Work and Energy

* $[Dimensions] \xrightarrow{\text{SI units}} [L]\rightarrow m$, $[M]\rightarrow kg$, $[T]\rightarrow s$

Mathematical representation

Translational or Linear Kinetic Energy $\frac{[M][L]^2}{[T]^2} \equiv KE_T = \frac{1}{2}m|\vec{v}_{cm}|^2$

Work $\frac{[M][L]^2}{[T]^2} \equiv W = \vec{F} \cdot \Delta\vec{r}$

"DOT PRODUCT". IT IS A VECTOR OPERATION

Work from a constant force and straight line displacement only. If not a constant force, use area under force vs position graph.

How to deal with dot product

$$W = |\vec{F}||\Delta\vec{r}|\cos(\theta)$$

Smallest angle between $\vec{F}$ and $\Delta\vec{r}$ when placed tail to tail.

Also

$W = F_{||}|\Delta\vec{r}|$ or $W = |\vec{F}|\Delta r_{||}$

$F_{||}$: Parallel component of force with respect to the displacement

$\Delta r_{||}$: Parallel component of displacement with respect to the force

Gravitational potential energy $\frac{[M][L]^2}{[T]^2} \equiv U^g = m\,g\,y$

- Valid for near surface of earth only.

y: Vertical distance above (+) or below (-) a horizontal reference axis.

Spring potential energy $\frac{[M][L]^2}{[T]^2} \equiv U^s = \frac{1}{2}\,k\,\Delta x^2$

Δx: Displacement from equilibrium position. $\Delta x = x_f - x_{eq}$

Work-energy

$$W_{external}^{C,NC} = \Sigma\Delta KE_T + \Sigma\Delta U^g + \Sigma\Delta U^s + \Sigma\Delta E^{Thermal} + \Sigma\Delta E^{Chemical} + \cdots$$

$\Delta KE_{system} = \Delta KE_{T1} + \Delta KE_{T2} + \ldots$

$W^{C}_{Internal} = -\Delta U^c$

$-\Delta U^g \ldots -\Delta U^s \ldots$

$W^{NC}_{Internal} = -\Delta E^{NC}$

$-\Delta E^{th} \ldots -\Delta E^{chem} \ldots$

$W^{C,NC}_{External} = \vec{F}^{C,NC}_{External} \cdot \Delta\vec{r}$

$W_{internal}$ - Mechanism for how system can transform energy within the system.

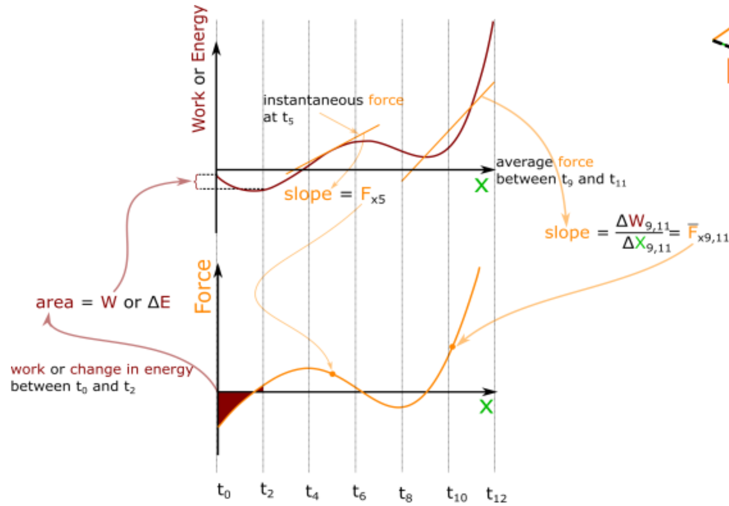
$W_{external}$ - Mechanism for how energy is transported into our out of a system.

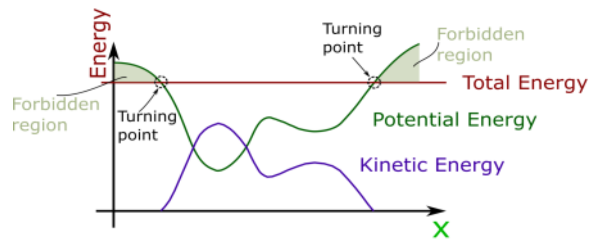
Graphical representation

Energy(x) or work(x)

Area / Slope

Force(x)





Total Energy = Kinetic Energy + Potential Energy

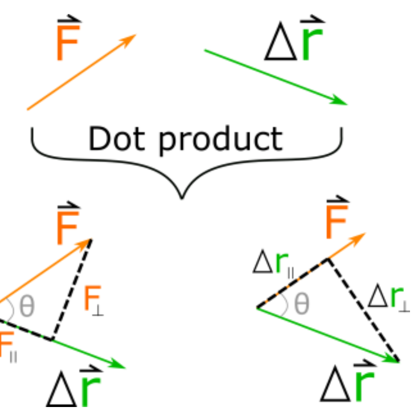
If $W_{ext} = 0$
Then Total Energy = Constant

Gradient "3-D slope"

$$\vec{F} = -\nabla U$$

$$\vec{\bar{F}} = -\left\langle \frac{\Delta U}{\Delta x}, \frac{\Delta U}{\Delta y}, \frac{\Delta U}{\Delta z} \right\rangle = \left\langle \bar{F}_x, \bar{F}_y, \bar{F}_z \right\rangle$$

Physical representation



Work is positive if:

$0° < \theta < 90°$
$\theta = 0°$

Work is negative if:

$90° < \theta < 180°$
$\theta = 180°$

Work is 0 if:

$\theta = 90°$