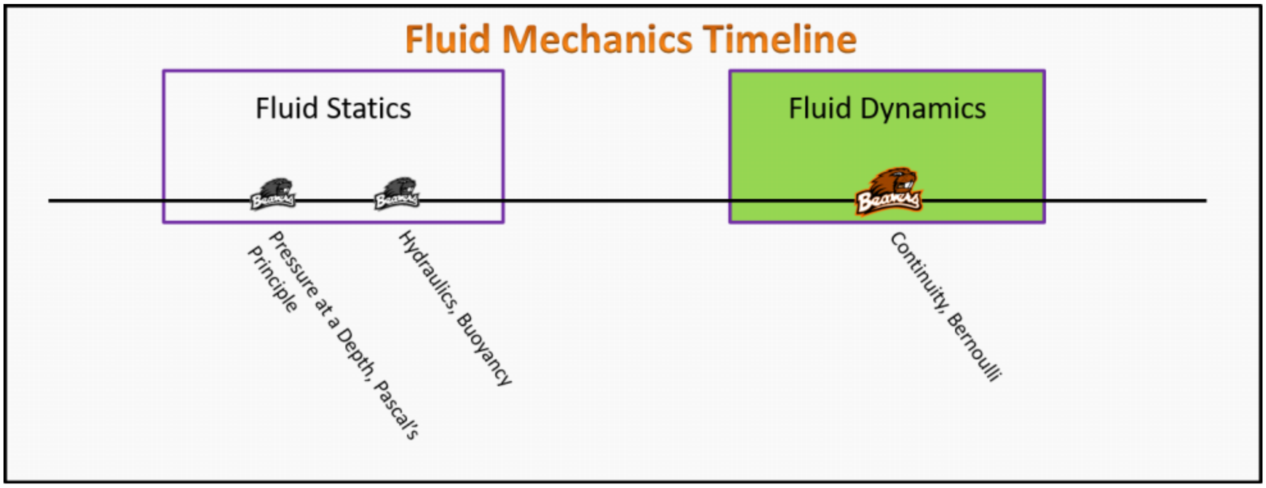
FD.2.L1 - Solutions

Friday, February 15, 2019 7:06 AM

Fluid Mechanics
Foundation Stage (FD.2.L1)

Lecture 1
Continuity, Bernoulli



Textbook Chapters (* Calculus version)

- **BoxSand** :: KC videos (Continuity ; Bernoulli's Principle)
- **Knight** (College Physics : A strategic approach 3rd) :: 13.5 ; 13.6
- ***Knight** (Physics for Scientists and Engineers 4th) :: 14.5
- **Giancoli** (Physics Principles with Applications 7th) :: 10-8 ; 10-9 ; 10-10

Warm up

FD.2.L1-1:

Description: Describe what pressure means in terms of forces and areas.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: We wish to describe energies and work for a fluid moving through an enclosed horizontal pipe.

(a) What is the translational kinetic energy of a certain mass of fluid ($\mathbf{m_f}$) moving through the pipe?

(1) $1/2\ \mathbf{m_f\, v^2}$
(2) $\mathbf{m_f\, v}$
(3) $1/2\ \mathbf{I_{f,cm}\, \omega^2}$
(4) $\mathbf{I_{f,cm}\, \omega}$

(b) What is the gravitational potential energy of a certain mass of fluid ($\mathbf{m_f}$) as it moves through the pipe?

(1) 0
(2) $1/2\ \mathbf{m_f\, g}$
(3) $\mathbf{m_f\, g}$
(4) $\mathbf{m_f\, g\, y}$

(c) Recall that work is $\mathbf{W} = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}}$. We know that there is a force ($\vec{\mathbf{F}}^{\Delta \mathbf{P}}$) associated with pressure differences; what is the work due to this pressure difference if the fluid is displaced horizontally to the right by some amount $\Delta\mathbf{x}$?

(1) **A P**
(2) **A ΔP**
(3) **A ΔP x**
(4) **A ΔP Δx**

$F^{\Delta P} = A\Delta P$

(d) Re-write the translational kinetic energy of a certain mass of fluid in terms of density ($\boldsymbol{\rho_f}$) and volume ($\mathbf{V}$).

$m = \rho V$

$\frac{1}{2}\rho V v^2$

(e) Re-write the gravitational potential energy of a certain mass of fluid in terms of density ($\boldsymbol{\rho_f}$) and volume ($\mathbf{V}$).

$\rho V g y$

(f) Re-write the work due to the pressure difference across the fluid in terms of volume ($\mathbf{V}$).

$A\Delta x = V$

$\Delta P V$

< 1

(g) Energy density is defined as energy/volume. What are the energy densities for the translational kinetic energy term, gravitational potential energy term, and work due to pressure difference term from the above parts of this problem?

$\frac{1}{2}\rho v^2$ $\rho g y$ ΔP

energy density conservation $\Rightarrow$ $\underbrace{P + \rho g y}_{\text{this much we have explored before!}} + \frac{1}{2}\rho v^2 = \text{const}$ in a fluid

with $P_1 = P_0 + \rho g d \Rightarrow P_1 + \rho g(-d) = P_0$

where height y = negative depth $(-d)$

Selected Learning Objectives

1. Coming soon to a lecture template near you.

Key Terms

- Archimedes Principle
- Buoyant force

Key Equations

$\dot{m}_{in} = \dot{m}_{out}$	$\mathbf{Q = v\,A}$	$v_1 A_1 = v_2 A_2 = v_3 A_3 = \cdots$

$$\mathbf{P_1 + \frac{1}{2}\rho_f v_1^2 + \rho_f\, g\, y_1 = P_2 + \frac{1}{2}\rho_f v_2^2 + \rho_f\, g\, y_2 = \cdots}$$

Key Concepts

- Coming soon to a lecture template near you.

Questions

Act I: Continuity

FD.2.L1-2:

Description: Identify which PV diagram represents an isochoric process. (2 minutes + 2 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Did someone say dimensions?

(a) What are the dimensions of volume flow rate?

(1) mass per time
(2) volume * velocity per time
(3) volume * time
(4) $\boxed{\text{volume per time}}$

(b) What are the dimensions of **Q** = **v A** ? flux!

(1) $\boxed{\frac{[L]^3}{[T]}}$ = Vol/time!

(2) $\frac{[L]}{[T]}$

(3) $\frac{[M]}{[T]}$

(4) $\frac{[L]}{[T]^2}$

$\left(\frac{m}{s}\right)(m^2)$

$\frac{m^3}{s}$ or $\frac{[L]^3}{[T]}$

FD.2.L1-3:

Description: Proportional reasoning with ideal gas law. (3 minutes)

Learning Objectives: [1, 12, 13]

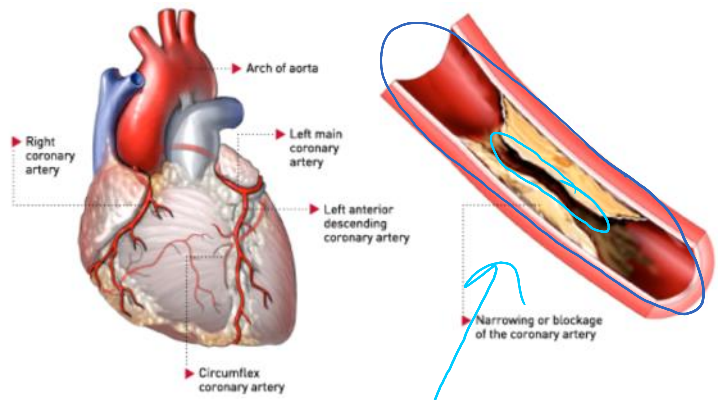
Problem Statement: Blood flows through a coronary artery that is partially blocked by deposits along the artery wall.

(a) Circle the part of the artery where the flux (volume of blood per unit time) is the largest at any given snapshot in time.

Same volume of fluid enters as leaves (otherwise bad consequences!)

$$Av = \frac{Vol}{time} = const$$

$\Rightarrow$ every where has same vol. flow rate (flux)



(b) Circle the part of the artery where the velocity of the blood is the largest.

$Av = const \Rightarrow$ if $A\downarrow$ $v\uparrow$

FD.2.L1-4:

Description: Determine signs of first law quantities. (5 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Consider a faucet with a diameter of 9.525 mm, and the speed of the water exiting the faucet is about 1.4 m/s.

(a) What is the volume flow rate, in SI units, at the exit location of the faucet?

$$Av = \pi \left(\frac{d}{2}\right)^2 v = 9.98\times10^{-5}\ m^3/s$$

(b) What is the volume flow rate of the water in the pipes under the sink which have a diameter of 12.7 mm?

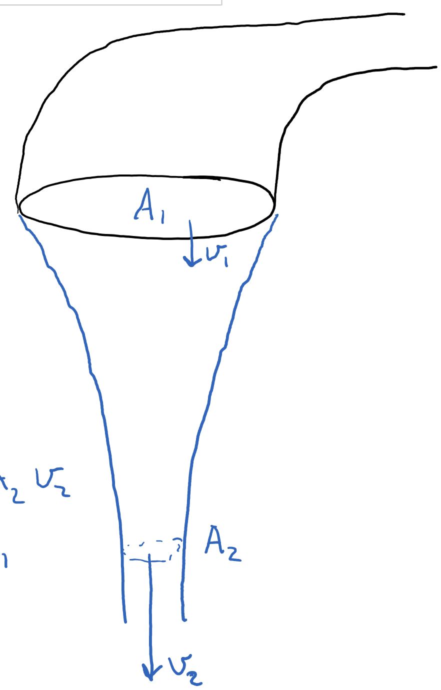
$$Av = \text{const} \Rightarrow 9.98 \times 10^{-5}\ m^3/s$$

FD.2.L1-5:

Description: Identify which diagram represents an isothermal process. (2 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Why does the diameter of water stream decrease as the water falls from the faucet?.



$$mgh \rightarrow \frac{1}{2}mv^2$$

$$\Rightarrow v_{water} \text{ is increasing} \Rightarrow A_1 v_1 = A_2 v_2$$

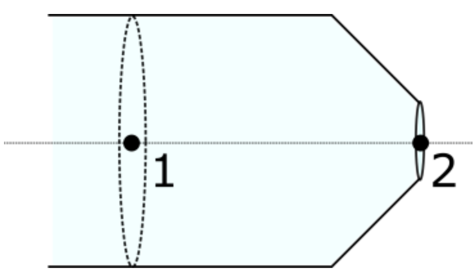
$$\Rightarrow A_2 < A_1$$

FD.2.L1-6:

Description: Proportional reasoning with ideal gas law. (2 minutes + 4 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Your little brother is standing at just the right distance away from you that the water in a garden hose does not reach him. Luckily you have taken physics, so you cover the end of the 8.00 mm radius hose such that you effectively create a 4.00 mm radius circular opening at the end. The water was coming out of the unobstructed hose at 3 m/s. What is the speed of the water coming out of the hose when you cover it with your thumb?



$$A_1 v_1 = A_2 v_2$$

$$\pi r_1^2 v_1 = \pi r_2^2 v_2$$

$$\Rightarrow v_2 = \frac{v_1 r_1^2}{r_2^2} = (3\ m/s)\frac{(0.008)^2}{(0.004)^2}$$

$$\boxed{v_2 = 12\ m/s}$$

Act II: Bernoulli's Principle

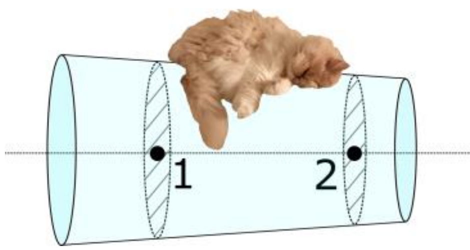
FD.2.L1-7:

Description: Determine signs of first law quantities. (5 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Water flows through a horizontal pipe as shown in the figure. What can be said about the pressure at points **1** and **2**?

(1) $P_1 > P_2$
(2) $P_1 < P_2$
(3) $P_1 = P_2$
(4) None of the above.



$y_1 = y_2$

$$P_1 + \cancel{\rho g y_1} + \frac{1}{2}\rho v_1^2 = \underbrace{P_2}_{\text{smaller than } P_1} + \cancel{\rho g y_2} + \underbrace{\frac{1}{2}\rho v_2^2}_{\text{bigger}}$$

$A_1 v_1 = A_2 v_2$

$A_2 \downarrow \Rightarrow v_2 \uparrow$

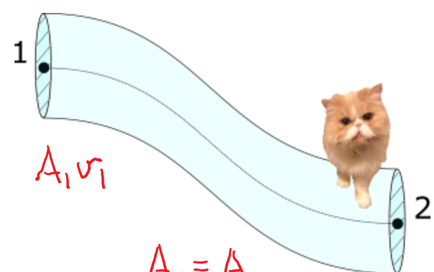
FD.2.L1-8:

Description: Identify which statements best represents an adiabatic curve on PV diagram (3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Water flows through a constant diameter pipe as shown in the figure below. What can be said about the pressures at locations **1** and **2**?

(1) $P_1 > P_2$
(2) $P_1 < P_2$
(3) $P_1 = P_2$
(4) P_1 could be less then or greater than P_2 depending on **h**.



$A_1 v_1$

$A_1 = A_2$

$\Rightarrow v_1 = v_2$

$A_2 v_2$

$$\underbrace{P_1}_{\text{smaller}} + \underbrace{\rho g y_1}_{\text{bigger}} + \cancel{\frac{1}{2}\rho v_1^2} = \underbrace{P_2}_{\text{bigger}} + \underbrace{\rho g y_2}_{\text{smaller}} + \cancel{\frac{1}{2}\rho v_2^2}$$

$$y_1 > y_2 \Rightarrow P_2 > P_1$$

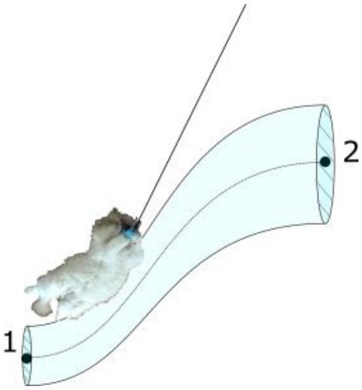
FD.2.L1-9:

Description: Identify proportionality for adiabatic process. (3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Water flows from location **1** to **2** in a pipe system shown below. The cross-sectional area at **1** is less than at **2** and location **2** is higher in elevation than at **1**. Which of the following statements are *plausible*? The drawing is not to scale.

(1) The speed of the water at **1** is less than at **2**.
(2) The speed of the water at **1** is equal to that at **2**.
(3) The speed of the water at **1** is greater than at **2**.
(4) The pressure at **1** is less than that at **2**.
(5) The pressure at **1** is equal to that at **2**.
(6) The pressure at **1** is greater than that at **2**.



$$P_1 + \underbrace{\rho g y_1}_{\text{smaller}} + \underbrace{\tfrac{1}{2}\rho v_1^2}_{\text{bigger}} = P_2 + \underbrace{\rho g y_2}_{\text{bigger}} + \underbrace{\tfrac{1}{2}\rho v_2^2}_{\text{smaller}}$$

$y_2 > y_1$

Competing effects

$\Rightarrow P_1 \gtrless P_2$ depends on strength of these effects!

$A_1 v_1 = A_2 v_2$

$A_1 < A_2$

$\Rightarrow v_1 > v_2$

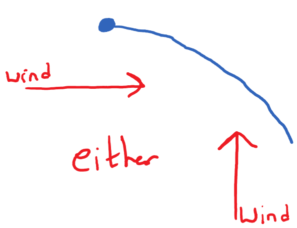
FD.2.L1-10:

Description: Determine signs of first law quantities. (5 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Consider holding a normal piece of paper horizontally in front of your mouth. Which of the following actions would result in the paper moving upwards?

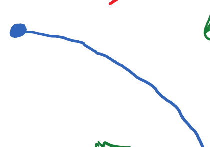
(1) Blow air across the top.
(2) Blow air across the bottom.
(3) Blow air upwards from the bottom.
(4) Blow air downwards from the top.



$\Rightarrow F_{up}^{N}$ or $F_{up}^{\Delta \vec{p}}$

momentum exchange!

Wind



$P_{above} + \rho g y + \underbrace{\tfrac{1}{2}\rho v^2}_{v \neq 0}$

same as

same as

$P_{below} + \cancel{\rho g y} + \cancel{\tfrac{1}{2}\rho v^2}^{\,0}$

these must be = b/c energy density is conserved.

$\Rightarrow P_{above} < P_{below}$

$\Rightarrow F = A\Delta P$!

FD.2.L1-11:

Description: Sketch an isobaric process. (2 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Consider a device as shown below where an air pump is used to bring air in from the left to the right. The left most tube is far enough away from the other tubes that the air around it does not get disturbed by the air pump.

(a) Rank the pressures across the top of each tube.

$P + \rho g y + \frac{1}{2}\rho v^2$

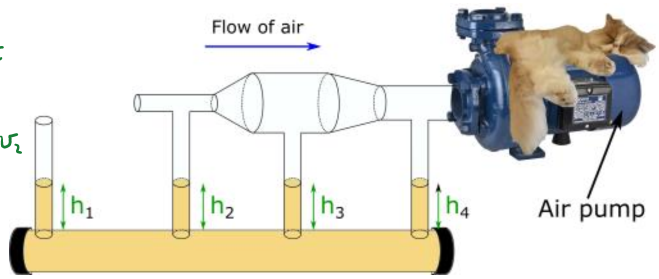
$A v = \text{const}$

$\Rightarrow A\uparrow \ v\downarrow$

$\Rightarrow v_1 < v_3 < v_4 < v_2$

$v\uparrow \Rightarrow P\downarrow$

$\Rightarrow \boxed{P_2 < P_4 < P_3 < P_1}$



(b) Rank the heights that the liquid rises to in each tube.

$F = A\Delta P = A(\underbrace{P_{bot}}_{\text{same for each}} - P_{top})$

$\Rightarrow P_{top}\downarrow \Rightarrow F\uparrow \Rightarrow h\uparrow$

$\Rightarrow \boxed{h_1 < h_3 < h_4 < h_2}$

FD.2.L1-12:

Description: Proportional reasoning with ideal gas law. (4 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Water flows steadily from an open tank near the surface of the earth filling a bow which Bernoulli is drinking from. The elevation of location **1** is 10.0 meters, and the elevation of points **2** and **3** is 2.00 m. The cross-sectional area at location **2** is 4.80×10^{-2} m^2 ; at location **3** where the water is discharged it is 1.60×10^{-2} m^2. The cross-sectional area of the tank is very large compared with the cross-sectional area of the pipes at 50.3 m^2.

(a) What is approximately true about the pressure and velocity at each location?

$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2$ ($v_1 \sim 0$)

$\parallel$

$P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

$\parallel$

$P_3 + \rho g y_3 + \frac{1}{2}\rho v_3^2$

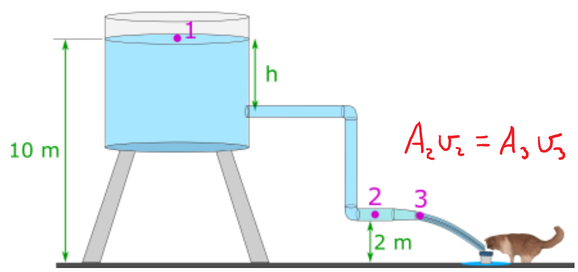
same

same

P₁ ≈ atm **v₁** ≈ 0

P₂ ≈ > atm **v₂** ≈ < v_3

P₃ ≈ atm **v₃** ≈ > v_2



$A_2 v_2 = A_3 v_3$

(b) Determine the flow rate.

$\rho g y_1 = \rho g y_3 + \frac{1}{2}\rho v_3^2$

$\rho g(y_1 - y_3) = \frac{1}{2}\rho v_3^2$

$\Rightarrow v_3 = \sqrt{2g(y_1 - y_3)}$

$Q = A_3 v_3 = A_3\sqrt{2g(y_1 - y_3)}$

$= \boxed{0.2 \text{ m}^3/\text{s}}$

(c) How long would it take to fill a 1 liter bottle?

$1 \text{ liter} = \frac{1}{1000} \text{ m}^3$

$\boxed{6 \times 10^{-4} \text{ sec}}$

(d) How long would it take to drain the tank if it's volume is roughly 302,000 liters? $= 302 \text{ m}^3$

(1) Less than 1510 s

(2) More than 1510 s

(3) Equal to 1510 s

Flow rate depends on y_1!

$\frac{302 \text{ m}^3}{0.2 \text{ m}^3/\text{s}} = 1510 \text{ s}$ if flow rate were const.

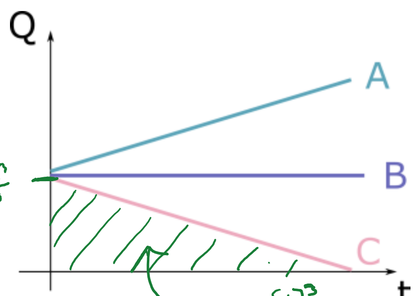
(e) Below are three graphs representing the flow rate as a function of time. Which graph do you think best represents the real flow rate?

$$\boxed{6\times10^{-4}\ \text{sec}}$$

(3) Equal to 1510 s

$\frac{302\ m^3}{0.2\ m^3/s} = 1510\ s$ if flow rate were const.

(e) Below are three graphs representing the flow rate as a function of time. Which graph do you think best represents the real flow rate?



$0.2\frac{m^3}{s}$

Area $= \frac{[L]^3}{[T]}[T] =$ Volume!

C

flow rate decreases with decreasing y_1

(f) How long would it take to drain the tank if it's volume is roughly 302,000 liters?

$= 302\ m^3$

$$\frac{1}{2}\left(0.2\frac{m^3}{s}\right)(t_f) = 302\ m^3 \qquad t_f = \frac{604\ m^3}{0.2\ m^3/s} = 3020\ \text{sec}$$

(g) Water flows steadily from an open tank near the surface of the earth filling a bow which Bernoulli is drinking from. The elevation of location **1** is 10.0 meters, and the elevation of points **2** and **3** is 2.00 m. The cross-sectional area at location **2** is 4.80 x 10^{-2} m^2 ; at location **3** where the water is discharged it is 1.60 x 10^{-2} m^2. The cross-sectional area of the tank is very large compared with the cross-sectional area of the pipes at 50.3 m^2. **What is the pressure at location 2?**

same as

$$P_3 + \cancel{\rho g y_3} + \frac{1}{2}\rho v_3^2 = P_2 + \cancel{\rho g y_2} + \frac{1}{2}\rho v_2^2$$

$$P_{atm} + \frac{1}{2}\rho v_3^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_2 = P_{atm} + \frac{1}{2}\rho(v_3^2 - v_2^2) = \boxed{171{,}000\ Pa}$$

$$A_2 v_2 = A_3 v_3$$

$$v_2 = \frac{A_3}{A_2} v_3 = \frac{A_3}{A_2}\sqrt{2g(y_1 - y_3)}$$

$$= \left(\frac{1.6}{4.8}\right)(12.5\ m/s)$$

$$v_2 = 4.18\ m/s$$

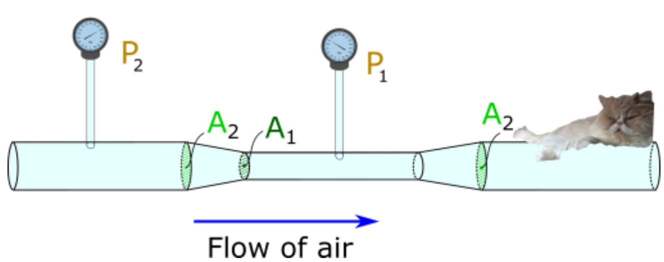
FD.2.L1-13:

Description: Determine signs of first law quantities. (5 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A Venturi meter is a device for measuring the speed of fluid within a pipe. The drawing shows a gas flowing at a speed of $\mathbf{v_2}$ through a horizontal section of pipe whose cross-sectional area is $\mathbf{A_2}$ = 0.0700 m^2. The gas has a density of 1.30 kg/m^3. The Venturi meter has a cross-sectional area in the center of $\mathbf{A_1}$ = 0.050 m^2. The pressure difference between the two sections is $\mathbf{P_2}$ - $\mathbf{P_1}$ = 120 Pa.

(a) Find the speed $\mathbf{v_2}$ of the gas in the larger pipe.



$$P_1 + \cancel{\rho g y_1} + \frac{1}{2}\rho v_1^2 = P_2 + \cancel{\rho g y_2} + \frac{1}{2}\rho v_2^2$$

$$P_2 - P_1 = \frac{1}{2}\rho(v_1^2 - v_2^2) = 120\ Pa$$

$$A_1 v_1 = A_2 v_2$$

$$v_1 = \frac{A_2}{A_1} v_2$$

$$\Rightarrow \frac{A_2^2}{A_1^2} v_2^2 - v_2^2 = \frac{240}{\rho}$$

$$v_2^2 = \frac{1}{\left(\frac{A_2^2}{A_1^2} - 1\right)}\frac{240}{\rho} = 192\frac{m^2}{s^2} \Rightarrow \boxed{v_2 = 13.9\ m/s}$$

(b) Find the volume flow rate of the gas.

$Q = A_2 v_2 = 0.97 \, m^3/s$

FD.2.L1-14:

Description: Determine signs of first law quantities. (5 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Planes and planes and planes.

(a) How does an airfoil, like the wings of a plane, fly?

(1) Bernoulli's principle. $\Rightarrow v\uparrow \; P\downarrow$

(2) Momentum transfer.

(3) Continuity. $\Rightarrow v_{top} > v_{bot}$

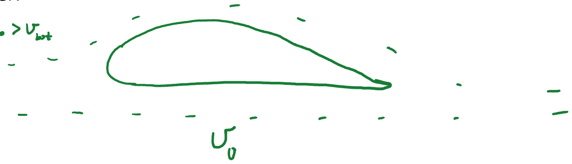
(4) Magic.

(5) Friction.

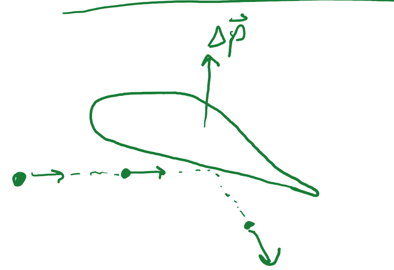
(6) Coffee.

(7) Money.

$v_- > v_0 \Rightarrow P < P_0 \Rightarrow F^{\Delta P} = A\Delta P$



Momentum transfer



(b) Of the options provided above, three are the main ideas that can be used to describe where lift comes from. Discuss with your neighbors which three of those are and how they contribute to lift.

Conceptual questions for discussion

1. Coming soon.

Hints

FD.2.L1-1: No hints.

FD.2.L1-2: No hints.

FD.2.L1-3: No hints.

FD.2.L11-4: No hints.

FD.2.L1-5: No hints.

FD.2.L1-6: No hints.

FD.2.L1-7: No hints.

FD.2.L1-8: No hints.

FD.2.L1-9: No hints.

FD.2.L1-10: No hints.

FD.2.L1-11: No hints.

FD.2.L1-12: No hints.

FD.2.L1-13: No hints.

FD.2.L1-14: No hints.