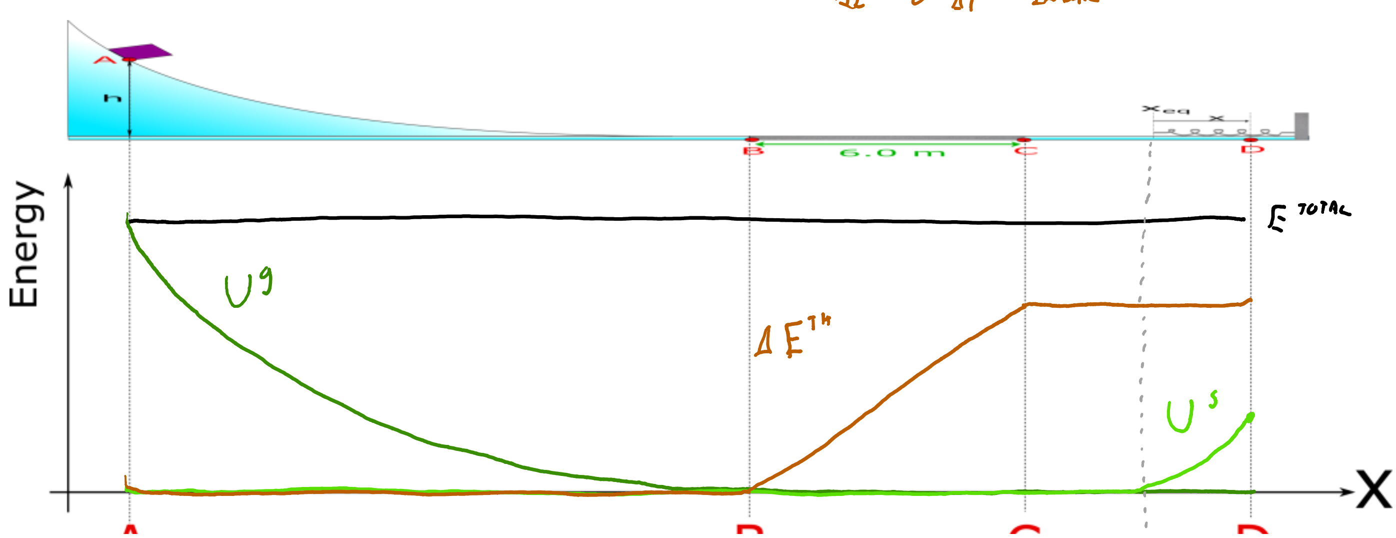
$$(10\text{ kg})(9.8\text{ m/s}^2)y_A = \tfrac{1}{2}(2230\text{ N/m})(0.3\text{ m})^2 + 0.328(10\text{ kg})(9.8\text{ m/s}^2)(6\text{ m})$$

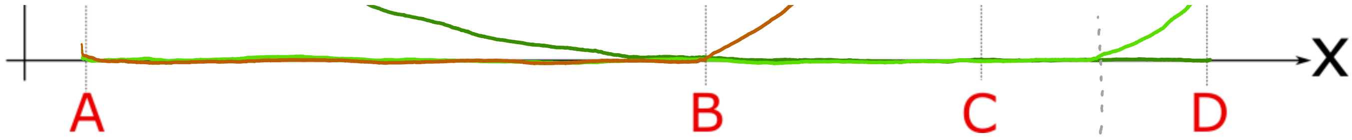
$$\boxed{y_A \approx 3.00\text{ m}}$$

$\Delta E^{th} = +F^{fk}\Delta r$ w/ F^{fk} const

$\Delta E^{th} \propto \Delta r \sim$ LINEAR

(d) Sketch a plot of the energies of the system on the axis provided below.





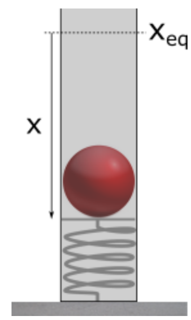
Act III: More practice

CE.2.L2-5:

Description: Apply conservation of energy for a mass changing height with springs involved (2 minutes + 2 minutes)

Learning Objectives: [1, 3, 5, 6, 7, 8, 9, 14]

Problem Statement: A 0.5-kg ball is pressed up against an ideal spring in the vertical position as shown in the figure below. The spring is initially compressed 25 cm from its equilibrium position. When the ball is released from rest, it reaches a maximum height of 2.30 meters above the equilibrium position of the spring.



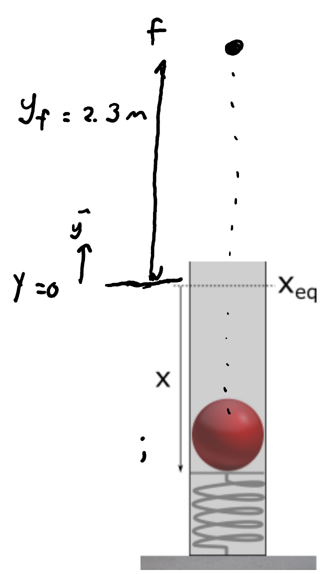
(a) Define your system.

(1) Ball
(2) Ball + spring
(3) Ball + earth
(4) Ball + earth + spring

(b) If two snap shots are taken, one when the spring is completely compressed and another at the maximum height, are there any external or internal non-conservative work between those two snapshots?

(1) Yes
(2) No

(c) A 0.5-kg ball is pressed up against an ideal spring in the vertical position as shown in the figure below. The spring is initially compressed 25 cm from its equilibrium position. When the ball is released from rest, it reaches a maximum height of 2.30 meters above the equilibrium position of the spring. **What is the spring constant of the spring?**



$$KE_i + U^g_i + U^s_i + \Sigma W_{ext} = KE_f + U^g_f + U^s_f$$

$$U^g_i + U^s_i = U^g_f$$

$$mgy_i + \frac{1}{2}kx^2 = mgy_f$$

$$(0.5kg)(9.8m/s^2)(-0.25m) + \frac{1}{2}k(0.25m)^2 = (0.5kg)(9.8m/s^2)(2.3m)$$

$$(0.5\,kg)(9.8\,m/s^2)(-0.23\,m) + \frac{1}{2}k(0.25\,m)^2 = (0.5\,kg)(9.8\,m/s^2)(2.3\,m)$$

$$\boxed{k \approx 400\ N/m}$$

Conceptual questions for discussion

1. Consider a system of a ball, the atmosphere, and the earth. If air resistant is not negligible, is there any internal or external non-conservative work? If there is none, explain why. If there is, explain what force is responsible.
2. Consider CE.2.L2-4: If the kinetic friction was external to the system, would energy be conserved of the box+spring+earth system?
3. If there are no external forces and no internal non-conservative forces, how many snap shots do you need to apply an energy conservation?

Hints

CE.2.L2-1: No hints.

CE.2.L2-2: No hints.

CE.2.L2-3: No hints.

CE.2.L2-4: No hints.

CE.2.L2-5: No hints.