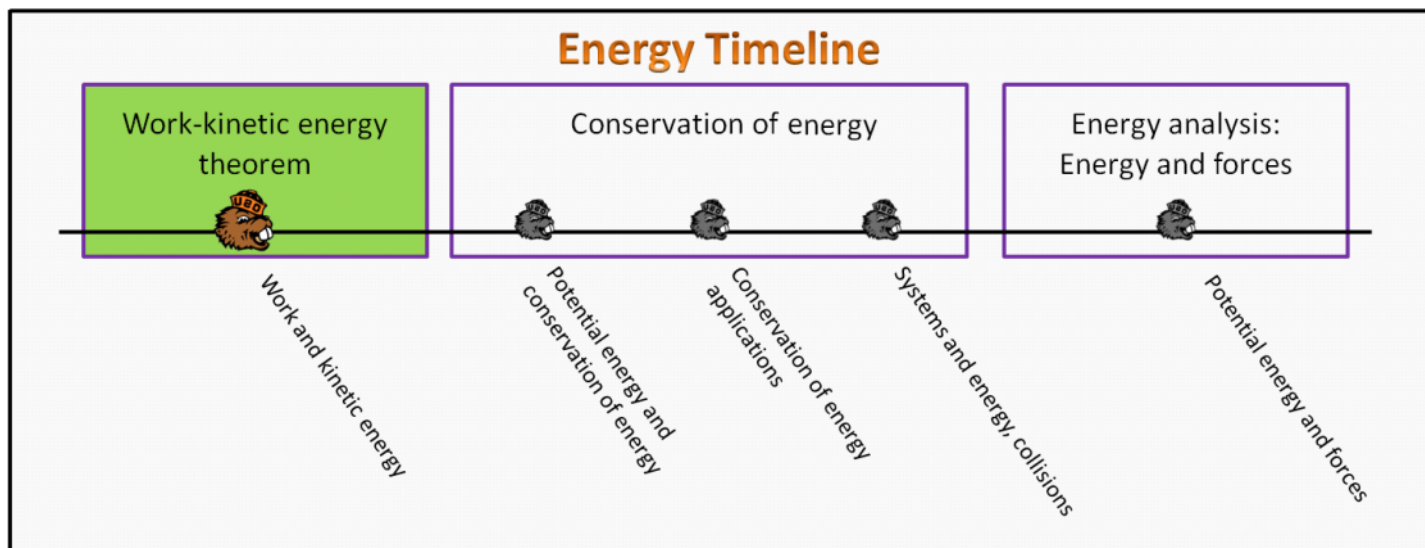


## Work-kinetic energy theorem Foundation Stage (WE.2.L1)

### lecture 1 Work and kinetic energy



**Textbook Chapters**

- **BoxSand** :: KC videos ( [work and kinetic energy](#) )
- **Giancoli** (Physics Principles with Applications 7<sup>th</sup>) :: 6-1 ; 6-3
- **Knight** (College Physics : A strategic approach 3<sup>rd</sup>) :: 10.2 ; 10.3
- **Knight** (Physics for Scientists and Engineers 4<sup>th</sup>) :: 9.1 ; 9.2

**Warm up**

**WE.2.L1-1:**

**Description:** Proportional reasoning with KE.

**Learning Objectives:** [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

**Problem Statement:** The kinetic energy of an object is defined as:  $KE = \frac{1}{2} m v^2$ , where **m** is the mass of the object and **v** is the speed of the object. If an objects speed increases by a factor of 3, by what factor does the KE change?

- (1) 1/9
- (2) 1/3
- (3) 1
- (4) 3
- (5) 9**

$$KE = \frac{1}{2} m v^2$$

*by m = const*

$$KE \propto v^2$$

If  $v \rightarrow 3v$   
 $\therefore \quad \quad \quad \rightarrow 9 \times KE$

$$\text{If } v \rightarrow 3v$$

$$\text{KE} \rightarrow 3^2 \text{KE}$$

## Selected Learning Objectives

1. Find the kinetic energy for a system.
2. Identify which quantities are scalars and which are vectors.
3. Define the work-energy theorem and explain how work is a mechanism to transfer energy between systems.
4. (UPMF) Differentiate between internal and external work.
5. Demonstrate that energy involves forces applied over distances and momentum involves forces applied over time.
6. (UPMF) Find the area under a net force vs displacement curve and set that equal to the work on the system.
7. Identify that work is a dot product between the force and the change in position.
8. Determine whether work is positive or negative based on forces and displacement.
9. Determine whether (net) work is positive or negative based on the change in kinetic energy.
10. Identify that a dot product depends on the magnitude of the two vectors and the smallest angle between them.
11. Explain how a dot product shows how much one vector points in the same direction as another using the projection method.
12. (UPMF) Use a vector operation diagram to visualize the dot product between two vectors.
13. Calculate work by applying the dot product in the mathematical form via magnitudes and smallest angles.
14. Calculate work by applying the dot product in the mathematical form of summation of products of components.
15. Construct the work-energy theorem in the mathematical representation to analyze physical systems.

## Key Terms

- o Work
- o Dot product
- o Kinetic energy
- o Work - kinetic energy theorem

## Key Equations

**Work**

Type of work and force:  
applied (generic)

Dot product  
(a vector operation)

$$W^A = \vec{F}^A \cdot \Delta \vec{r}$$

Work

Force

Displacement

**Kinetic Energy**

Kinetic Energy

Mass

Speed

$$KE = \frac{1}{2} m v^2$$

In words: Work done by force A is equal to the dot product of force A with the displacement that the force acts through.

In words: The kinetic energy of an object is equal to one half the mass of the object multiplied by the speed of the object squared.

Type of work and force: applied (generic)

Smallest angle between force and displacement

$$W^A = |\vec{F}^A| |\Delta \vec{r}| \cos(\theta)$$

Work      Force      Displacement

In words: The work done by force A is equal to the magnitude of force A times the magnitude of the displacement that the force acts through times the cosine of the smallest angle between force A and the displacement when placed tail-to-tail.

### Work - kinetic energy theorem

Work

Change in Kinetic Energy

$$\sum W_{\text{external to system}} = \Delta KE$$

Net (i.e. add up all of the \_)

OR

Initial Kinetic Energy

Work

Final Kinetic Energy

$$KE_i + \sum W_{\text{external to system}} = KE_f$$

Net (i.e. add up all of the \_)

In words: The final kinetic energy of a system is equal to the initial kinetic energy of the system plus the net external work.

In words: The net external work on a system is equal to the change in kinetic energy of the system.

### Key Concepts

- Work is a scalar quantity. Remember that scalars can be positive or negative.
- The dot product is a vector operation that takes in two vectors, and returns a scalar.
- Conceptually, the dot product asks how parallel two vectors are relative to each other. The larger the value of the dot product, the more parallel until the dot product becomes its maximum value, then the two vectors are perfectly parallel to each other. If the dot product is zero, then the two vectors are perpendicular to each other.
- The angle that goes into the cosine in the dot product, is the angle between the two vectors when placed tail to tail. It is a good idea to draw a vector operation to find this angle when the vectors are not parallel or perpendicular.
- The definition of work as seen in the key equations above is only valid if the force and displacement are constant.
- Kinetic energy is a positive scalar that represents the energy associated with the translational motion of the center of mass of an object.
- The work - kinetic energy theorem relates the net external work and the change in kinetic energy of an object. If the kinetic energy of an object increases, then the net external work on the object is positive. If the kinetic energy of an object decreases, then the net external work on the object is negative.
- If you have information about forces and distances for an object, then the work - kinetic energy theorem is a good place to start when trying to analyze the motion of the object. Similarly, if you have information about forces and time for an object, then the impulse - momentum theorem is a good place to start when trying to analyze the motion of the object.
- For 1-D motion, the area under the net force vs position graph is the work done on the object. Recall the work - kinetic energy theorem, thus the area under the next force vs position graph for 1-D motion is also the change in kinetic energy of the object.

### Act I: Work

### Questions

WE.2.L1-2:

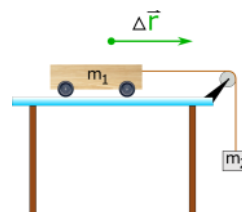
**Description:** Identify relevant physics analysis tools given a scenario. (2 minutes + 2 minutes + 2 minutes + 2 minutes )

**Learning Objectives:** [5]

**Problem Statement:** Which of the following physics analysis procedures can be used to analyze the given scenarios?

(a) A cart on a table with negligible friction has an ideal rope attached to it which is draped over an ideal pulley and connected to a hanging mass. The cart starts from rest and goes through a displacement of  $\Delta \vec{r}$ .

- (1) Kinematics analysis
- (2) Force analysis
- X (3) Impulse and momentum analysis *~ NO DIRECT INFO OF  $\Delta t$*
- X (4) Conservation of momentum analysis  *$\Delta \vec{P} \neq 0$*
- (5) Work and kinetic energy analysis

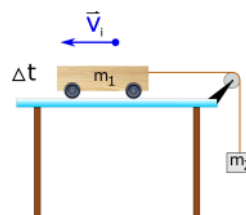


w/  $\vec{v}$ :  $+ \Delta \vec{r} + \sum \vec{F} = m \vec{a}$

OR w/KE:  $+ \Delta \vec{r} + \sum W$

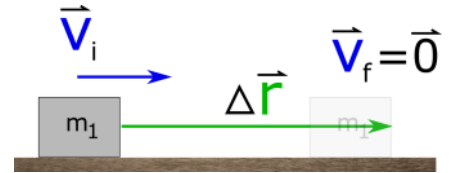
(b) A cart on a table with negligible friction has an ideal rope attached to it which is draped over an ideal pulley and connected to a hanging mass. The cart has an initial velocity to the left and the tension acts on the cart for some amount of time  $\Delta t$  before coming to a rest.

- (1) Kinematics analysis
- (2) Force analysis
- (3) Impulse and momentum analysis
- X (4) Conservation of momentum analysis *~  $\Delta \vec{P} \neq 0$*
- X (5) Work and kinetic energy analysis *~ NO DIRECT INFO OF  $\Delta \vec{r}$*



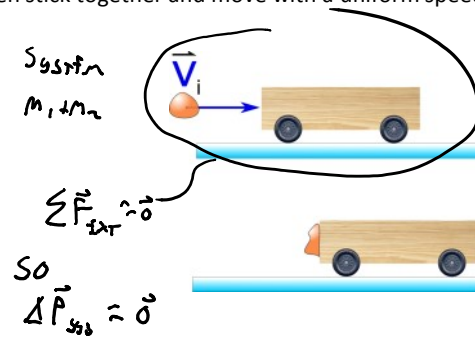
(c) A wood box slides across a carpet, going through some displacement  $\Delta\vec{r}$  before coming to rest.

- (1) Kinematics analysis
- (2) Force analysis
- (3) Impulse and momentum analysis
- (4) Conservation of momentum analysis
- (5) Work and kinetic energy analysis



(d) A clay ball is thrown at a cart with negligible friction. The two then stick together and move with a uniform speed to the right.

- (1) Kinematics analysis
- (2) Force analysis
- (3) Impulse and momentum analysis
- (4) Conservation of momentum analysis
- (5) Work and kinetic energy analysis



**WE.2.L1-3:**

**Description:** Find work from tension for a 1-D system. (4 minutes + 2 minutes)

**Learning Objectives:** [7, 8, 10, 12, 13]

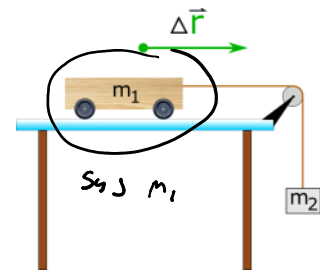
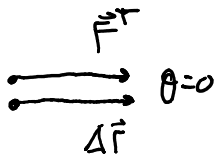
**Problem Statement:** Consider a system of just the cart. Calculate the work done by tension on the cart.

(a) A cart on a table with negligible friction has an ideal rope attached to it which is draped over an ideal pulley and connected to a hanging mass. The cart starts from rest and goes through a displacement of 2 meters to the right with a constant tension of 5 N to the right.

- (1) 3 Nm
- (2) -3 Nm
- (3) 6 Nm
- (4) -6 Nm
- (5) 10 Nm
- (6) -10 Nm

$$\begin{aligned}
 W^T &= \vec{F}^T \cdot \Delta \vec{r} \\
 &= |\vec{F}^T| |\Delta \vec{r}| \cos \theta \\
 &= (5\text{ N})(2\text{ m}) \cos(0) \\
 &= 10\text{ Nm}
 \end{aligned}$$

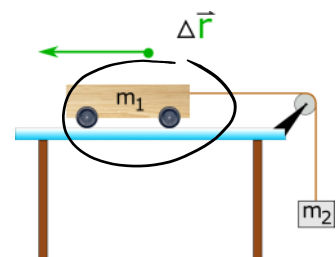
VEC. OP.



(b) A cart on a table with negligible friction has an ideal rope attached to it which is draped over an ideal pulley and connected to a hanging mass. The cart has an initial velocity to the left and the tension, 5 N, acts on the cart through a displacement of 2 meters to the left.

- (1) 3 Nm
- (2) -3 Nm
- (3) 6 Nm
- (4) -6 Nm
- (5) 10 Nm
- (6) -10 Nm

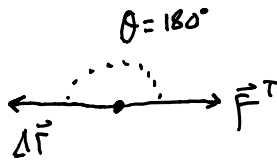
$$\begin{aligned}
 W^T &= |\vec{F}^T| |\Delta \vec{r}| \cos \theta \\
 &= (5\text{ N})(2\text{ m}) \cos(180) \\
 &= -10\text{ Nm}
 \end{aligned}$$



$\odot -10 \text{ Nm}$

$\approx -10 \text{ Nm}$

UEC.  $\infty$ .



**WE.2.L1-4:**

**Description:** Rank magnitude of work given a scaled FBD and direction of motion. (4 minutes)

**Learning Objectives:** [7, 10, 11, 12]

**Problem Statement:** Three tug boats are pulling OSU's new Regional Class Research Vessel at a constant velocity from the bow of the boat. The image below shows all the forces acting on the RCRV: Tensions from the tug boats, gravity, and buoyant force. Which force does the most magnitude of work on the RCRV?

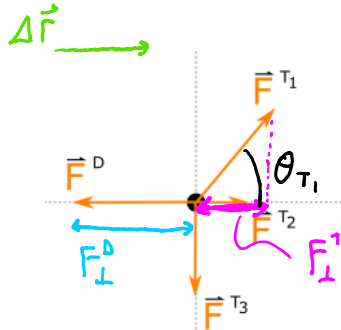
- X (1)  $|W^{T_1}|$
- X (2)  $|W^{T_2}|$
- (3)  $|W^{T_3}|$
- (4)  $|W^g|$
- (5)  $|W^B|$
- (6)  $|W^D|$
- X (7)  $|W^{T_1}| = |W^D|$

$\omega |F^{T_2}| < |F^D|$

AND  $|\cos \theta_{T_2}| = |\cos \theta_D|$

$|W^{T_2}| < |W^D|$

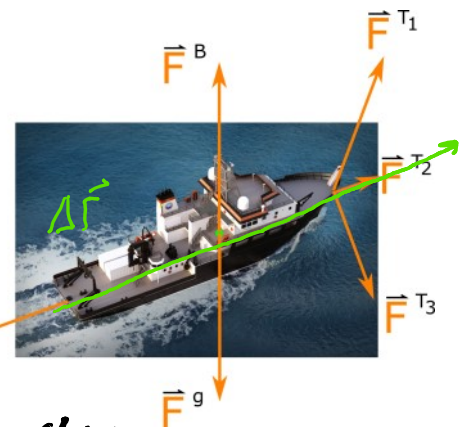
FBD Top-down view



$\omega |F^{T_1}| = |F^D|$

AND  $|\cos \theta_{T_1}| < |\cos \theta_D|$

$|W^{T_1}| < |W^D|$



...OR...  
 $F_{\perp}^{T_1} < F_{\perp}^D$   
 $\omega \Delta F$  SAME  
 $|W^{T_1}| < |W^D|$

**WE.2.L1-5:**

**Description:** Find net work on an object given forces and displacement. (6 minutes)

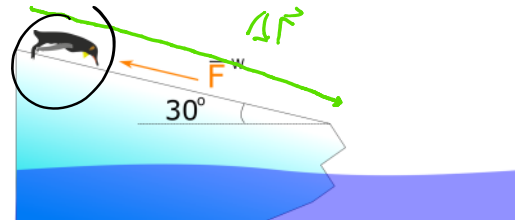
**Learning Objectives:** [7, 10, 12, 13]

**Problem Statement:** Puddles the Penguin has a mass of 10 kg and slides down a 3 meter icy slope while there is a constant wind force of magnitude 8 N up the incline acting on her. Calculate the net work on Puddles.

- (1) 123 N·m
- (2) 147 N·m
- (2) 173 N·m
- (3) -23 N·m

$$\sum W_{\text{Ext}} = W^N + W^g + W^w$$

$$\vec{F}^w \perp \Delta \vec{r}$$



$$= |\vec{F}^g| |\Delta \vec{r}| \cos \theta_g + |\vec{F}^w| |\Delta \vec{r}| \cos \theta_w$$

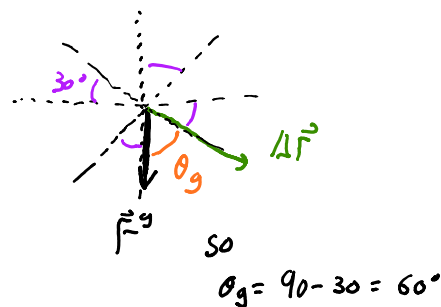
$$= mg |\Delta \vec{r}| \cos(60) - |\vec{F}^w| |\Delta \vec{r}|$$

$$= (10 \text{ kg})(9.8 \text{ m/s}^2)(3 \text{ m}) \cos(60) - (8 \text{ N})(3 \text{ m})$$

$$= 147 - 24$$

$$\boxed{\sum W = 123 \text{ Nm}}$$

Vec. OP.



**WE.2.L1-6:**

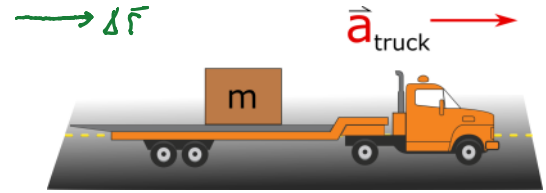
**Description:** Identify which forces produce positive work. (4 minutes)

**Learning Objectives:** [8]

**Problem Statement:** A truck, starting from rest, accelerates to the right with a box in the bed of the truck as shown in the image below. Which of the forces do positive work?



- ① friction from the road on the truck +
- ② friction from the truck on the box +
- ③ normal force from the truck on the box 0
- ④ friction from the box on the truck -
- ⑤ normal force from the box on the truck 0



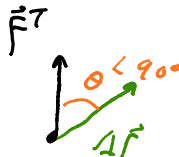
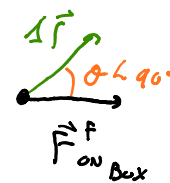
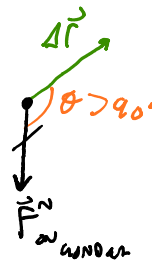
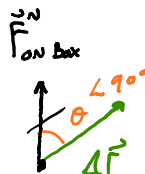
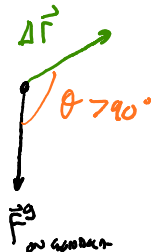
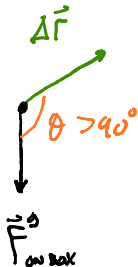
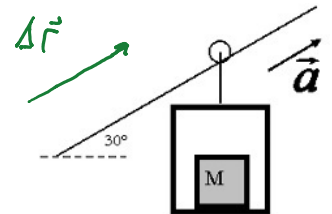
**WE.2.L1-7:**

**Description:** Identify which forces produce positive work. (4 minutes)

**Learning Objectives:** [8]

**Problem Statement:** A gondola, starting from rest, accelerates up and to the right as shown in the figure below. Which of the forces do positive work?

- (1) gravity on the box -
- (2) gravity on the gondola -
- Ⓐ normal force on the box +
- (4) normal force on the gondola -
- Ⓑ friction on the box +
- Ⓒ tension from cable on box +



**Act II: Kinetic Energy**

**WE.2.L1-8:**

**Description:** Identify vectors and scalars. (2 minutes)

**Learning Objectives:** [2]

**Problem Statement:** Which of the following quantities are vectors?

- ① Forces  $\checkmark$
- ② Displacement  $\checkmark$
- ③ Work  $\checkmark$
- ④ Kinetic energy  $\checkmark$
- ⑤ Potential energy  $\checkmark$

**WE.2.L1-9:**

**Description:** Proportional reasoning with kinetic energy. (4 minutes)

**Learning Objectives:** []

$$m_2 = 2m_1$$

**Problem Statement:** Two marbles, one twice as heavy as the other, are dropped to the ground from the roof of a building. Just before hitting the ground, the heavier marble has

- (1) as much kinetic energy as the lighter one
- ② twice as much kinetic energy as the lighter one
- (3) half as much kinetic energy as the lighter one
- (4) four times as much kinetic energy as the lighter one
- (5) impossible to determine without known masses and speeds

$$KE = \frac{1}{2} m v^2$$

$$\omega / v = \text{CONST}$$

$KE \propto m$

1st KINEMATICS

$$\omega / \vec{v}_1 = \vec{v}_2$$

$$\dagger \Delta \vec{r}_1 = \Delta \vec{r}_2$$

$$\dagger \vec{a}_1 = \vec{a}_2$$

THUS  $\vec{v}_{f1} = \vec{v}_{f2}$

**WE.2.L1-10:**

**Description:** Find kinetic energy given mass and velocity. (3 minutes)

**Learning Objectives:** [1]

**Problem Statement:** A 1 kg marble has a velocity of  $\langle -3, -4 \rangle$  m/s. What is its kinetic energy?

- (1)  $\langle 7.5, 10 \rangle$  J
- (2)  $\langle -7.5, -10 \rangle$  J
- (3) 12.5 J
- (4) -12.5 J
- (5) 17.5 J
- (6) -17.5 J

$$\begin{aligned} KE &= \frac{1}{2} m v^2 \quad \text{SPEED!} \\ &= \frac{1}{2} m |\vec{v}|^2 \\ &= \frac{1}{2} (1 \text{ kg}) \left( \sqrt{(-3)^2 + (-4)^2} \text{ m/s} \right)^2 \\ &= \frac{1}{2} (1 \text{ kg}) (5 \text{ m/s})^2 \\ &= 12.5 \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} \end{aligned}$$

**Act III: Work and Kinetic Energy**

**WE.2.L1-11:**

**Description:** Determine which forces produce positive and negative work. (4 minutes)

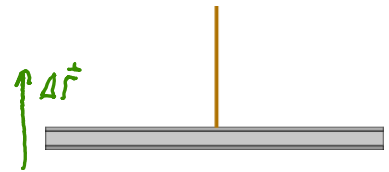
**Learning Objectives:** [8, 9]

**Problem Statement:** A crane raises a steel girder into place at a construction site. The girder moves with constant speed. Consider the work  $W^g$  done by gravity and the work  $W^T$  done by the tension in the cable. Which of the following is correct?

- f (1)  $W^g$  and  $W^T$  are both zero
- f (2)  $W^g$  is negative and  $W^T$  is negative
- † (3)  $W^g$  is negative and  $W^T$  is positive

↑  $\Delta \vec{r}$

- F (1)  $W^g$  and  $W^T$  are both zero
- f (2)  $W^g$  is negative and  $W^T$  is negative
- † (3)  $W^g$  is negative and  $W^T$  is positive
- f (4)  $W^g$  is positive and  $W^T$  is positive
- F (5)  $W^g$  is positive and  $W^T$  is negative
- f (6) Kinetic energy is increasing
- f (7) Kinetic energy is decreasing
- T (8) Kinetic energy remains constant



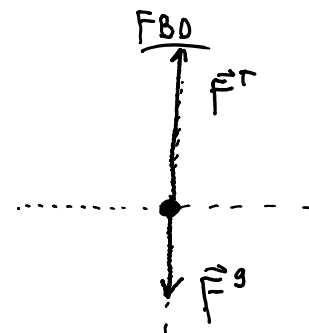
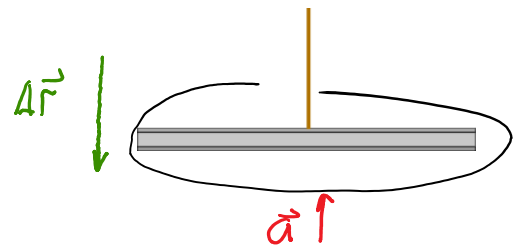
**WE.2.L1-12:**

**Description:** Determine which forces produce positive and negative work, rank magnitudes of work, and determine if kinetic energy is changing or constant. (6 minutes)

**Learning Objectives:** [8,9]

**Problem Statement:** A crane raises a steel girder into place at a construction site. The girder is moving downwards and slowing down. Which of the following statements about the girder are true?

- F (1) Kinetic energy is increasing
- T (2) Kinetic energy is decreasing
- F (3) Kinetic energy remains constant
- f (4) The work from the cable is positive
- T (5) The work from the cable is negative
- T (6) The work from gravity is positive
- F (7) The work from gravity is negative
- T (8) The magnitude of work from the cable is greater than that from gravity
- F (9) The magnitude of work from the cable is less than that from gravity
- f (10) The magnitude of work from the cable is equal to that from gravity



**WE.2.L1-13:**

**Description:** Determine sign of work given initial velocity, forces, and coefficients of friction. (4 minutes + 2 minutes)

**Learning Objectives:** [3, 5]

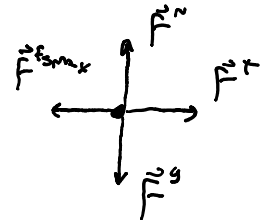
**Problem Statement:** A 2 kg block, initially at rest is being pulled horizontally by a 5N tension force. The coefficient of static and kinetic friction between the block and the surface is 0.5 and 0.2 respectively.

(a) Which of the following quantities are contributing to increase the kinetic energy?

- (1) work from gravity  $\emptyset$
- (2) work from the normal force  $\emptyset$
- (3) work from the tension  $\emptyset$
- (4) work from the friction  $\emptyset$

CHECK IF  $\vec{F}^T$  IS LARGE ENOUGH TO MOVE BLOCK

$$\begin{aligned} |\vec{F}^{f_{smax}}| &= \mu_s |\vec{F}^N| \\ &= \mu_s mg \\ &= (0.5)(2)(9.8) \\ &= 9.8 \text{ N} // \end{aligned}$$



So ... STILL STATIONARY  
 $\Delta \vec{K} = \vec{0}$

(b) Now sliding in the direction of tension, which of the following quantities are contributing to decreasing the kinetic energy.

- (1) work from gravity  $\emptyset$
- (2) work from the normal force  $\emptyset$
- (3) work from the tension  $+$
- (4) work from the friction  $-$



**WE.2.L1-14:**

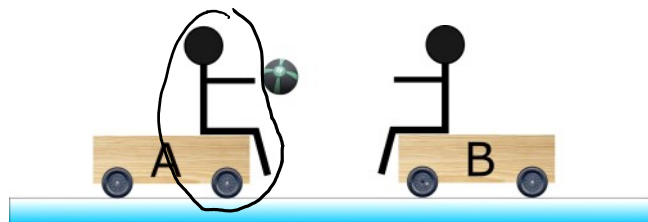
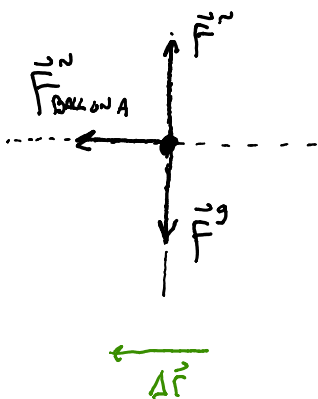
**Description:** Determine sign of work. (3 minutes + 3 minutes)

Learning Objectives: [8, 9]

**Problem Statement:** Two people are on low friction carts, throwing a medicine ball back and forth, speeding up the rate at which they move away from each other.

(a) What is the sign of the work of the ball on person A while they catch the ball?

- (1) positive
- (2) negative
- (3) zero



(b) What is the sign of the work of the ball on person A while they throw the ball?

- (1) positive
- (2) negative
- (3) zero

SAME FBD! .. OR...

$$\sum W_{Ext} = \Delta KE$$

$$\cancel{W^W} + \cancel{W^g} + W^{Ball} = \Delta KE$$

$$W^{Ball} = \Delta KE$$

(+)

**WE.2.L1-15:**

**Description:** Rank final kinetic energy given forces and displacement. (8 minutes)

Learning Objectives: [5, 15]

**Problem Statement:** The diagram depicts two pucks on a frictionless table. Puck 2 is four times as massive as puck 1. Starting from rest, the pucks are pushed across the table by two equal forces. The forces act on both of them all the way to the finish line. Which puck will have a greater kinetic energy upon reaching the finish line?

- (1) Puck 1
- (2) Puck 2

$$KF \cdot t \sin \theta = KE$$



- (1) Puck 1
- (2) Puck 2
- (3) Both will have the same
- (4) Not enough information to decide.

$$KE_i + \sum W_{ext} = KE_f$$

$$W^m + W^g + W^F = KE_f$$

$$W^m = W^g = 0 \text{ for both}$$

$$W^F = KE_f$$

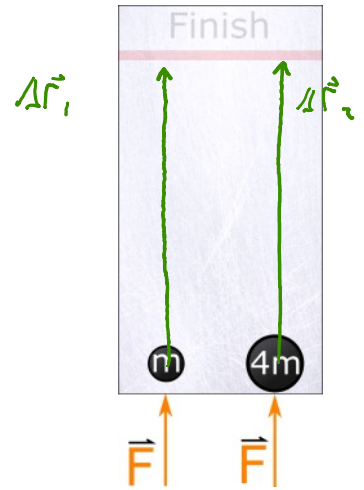
$$W / \Delta \vec{r}_i = \Delta \vec{r}_2$$

$$\uparrow \vec{F}_1 = \vec{F}_2$$

$$\uparrow \theta_1 = \theta_2$$

$$|\vec{F}| |\Delta \vec{r}| \cos \theta = KE_f$$

$$KE_{f1} = KE_{f2}$$



KE & P CONNECTION

$$KE = \frac{1}{2} m v^2 = \frac{p^2}{2m}$$

RECALL IMPULSE - MOMENTUM

$$\sum \vec{F}_{ext} \Delta t = \Delta \vec{p}$$

w/  $t_1 < t_2$

$$\Delta \vec{p}_1 < \Delta \vec{p}_2$$

**WE.2.L1-16:**

**Description:** Rank final kinetic energy given forces and time. (8 minutes)

**Learning Objectives:** [8, 15]

**Problem Statement:** The diagram depicts two pucks on a frictionless table. Puck 2 is four times as massive as puck 1. Starting from rest, the pucks are pushed across the table by two equal forces. The forces act on both of them for 6.0 seconds. Rank the final kinetic energy of the two pucks.

- (1)  $KE_{f1} > KE_{f2}$
- (2)  $KE_{f1} < KE_{f2}$
- (3)  $KE_{f1} = KE_{f2}$
- (4) Not enough information to decide.

$$KE_i + \sum W_{ext} = KE_f$$

$$W / \sum \vec{F}_i = \sum \vec{r}_2$$

AND  $m_1 < m_2$

THUS  $a_1 > a_2$

THUS w/  $\Delta t_1 = \Delta t_2$

$$\uparrow \vec{v}_{i1} = \vec{v}_{i2} = \vec{0}$$

$$\Delta \vec{r}_1 > \Delta \vec{r}_2$$

$$|\vec{F}| |\Delta \vec{r}| \cos \theta = KE_f$$

$$\theta_1 = \theta_2$$



RECALL IMPULSE - MOMENTUM

w/  $t_1 = t_2$

$v_1, v_2 \rightarrow$

$$\Delta \vec{r}_1 > \Delta \vec{r}_2$$

$$\text{So } KE_{F1} > KE_{F2}$$

RECALL IMPULSE-MOMENTUM

$$w/\Delta t_1 = \Delta t_2$$

$$\Delta \vec{p}_1 = \Delta \vec{p}_2$$

So ...

$$KE = \frac{p^2}{2m} \quad w/ \quad p = \text{CONST}$$

$$KE \propto \frac{1}{m} \quad w/ \quad m_1 < m_2$$

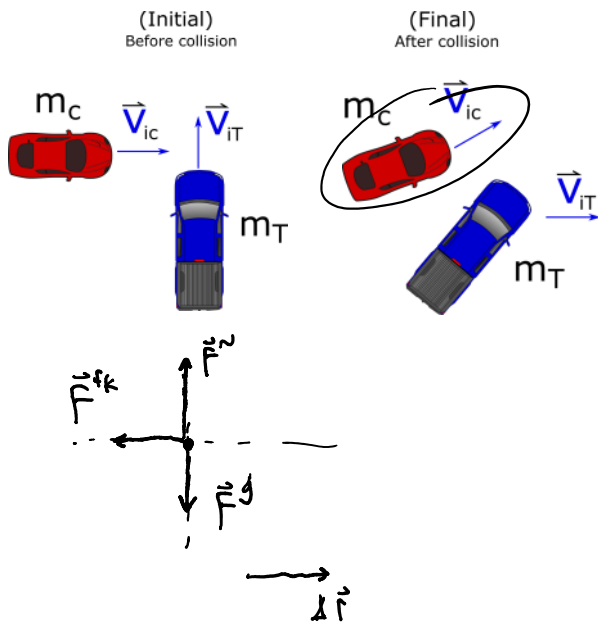
$$KE_1 > KE_2$$

**WE.2.L1-17:**

**Description:** Construct work - kinetic energy equation to find distance object slides to rest. (10 minutes)

**Learning Objectives:** [15]

**Problem Statement:** A car of mass 1300 kg traveling horizontally to the right as seen from above collides with a 2300 kg truck which was traveling 9.0 m/s vertically upwards as seen from above. The moment after the collision the truck is moving horizontally to the right at a speed of 7 m/s. The moment after the collision the car is traveling up and to the right at a speed of 17.57 m/s. The coefficient of kinetic friction between the road and the car's tires is 0.70. How far does the car slide before coming to a rest?



$$KE_{C_i} + \sum W_{Ext} = KE_{C_f}$$

$$\frac{1}{2} m_C v_{ic}^2 + W^{N^0} + W^{g^0} + W^F = 0$$

$$\frac{1}{2} m_C v_{ic}^2 + (\vec{F}^{fk} \parallel \Delta \vec{r}) \cos \theta = 0$$

$$\frac{1}{2} m_C v_{ic}^2 - \mu_k m_C g |\Delta \vec{r}| = 0$$

$$|\Delta \vec{r}| = \frac{v_{ic}^2}{2\mu_k g} = \frac{(17.57 \text{ m/s})^2}{2(0.7)(9.8)}$$

$$|\Delta \vec{r}| \approx 22.5 \text{ m}$$



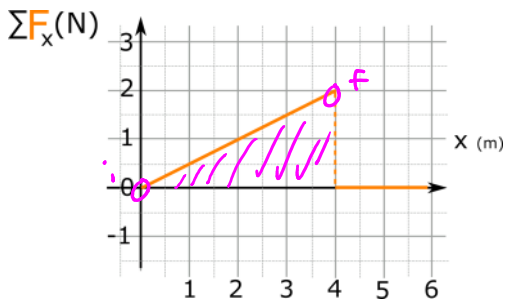
### Act IV: Graphical representation

#### WE.2.L1-18:

**Description:** Find final kinetic energy given a net force vs displacement graph. (4 minutes)

**Learning Objectives:** [6]

**Problem Statement:** Darin the Duck is moving in a straight line in a river. The net force on Darin is shown on the graph below. If Darin has 2.0 J of Kinetic energy as he passes  $x = 0$ , what is his kinetic energy when he reaches  $x = 4$  m?



$$\frac{1}{2}(4\text{m})(2\text{N})$$

$$4\text{J}$$

$$KE_i + \underbrace{\sum W_{\text{ext}}}_{\text{AREA}} = KE_f$$

$$2\text{J} + 4\text{J} = KE_f$$

$$KE_f = 6\text{J}$$

#### WE.2.L1-19:

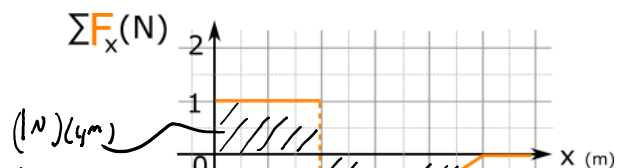
**Description:** Identify systems with negligible impulse. (4 minutes + 6 minutes)

**Learning Objectives:** [6, 15]

**Problem Statement:** Dianne the Duck is moving along a straight line in a river. The net force on Dianne is shown on the graph below.

(a) What is the total work done on Dianne?

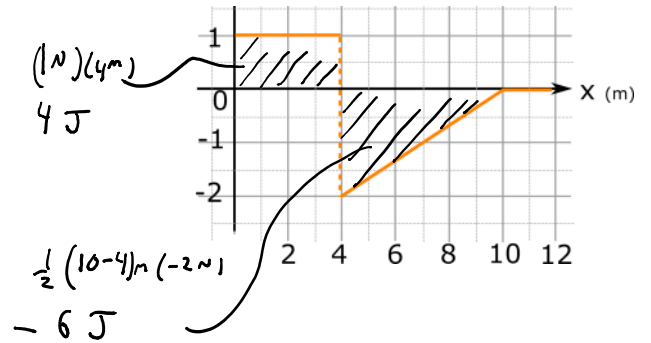
$$\overbrace{\sum W_{\text{ext}}} = 4\text{J} - 6\text{J}$$



HVZ

$$\sum W = 4 \text{ J} - 6 \text{ J}$$

$$\boxed{\sum W = -2 \text{ J}}$$



(b) If Dianne is 1.25 kg and initially started with a speed of 2.828 m/s when crossing  $x = 0$ , what is her speed as she reaches  $x = 10 \text{ m}$ ?

$$KE_i + \sum W_{\text{Ext}} = KE_f$$

$$\frac{1}{2} M V_i^2 + (-2 \text{ J}) = \frac{1}{2} M V_f^2$$

$$\frac{1}{2} (1.25 \text{ kg}) (2.828 \text{ m/s})^2 - 2 \text{ J} = \frac{1}{2} (1.25 \text{ kg}) V_f^2$$

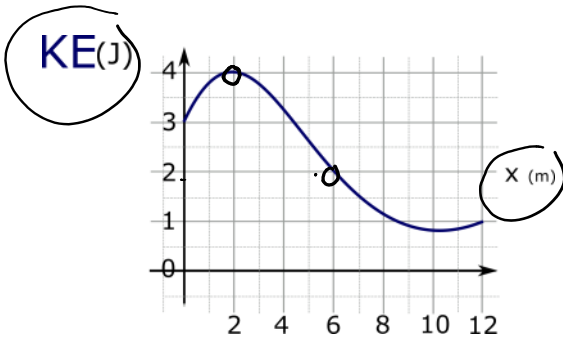
$$\boxed{V_f \approx 2.19 \text{ m/s}}$$

**WE.2.L1-20:**

**Description:** Find net work given a kinetic energy vs position graph. (3 minutes)

**Learning Objectives:** [3, 5]

**Problem Statement:** The kinetic energy of Pete the Pelican is shown on the graph below as he flies along a straight path. How much work is done on Pete between  $x = 2 \text{ m}$  and  $x = 6 \text{ m}$ ?



$\sim$  i or F? Don't know

$KE @ x=2 = 4 \text{ J}$

$KE @ x=6 = 2 \text{ J}$

$$\sum W_{\text{Ext}} = \Delta KE$$

$$\boxed{\sum W_{\text{Ext}} = \pm 2 \text{ J}}$$

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## Conceptual questions for discussion

1. Can a normal force on an object ever do work? If so, provide an example. If not, explain the reason why.
  2. Can static friction ever do work on an object? If so, provide an example. If not, explain the reason why.
  3. Consider pushing on a wall of a house. Are you doing work on the house? Support your answer with the work - kinetic energy theorem.
- 

## Hints

**WE.2.L1-1:** No hints.

**WE.2.L1-2:** No hints.

**WE.2.L1-3:** No hints.

**WE.2.L1-4:** No hints.

**WE.2.L1-5:** Draw a vector operation to find the angle that go into the dot product.

**WE.2.L1-6:** No hints.

**WE.2.L1-7:** No hints.

**WE.2.L1-8:** No hints.

**WE.2.L1-9:** Think kinematics first: if two balls of different mass are dropped from rest, how are their speeds, before they hit the ground, related?

**WE.2.L1-10:** We say "kinetic energy is one half  $m v^2$ ", but the  $v$  in kinetic energy is the speed.

**WE.2.L1-11:** No hints.

**WE.2.L1-12:** No hints.

**WE.2.L1-13:** No if you push on a stationary box, does it necessarily move? What is the maximum force you can apply to a stationary box just before it begins to slide?

**WE.2.L1-14:** No hints.

**WE.2.L1-15:** Start with work - kinetic energy equation. Look for quantities that are the same.

**WE.2.L1-16:** Can you use forces and kinematics to determine which puck travels further?

**WE.2.L1-17:** Construct a work - kinetic energy equation.

**WE.2.L1-18:** No hints.

**WE.2.L1-19:** What is the significance of the negative area? Can work be negative?

**WE.2.L1-20:** Look at the axis carefully.