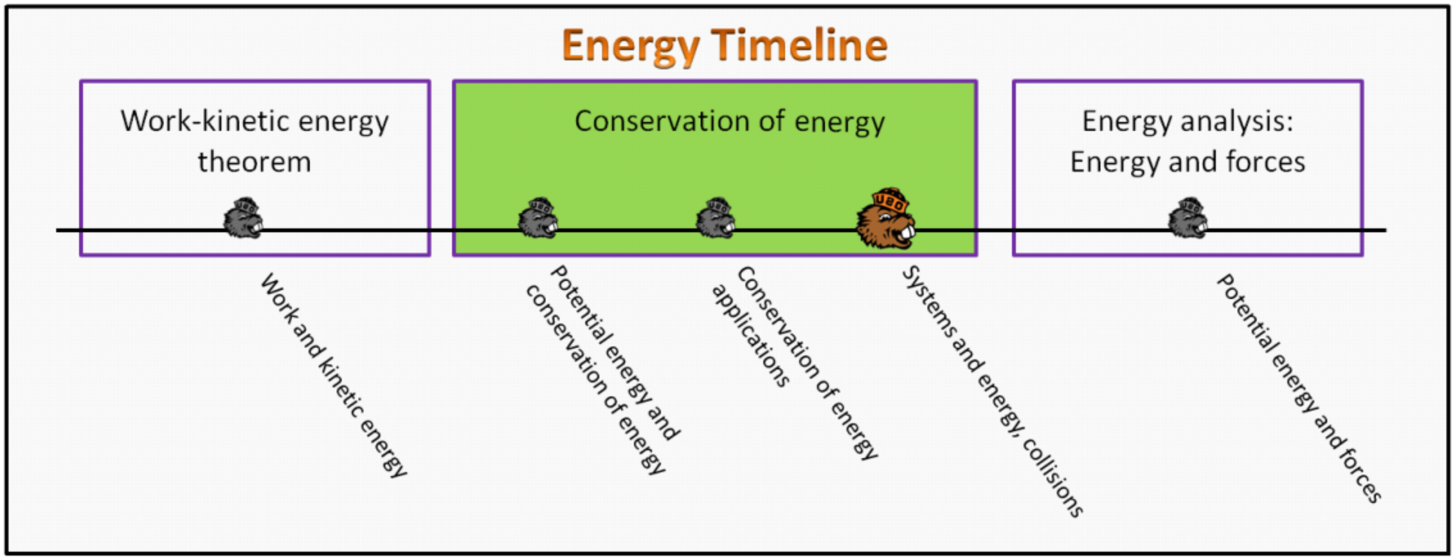
CE.2.L3.sols: Systems and Energy, Collisions

Monday, January 22, 2018 5:44 PM

Conservation of Energy
Foundation Stage (CE.2.L3)

lecture 3
Systems and energy, collisions



Textbook Chapters (* Calculus version)

- **BoxSand** :: KC videos (conservation of energy)
- **Giancoli** (Physics Principles with Applications 7th) :: 6-8 ; 7-4 ; 7-5 ; 7-6
- **Knight** (College Physics : A strategic approach 3rd) :: 10.1 ; 10.7
- ***Knight** (Physics for Scientists and Engineers 4th) :: 9.1 ; 11.3

Warm up

CE.2.L3-1:

Description: Given energy transformations and transfers, construct a work-energy equation..

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: Consider a system where spring potential energy and gravitational potential energy get converted into kinetic energy and an increase in thermal energy with no external work. Which of the following expressions is a correct application of the work-energy theorem for this scenario?

(1) $KE_i + U_i^s = KE_f$

$U^s + U^g \longrightarrow KE + E^{Th}$

(2) $U_i^g = KE_f$

(3) $U_i^g + U_i^s = KE_f + \Delta E^{Th}$

(4) $U_i^g + U_i^s \, KE_f + \Delta E^{Th} = 0$

Selected Learning Objectives

1. Define a system and identify internal and external objects, forces, and work.
2. Show that conservative forces yield work that is independent of the path taken.
3. Identify conservative and non-conservative forces and work.
4. (UPMF) Use the independence of path for conservative work to create a function of position to account for conservative work. We call that function a potential energy function for all internal conservative work.
5. Construct the final form of the work-energy theorem using the concept of potential energy.
6. Show that energy is conserved for systems where the net external work is zero.
7. Show that application of an energy analysis involves bookkeeping an initial and final state of the system.
8. Use the gravitational potential energy function, for near Earth objects, in an energy analysis.
9. Use the spring potential energy function for Hooke's law springs in an energy analysis.
10. Construct the graphical representation depicting the kinetic, potential, thermal, and total energy as a function of position.
11. (UPMF) When non-conservative work is internal to a system, there is a form of energy associated with that work and it is not called potential energy. E.g. work done by friction converting macroscopic kinetic energy to microscopic kinetic energy (aka thermal energy)
12. Identify other forms of energy such as thermal, chemical potential, electric potential, sound, light, … etc.
13. Identify energy transformations within a system and energy transfers into/out of a system.
14. Apply a conservation of energy equation for a system from its initial to final state.
15. Apply conservation of energy to system with multiple internal objects.
16. Define elastic collisions and apply a conservation of energy and momentum analysis to the collision.
17. Define inelastic collisions and apply a conservation of energy and momentum analysis to the collision.
18. Derive the relationship between kinetic energy and momentum.

Key Terms

- Internal energy transformation (i.e. energy transformation)
- External energy transfer (i.e. energy transfer)
- Chemical energy
- Sound energy
- Many many body systems
- Elastic collision
- Inelastic collision

Key Equations

$$E^{Total} = KE + U^{g} + U^{s} + E^{Th} + E^{Chem} + E^{Sound} + E^{Light} + \ldots$$

In words: The total energy of a system is equal to the sum of kinetic energy, gravitational potential energy, spring potential energy, thermal energy, chemical energy, sound energy, light energy, and other forms of energy that will be covered futher into our studies of physics.

Key Concepts

- Internal energy transformations are the exchanges of energy from one form to another within a system. For example, gravitational potential energy can be transformed into kinetic energy in a system. Recall that the conservation of energy is a bookkeeping system that states the total energy of an isolated system remains a constant value. Even though the total energy remains a constant value, the different forms of energy that make up a system can change so long as the total energy is constant.
- External energy transfers refers to the energy put into a system or taken out of a system when there is net external work. For example, consider a book and earth system. If I come along and lift to book up to a higher location, I put energy into the book earth system.
- As molecular bonds rearrange during chemical reactions, energy can be released or absorbed. Some examples of substances that have potential to transform chemical energy into other forms of energy are: food, gasoline, batteries, chemical reactions, etc..
- The atmosphere is made up of molecules that are able to wiggle back and forth. The wiggle motion is considered sound energy; basically it's tiny vibrational kinetic energy of the air molecules. If you wiggle some of the molecules at one location, their motion influences the motion of the air molecules around them causing nearby air molecules to wiggle. So the initial wiggle is propagated from one location to another, or in other words the sound energy propagates from one location to the next. This is how we hear someone from a distance for example.
- Informally, many many body systems means that more than one object in a system has kinetic energy that changes.
- In an elastic collision the kinetic energy before the collision is the same as the kinetic energy after the collision.
- In an inelastic collision the kinetic energy before the collision is greater than kinetic energy after the collision. Basically, some (or all) of the kinetic energy before the collision is transformed into other forms of energy.

Act I: Internal energy transformations and external energy transfers

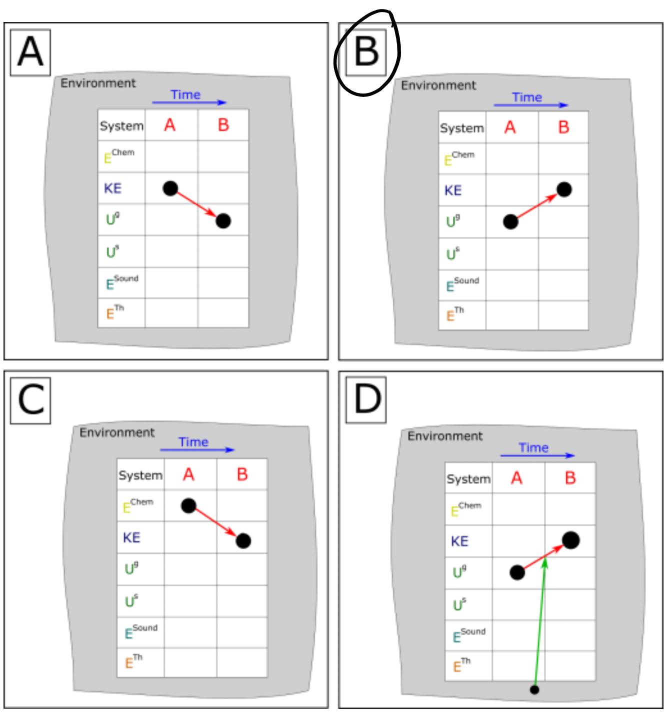
Questions

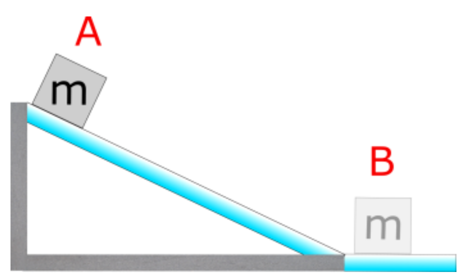
CE.2.L3-2:

Description: Identify energy transformations and transfers. (2 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A box starting from rest slides down a frictionless ramp as seen in the figure below where two snapshots are taken at location **A** and **B**. Consider a system that contains the box and the earth. Which diagram below correctly shows the energy transformations and transfers as the system evolves from **A** to **B**?



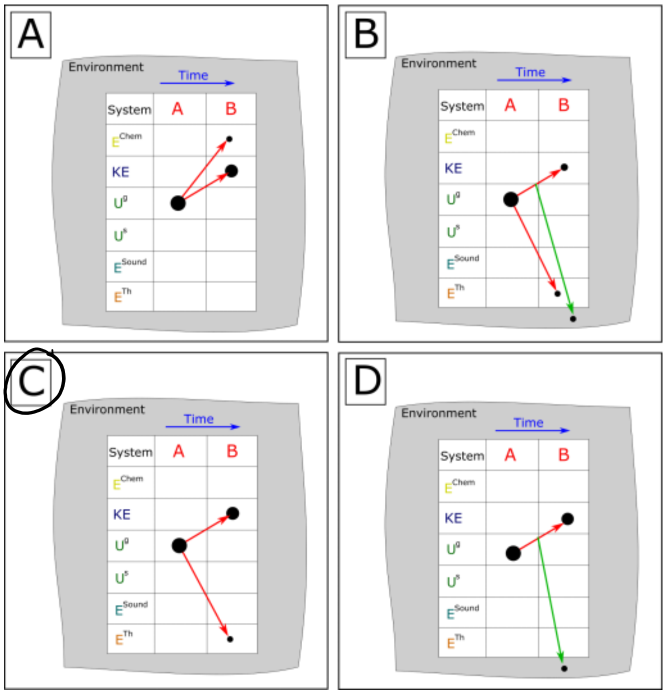


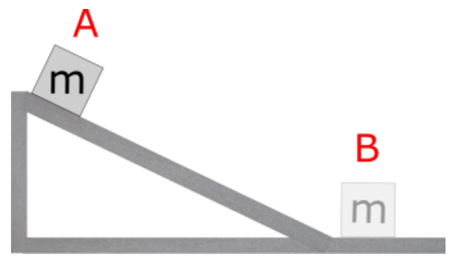
CE.2.L3-3:

Description: Identify energy transformations and transfers. (3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A box starting from rest slides down a ramp with friction as seen in the figure below where two snapshots are taken at location **A** and **B**. Consider a system that contains the box, earth, and ramp. Which diagram below correctly shows the energy transformations and transfers as the system evolves from **A** to **B**?



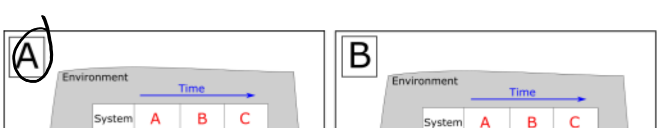


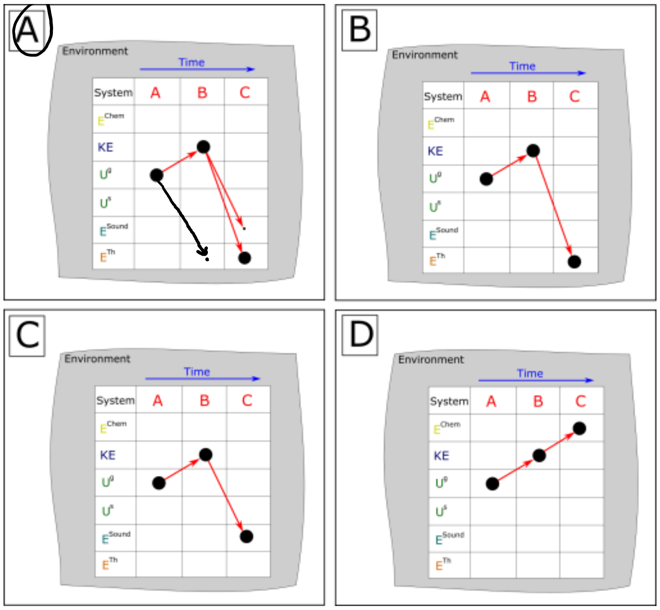
CE.2.L3-4:

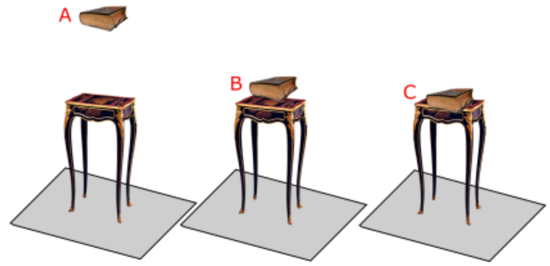
Description: Identify energy transformations and transfers. (3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A book is released from rest above a table as seen in the figure below where three snapshots are taken at location **A**, **B,** and **C**. Snapshot **B** is taken the moment before the book hits the table. Consider a system that contains the book, earth, table, and atmosphere (i.e. air molecules). Which diagram below correctly shows the energy transformations and transfers as the system evolves from **A** to **B** and then from **B** to **C**?





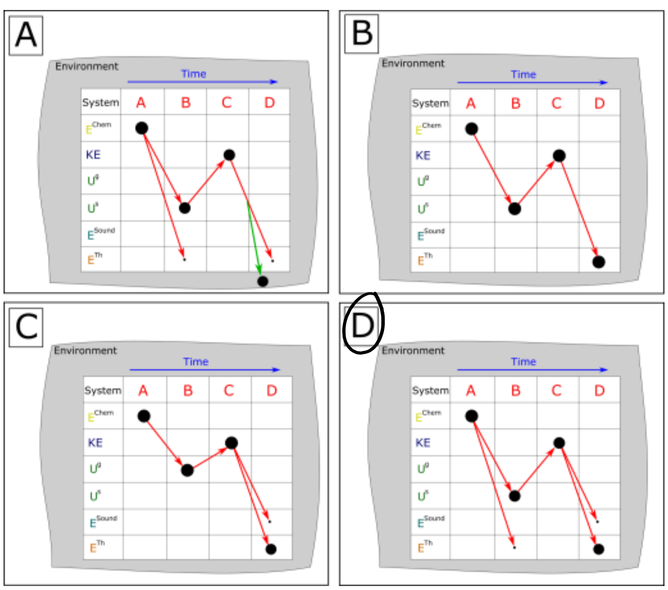


CE.2.L3-5:

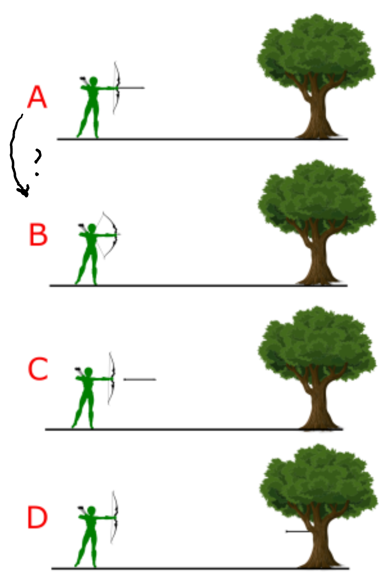
Description: Identify energy transformations and transfers. (4 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Artemis pulls back an arrow at a constant velocity with a bow, and then fires the arrow into a tree as seen in the figures below. Each figure is a snapshot in time. Consider a system that contains Artemis, earth, bow, arrow, tree, and atmosphere. Which diagram below correctly shows the energy transformations and transfers as the system evolves from **A** to **B** , then **B** to **C**, then **C** to **D**?



Molecular motors... ...microscopic forces over microscopic displacements... they use ATP to exert these forces and the ATP comes from the chemical breakdown of our food, so we call this chemical energy.

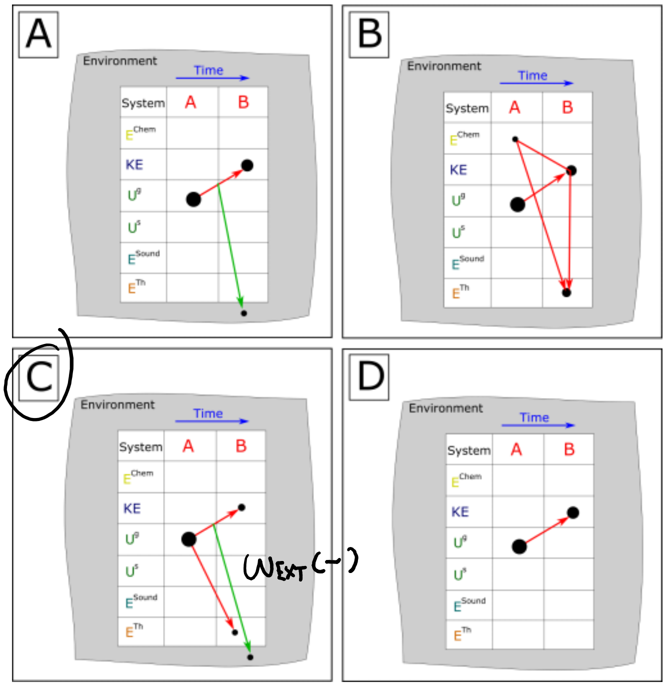


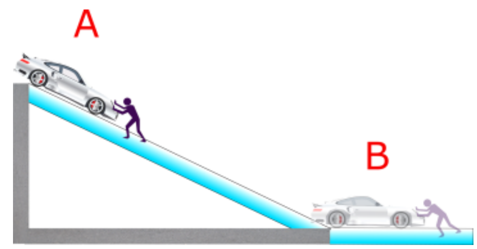
CE.2.L3-6:

Description: Identify energy transformations and transfers. (3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A car in neutral starting from rest begins to move down an incline while speeding up. A person begins to push on the front of the car in hopes to slow it down. Two snapshots are taken where **A** is taken when the car is at rest and **B** is taken a little time later as the car is moving down the incline and speeding up. Consider a system of the car, and the earth. Which diagram below correctly shows the energy transformations and transfers as the system evolves from **A** to **B**? (Ignore the rotation of the wheels).



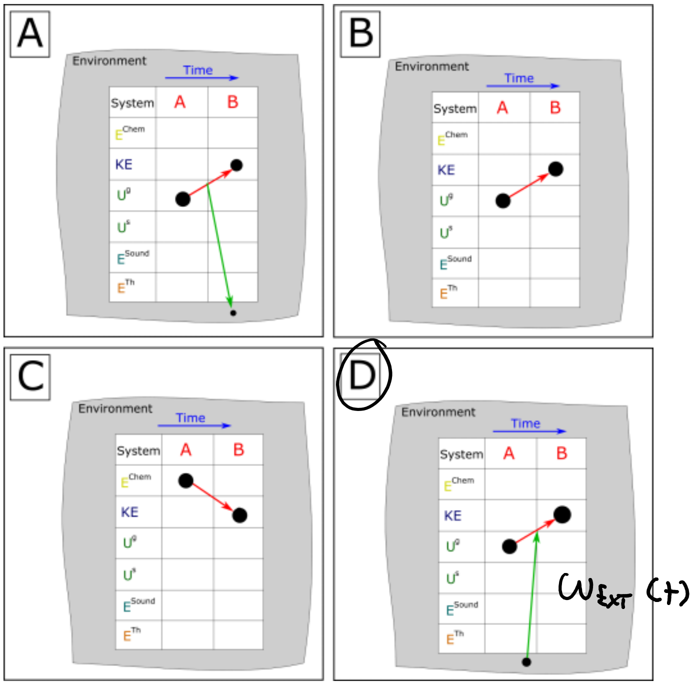


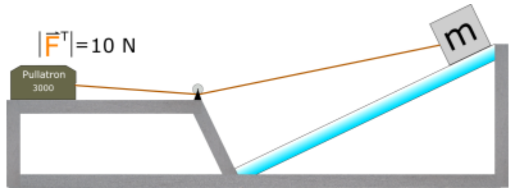
CE.2.L3-7:

Description: Identify energy transformations and transfers. (3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A deliver box, initially at rest, sits on top of a frictionless incline plane and is being pulled with a constant magnitude force from a Pullatron 3000. Consider a system that contains the box and earth. Which diagram below correctly shows the energy transformations and transfers as the system evolves from snapshot **A** to **B** ?





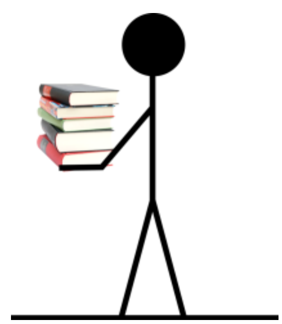
CE.2.L3-8:

Description: Identify energy transformations and transfers. (4 minutes + 2 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Three students are discussing the concept of work in regards to a situation where a dude is standing at rest holding some books.

(a) Which student do you agree with the most?



T (1) "The books aren't moving so there is no way any work is being done on them."

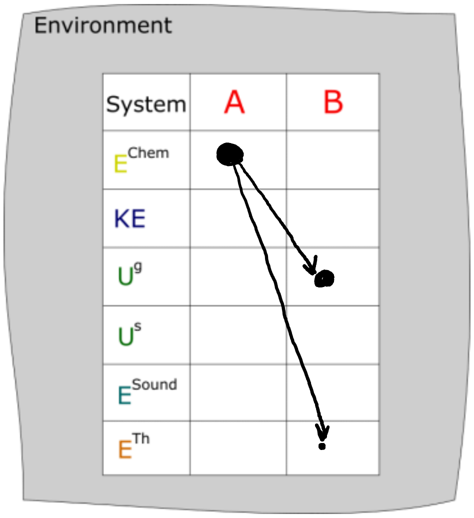
(2) "That doesn't make sense because the dudes arms would be getting tired and thus he must be losing energy, if energy is being lost then work must be done on the dude."

(3) "The only force connecting the books and that dude are the normal force, so if the books have no work being done on them but the dude does have work being on him it can't be due to the normal force from the books."

(4) All statements are correct.

$E^{CHEM} \longrightarrow E^{TH}$

(b) The dude now begins to lift the books at a constant velocity upwards. With a system that includes the dude, earth, and the books, sketch the energy transformations and transfers as the system evolves from snapshot **A** to **B** where the books are at a higher height than they were at **A**.



$\Delta E^{chem} + \Delta U^{g} + \Delta E^{th} = 0$

CE.2.L3-9:

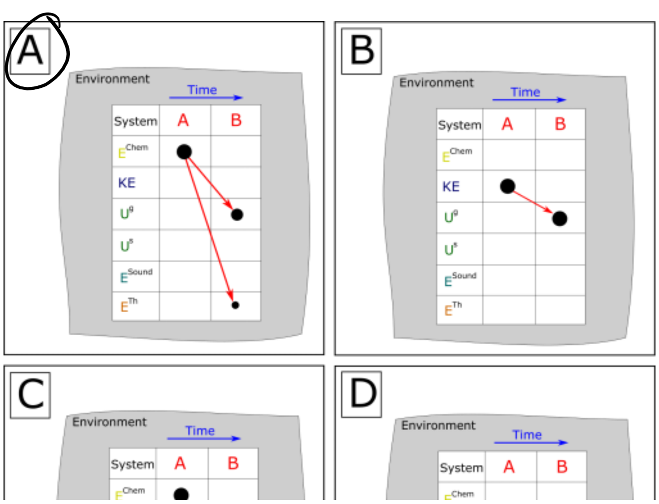
Description: Identify energy transformations and transfers. (2 minutes + 2 minutes)

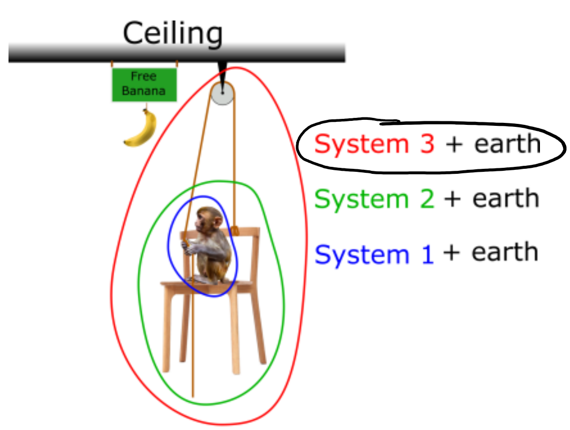
Learning Objectives: [1, 12, 13]

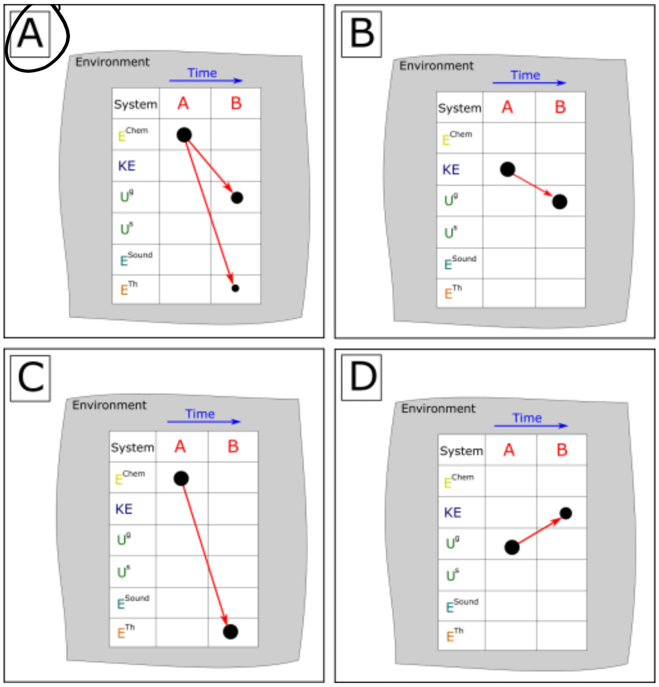
Problem Statement: Mavin the monkey lifts himself up while on a chair with the contraption shown in the image below.

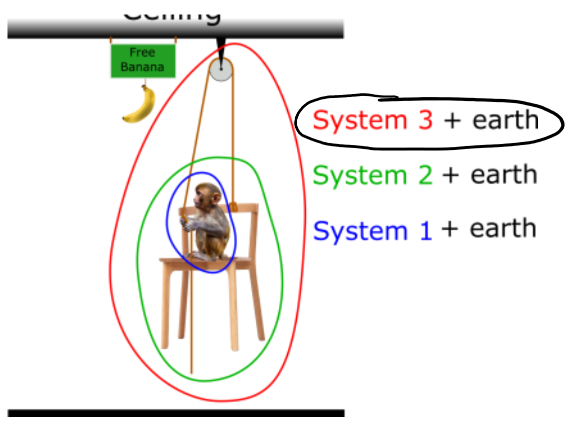
(a) Which system is there zero external work being done? System 3

(b) With your answer to part (a), which diagram below shows the correct energy transformations and within the system?









FOR FUN ... MA = 2

Act II: Many many body systems

CE.2.L3-10:

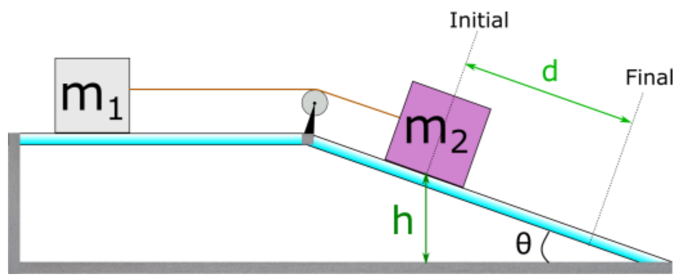
Description: Apply conservation of energy for a system with more than one object with changing KE. (3 minutes + 1 minute + 3 minutes + 1 minute + 6 minutes + 4 minutes)

Learning Objectives: [15]

Problem Statement: Two masses are connected via an ideal pulley as seen in the figure below. Mass $\mathbf{m_1}$ is on a horizontal surface and mass $\mathbf{m_2}$ is on an incline initially at rest. There is negligible friction between the masses and the surfaces they are on. We eventually wish to find the speed of $\mathbf{m_2}$ once it has traveled some distance **d** down the incline.

(a) Which physics tools would be useful in the analysis to find the speed of $\mathbf{m_2}$?

T (1) Kinematics and forces using separate systems (1)(2)
F (2) Conservation of momentum using combine system(12)
F (3) Impulse and momentum using separate systems(1)(2)
T (4) Work and energy using separate systems(1)(2)
T (5) Conservation of energy using combine system(12)



(b) How many systems do you see? 3 1, 2, 12

(c) Below there are three solution flowcharts to get to the final speed of $\mathbf{m_2}$. Each flowchart represents an physics analysis using a specific system. Match each flowchart with the physics tools from part (a).

$FBD \longrightarrow \Sigma\vec{F} \longrightarrow \Sigma W_{ext} \longrightarrow E_i + \Sigma W_{ext} = E_f \longrightarrow |\vec{v}_2|$

$FBD \longrightarrow \Sigma\vec{F} \longrightarrow \vec{a} \longrightarrow (\Delta x, v_{ix}, v_{fx}, \Delta t) \longrightarrow |\vec{v}_2|$

$\Sigma E_i + 0 = \Sigma E_f \longrightarrow |\vec{v}_2|$

(1) Kinematics and forces using separate systems (1)(2)

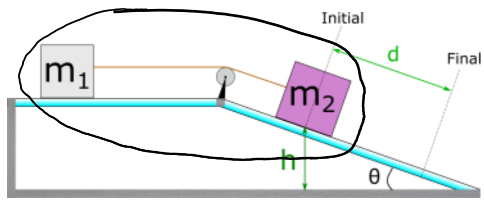
(2) Conservation of momentum using combine system(12)

(3) Impulse and momentum using separate systems(1)(2)

(4) Work and energy using separate systems(1)(2)

(5) Conservation of energy using combine system(12)

(d) Draw a line to define a system that would have zero external work being done on it.



(e) Which of the following statements are correct?

F (1) The kinetic energy of **m_1** doesn't change but **m_2** has gravitational potential energy converted into kinetic energy.

F (2) The gravitational potential energy of **m_1** and **m_2** remains constant as the kinetic energy from **m_1** is converted into kinetic energy of **m_2**.

F (3) The potential energy of both **m_1** and **m_2** is converted into kinetic energy of **m_1** and **m_2**.

T (4) Some potential energy from **m_2** is converted into kinetic energy in **m_1** and **m_2**.

(f) Two masses are connected via an ideal pulley as seen in the figure below. Mass m_1 is on a horizontal surface and mass m_2 is on an incline initially at rest. There is negligible friction between the masses and the surfaces they are on. **Find an expression for the speed squared for m_2 in terms of m_1, m_2, g, and h after it has traveled a distance d down the incline.**

(1) $\frac{2\, m_2\, g\, h}{m_1 + m_2}$

X (2) $\frac{2\, m_1\, g\, h}{m_1 + m_2}$

F (3) $\frac{2\, m_2\, g}{m_1 + m_2}$ $\frac{m}{s^2}$ X

F (4) $\frac{2\, m_1\, g}{m_1 + m_2}$ $\frac{m}{s^2}$ X

$KE_{i1} + KE_{i2} + U^g_{i2} + \Sigma W_{ext} = KE_{f1} + KE_{f2} + U^g_{f2}$

$U^g_{i2} = KE_{f1} + KE_{f2}$

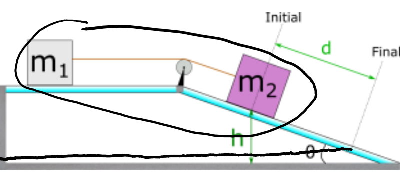
$m_2 g y_{i2} = \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2$

$m_2 g h = \frac{1}{2} m_1 V_f^2 + \frac{1}{2} m_2 V_f^2$

$2 m_2 g h = m_1 V_f^2 + m_2 V_f^2$

$2 m_2 g h = (m_1 + m_2) V_f^2$

$$V_f^2 = \frac{2 m_2 g h}{(m_1 + m_2)}$$



$\hat{y}$ y=0

Connection

$V_{1f} = V_{2f} = V_f$

(g) Using the correct expression from part (f), what does the proportional reasoning sense making strategy say will happen to the final speed of **m_2** if **m_1** gets larger?

(1) final speed of **m_2** will increase

$V_f^2 = \frac{2 m_2 g h}{\quad}$

speed of $\mathbf{m_2}$ if $\mathbf{m_1}$ gets larger?

$$V_f^2 = \frac{2m_2gh}{(m_1+m_2)}$$

(1) final speed of $\mathbf{m_2}$ will increase
(2) final speed of $\mathbf{m_2}$ will decrease
(3) final speed of $\mathbf{m_2}$ will remain the same
(4) since $\mathbf{m_1}$ is added to $\mathbf{m_2}$, we cannot determine how this will affect the final speed of $\mathbf{m_2}$

w/ g + h const

$$V_f^2 \propto \frac{m_2}{m_1+m_2}$$

IF $m_1 \uparrow$

THEN $m_1 + m_2 \uparrow$

SO $\frac{m_2}{(m_1+m_2)} \downarrow$

Act III: Elastic vs. Inelastic collisions

CE.2.L3-11:

Description: Identify elastic and inelastic collisions. (3 minutes)

Learning Objectives: [16, 17]

Problem Statement: Which of the following scenarios are elastic collisions?

(1) A lineman tackling a running back
(2) Two billiard balls bouncing off each other ~ SO VERY CLOSE BUT NOT NECESSARILY
(3) Two cars bouncing off each other
(4) Asteroid deflected by earth
(5) Electrons scattering off each other

CE.2.L3-12:

Description: Identify elastic and inelastic collisions. (3 minutes)

Learning Objectives: [16, 17]

Problem Statement: Which of the following scenarios is the most inelastic collision?

(1) A lineman tackling a running back
(2) Two billiard balls bouncing off each other
(3) Two cars bouncing off each other
(4) Asteroid deflected by earth
(5) Electrons scattering off each other

DEPENDS ON HOW THEY COLLIDE

BUT 2 CARS MOST LIKELY.

CE.2.L3-13:

Description: Apply conservation of energy with conservation of momentum when an inelastic collision is present. (2 minutes + 2 minutes + 5 minutes + 8 minutes)

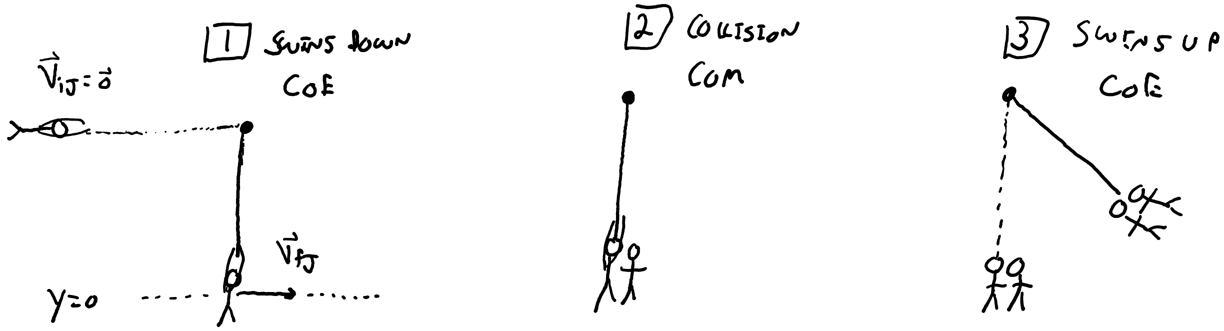
Learning Objectives: [17]

Problem Statement: Tarzan is in the path of a pack of stampeding elephants when Jane swings in to the rescue on a rope vine, hauling him off to safety. The length of the rope is 25 meters, and Jane starts her swing with the rope horizontal. Jane's mass is 54 kg and Tarzan's mass is 82 kg. We wish to find the maximum height the two will be above the ground after Jane saves Tarzan.

(a) How many stages would you break this problem into?

3

(b) What is the physics in each stage?



(c) Tarzan is in the path of a pack of stampeding elephants when Jane swings in to the rescue on a rope vine, hauling him off to safety. The length of the rope is 25 meters, and Jane starts her swing with the rope horizontal. Jane's mass is 54 kg and Tarzan's mass is 82 kg. **What is the speed of Jane right before she catches Tarzan?**

1 CoE

$$U^g_{iJ} + \cancel{KE_{iJ}}^{0} + \cancel{\Sigma W_{ext}}^{0} = KE_{fJ} + \cancel{U^g_{fJ}}^{0}$$

$$m_J g y_i = \frac{1}{2} m_J v_{fJ}^2$$

$$v_{fJ} = \sqrt{2 g y_i} = \sqrt{2(9.8\,m/s^2)(25\,m)} \approx 22.136\ m/s$$

(d) Tarzan is in the path of a pack of stampeding elephants when Jane swings in to the rescue on a rope vine, hauling him off to safety. The length of the rope is 25 meters, and Jane starts her swing with the rope horizontal. Jane's mass is 54 kg and Tarzan's mass is 82 kg. **What is the maximum height the two will be above the ground after Jane saves Tarzan?**

2) COM $\rightarrow \hat{x}$

$$\sum \vec{P}_i = \sum \vec{P}_f$$

$$m_J v_{iJx} + m_T v_{iTx} = (m_J + m_T) v_{fTJx}$$

$$v_{fTJx} = \frac{m_J}{(m_J + m_T)} v_{iJx}$$

OR

$$v_{fTJx} = \frac{m_J}{(m_J + m_T)} \sqrt{2 g y_i} \approx 8.789\ m/s$$

3) COE

$$KE_{iTJ} + U^g_{iTJ} + \sum W_{Ext} = KE_{fJT} + U^g_{fTJ}$$

$$\frac{1}{2}(m_T + m_J) v_{iTJ}^2 = (m_T + m_J) g y_f$$

OR

$$y_f = \frac{1}{2g} v_{iTJ}^2$$

$$y_f = \frac{m_J^2}{(m_J + m_T)^2} y_i \approx 3.94\ m$$

Conceptual questions for discussion

1. Find a roommate/friend/family-member and try to explain to them in your own words what conservation of energy is and what

energy transformations and transfers are. Perhaps use some simplified examples similar to act I questions to help communicate your understanding.

2. Create an energy flow chart similar to the ones in act I for a grandfathers clock that uses a falling weight to keep a pendulum swinging.
3. What are some of the most inelastic collisions you can think of? How much KE is left after the collisions in your example(s)?

Hints

CE.2.L3-1: No hints.

CE.2.L3-2: No hints.

CE.2.L3-3: No hints.

CE.2.L3-4: No hints.

CE.2.L3-5: No hints.

CE.2.L3-6: No hints.

CE.2.L3-7: No hints.

CE.2.L3-8: No hints.

CE.2.L3-9: No hints.

CE.2.L3-10: No hints.

CE.2.L3-11: No hints.

CE.2.L3-12: No hints.

CE.2.L3-13: No hints.

CE.2.L3-14: No hints.

CE.2.L3-15: No hints.

CE.2.L3-16: No hints.

CE.2.L3-17: No hints.

CE.2.L3-18: No hints.

CE.2.L3-19: No hints.

CE.2.L3-20: No hints.