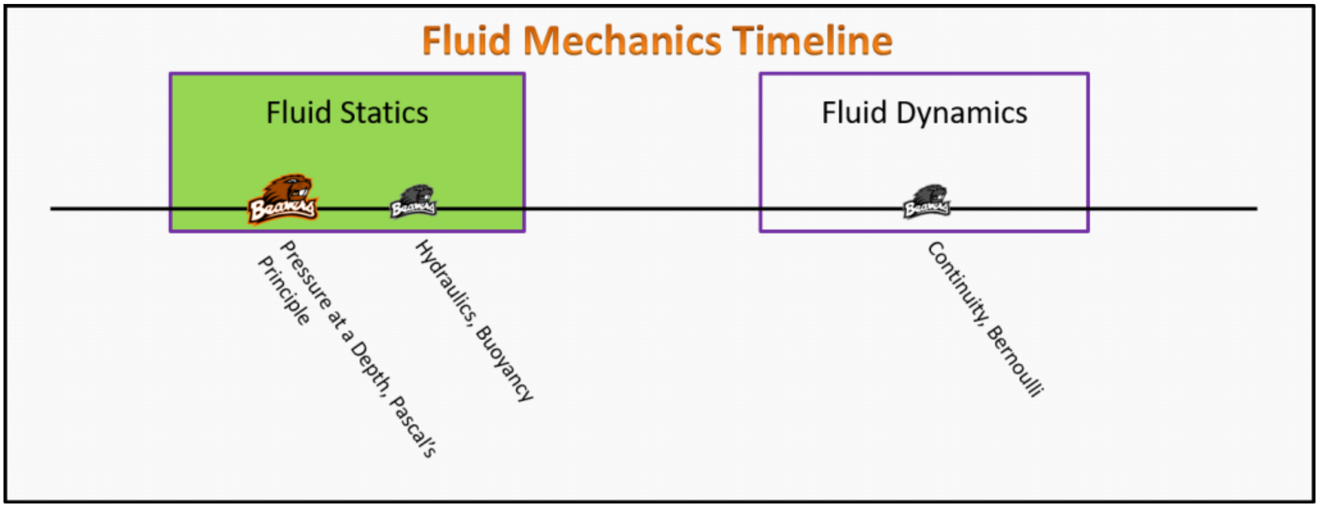
FS.2.L1 - Solutions

Monday, February 11, 2019 7:16 AM

Fluid Mechanics
Foundation Stage (FS.2.L1)

Lecture 1
Pressure at a Depth, Pascal's Principle



Textbook Chapters (* Calculus version)

- **BoxSand** :: KC videos (Density , Pressure , Hydrostatics)
- **Knight** (College Physics : A strategic approach 3rd) :: 13.1 ; 13.2 ; 13.3
- ***Knight** (Physics for Scientists and Engineers 4th) :: 14.1 ; 14.2 ; 14.3
- **Giancoli** (Physics Principles with Applications 7th) :: 10-1 ; 10-2 ; 10-3 ; 10-4 ; 10-5 ; 10-6

Warm up

FS.2.L1-1:

Description: Describe what pressure means in terms of forces and areas.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: Suppose you measure a 10 N force exerted by an object resting on a table. You then divide this force by the contact area between the object and the table; you measured 2 m^2. The calculated number 10N/2m^2 is called pressure, but how do you interpret the number 10/2? Basically what does this number mean to you, or what does the number tell you?

Selected Learning Objectives

1. Coming soon to a lecture template near you.

Key Terms

- Mass density
- Pressure
- Atmospheric pressure
- Gauge pressure
- Absolute pressure (a.k.a true pressure, or actual pressure)
- Hydrostatic pressure (a.k.a pressure at a depth)
- Force associated with pressure difference
- Pascal's principle

Key Equations

$\mathbf{P_1 = P_o + \rho_f\, g\, d}$	$\mathbf{P_{abs} = P_{atm} + P_g}$

Key Concepts

- Coming soon to a lecture template near you.

Questions

Act I: Definition of Pressure

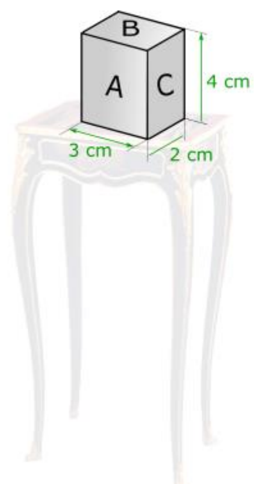
FS.2.L1-2:

Description: Identify which PV diagram represents an isochoric process. (2 minutes + 2 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A rectangular block is at rest on a table. Three faces of the block are labeled **A**, **B**, and **C**. The dimensions of each face are shown in the table below.

Face	Dimensions (L x W)
A	3 cm x 4 cm
B	2 cm x 3 cm
C	2 cm x 4 cm



(a) Rank the pressure exerted by the block on the table when it is resting on each labeled side.

F = same for all $\quad A_B < A_C < A_A$

$P = \frac{F}{A} \Rightarrow \boxed{P_A < P_C < P_B}$

(b) Calculate the largest pressure (in Pascals) of the block on the table if the block has a density of 8,050 kg/m³.

Block Volume $= (0.03)(0.04)(0.02) = 2.4\times10^{-5}\ m^3$

$F = \rho V g = \rho(L\times W\times H)g$

$P = F/A = \frac{F}{(L\times W)} = g\rho\frac{L\times W\times H}{L\times W} = g\rho H = \left(8050\ \frac{kg}{m^3}\right)(0.04\ m)\left(9.81\ \frac{m}{s^2}\right)$

$$P = F/A = \frac{F}{(L \times W)} = g\rho \frac{L \times W \times H}{L \times W} = g\rho H = \left(8050 \frac{kg}{m^3}\right)(0.04\,m)\left(9.81 \frac{m}{s^2}\right)$$

$$\boxed{P_{max} = 3{,}160\ Pa}$$

FS.2.L1-3:

Description: Proportional reasoning with ideal gas law. (3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: As a lab coordinator, you receive a report from the engineers building your lab that they will have -10 PSI at one of the stations.

(a) Is this the actual pressure at the station? Support your answer with an explanation.

(1) Yes
(2) No
(3) Unable to determine with given information.

negative absolute pressure is not possible. This must be a gauge pressure.

(b) What is the actual pressure at the station in Pascals? 1 PSI = 6895 Pa

$$P_{abs} = P_{atm} + P_{gauge}$$

$$P_{abs} = 101{,}325\ Pa + (-10\ Psi)\left(\frac{6895\ Pa}{1\ Psi}\right)$$

$$P_{abs} = 32{,}400\ Pa$$

Act II: Pressure Gradients and Differences

FS.2.L1-4:

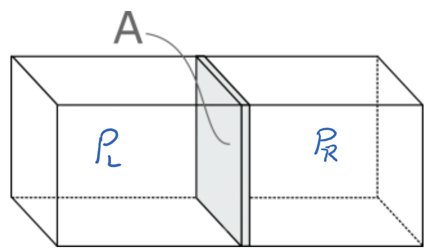
Description: Determine signs of first law quantities. (5 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Below shows a chamber with a piston, which is free to move and has a cross-sectional area **A**, in the center that separates the container into two sides.

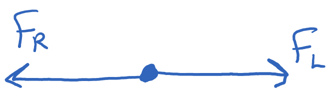
(a) Which of the following actions will move the piston to the right?

(1) Increase the pressure on the left.
(2) Increase the pressure on the right.
(3) Decrease the pressure on the left.
(4) Decrease the pressure on the right.



(b) Which of the following equations describes the force acting on the piston?

(1) **A·P**
(2) **A/P**
(3) **A·ΔP**
(4) **A/ΔP**



$F_R = P_R A_{pst}$ $F_L = P_L A_{pist}$

$$F_{net} = F_L - F_R = A_{pist}(P_L - P_R) = A\,\Delta P$$

FS.2.L1-5:

Description: Identify which diagram represents an isothermal process. (2 minutes)

Learning Objectives: [1, 12, 13]

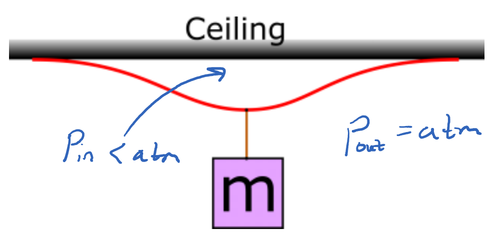
Problem Statement: A 10.0-cm-diameter suction cup is pushed against a smooth ceiling at a location near sea level. The mass of the suction cup is negligible compared to the mass hanging from it. Recall that P_{atm} = 101.3 kPa

(a) What is the pressure right outside of the suction cup?

(1) P_{atm}
(2) Less than P_{atm}
(3) Greater than P_{atm}

(b) What is the pressure inside the suction cup?

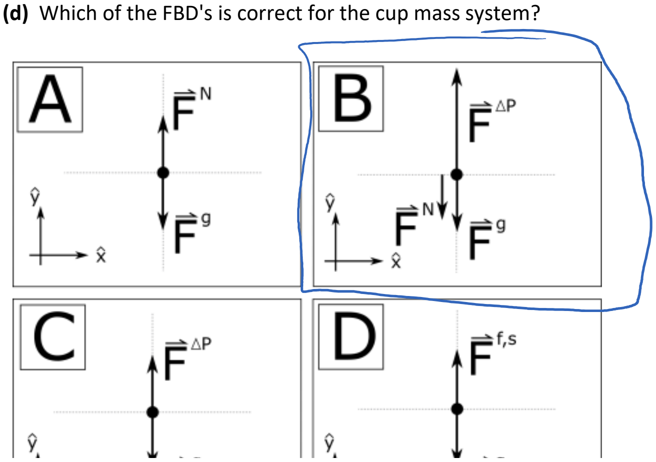
(1) P_{atm}
(2) Less than P_{atm}
(3) Greater than P_{atm}



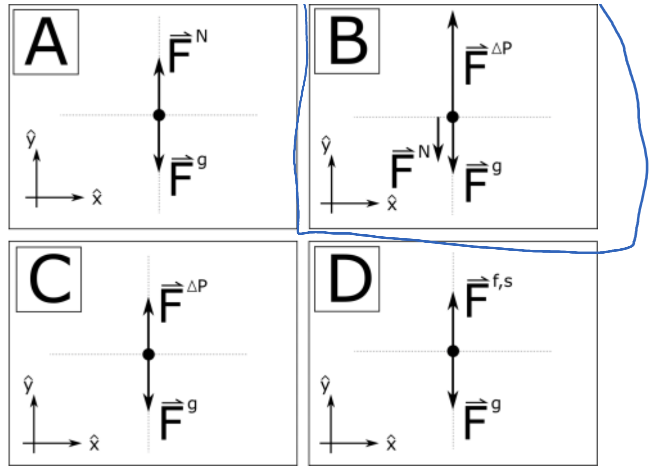
(c) What force is holding the mass-cup system up?

(1) Normal force
(2) Friction force
(3) Force from pressure difference
(4) Gravity

(d) Which of the FBD's is correct for the cup mass system?



(1) Normal force
(2) Friction force
(3) Force from pressure difference
(4) Gravity



(e) Which of the following expressions could be used to determine the force from the pressure difference?

(1) **A·P**
(2) **A/P**
(3) **A·ΔP**
(4) **A/ΔP**

$F = A(P_{out} - P_{in})$

(f) At the critical point where the cup is holding the largest mass possible, which of the following forces go to zero?

(1) Normal force
(2) Friction force
(3) Force from pressure difference
(4) Gravity

(g) A 10.0-cm-diameter suction cup is pushed against a smooth ceiling at a location near sea level. The mass of the suction cup is negligible compared to the mass hanging from it. Recall that $\mathbf{P_{atm}}$ = 101.3 kPa **Calculate the maximum mass the suction cup can hold before it peels off the ceiling.**

at max $F^N \to 0 \Rightarrow |F^g| = |F^{\Delta P}|$

$mg = A(P_{out} - P_{in})$, $P_{in} \to 0$

$mg = A\,P_{atm}$

$m_{max} = \frac{\pi r^2 P_{atm}}{g} = 81\ kg$ = very approximately one human

FS.2.L1-6:

Description: Proportional reasoning with ideal gas law. (2 minutes + 4 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Three students are discussing how human lungs given the simplified models of inhaling and exhaling below. Which student do you agree with the most?

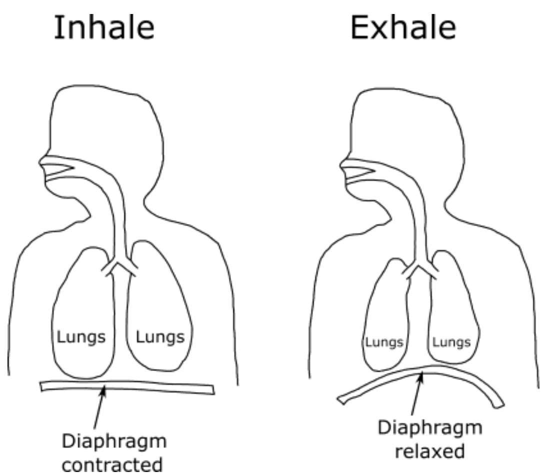
(1) It looks like the diagram contracts when we inhale, thus creating a larger volume in the chest cavity where the lungs are. If we model air as an ideal gas at roughly a constant temperature, then as the volume of the lungs increases the pressure also increases which sucks air into the lungs.

$PV = nRT$ (const) $\Rightarrow V\uparrow P\downarrow$

(2) I agree that as the diaphragm contracts the volume of the chest cavity and thus lungs increases. But if we model the air as an ideal gas, then this increase in volume would cause a decrease in pressure. Since the pressure outside out mouth is roughly atmospheric, and the pressure in our lungs ~~tension~~ decreases, air gets pushed into our lungs because of the pressure difference.

(3) I think we need to look at both the inhale and exhale pictures. During the inhale, the diaphragm pulls air down with it as it contracts, then on the exhale the diaphragm pushes the air up and back out of the mouth.

ok, too!



FS.2.L1-7:

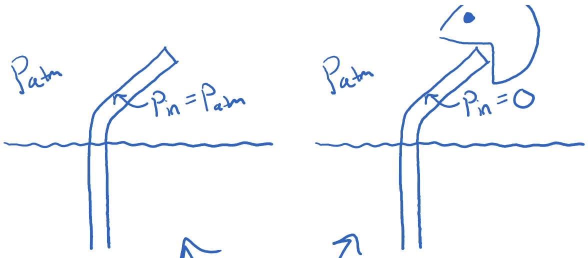
Description: Determine signs of first law quantities. (5 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Is it easier or harder to suck through a straw on the moon?

(1) Easier
(2) Harder
(3) Same effort
(4) Impossible

$P_{atm}^{moon} = 0$



On moon, no difference between these two!

on earth, Patm & low pressure inside straw/mouth give pressure difference that causes a force which lifts the water

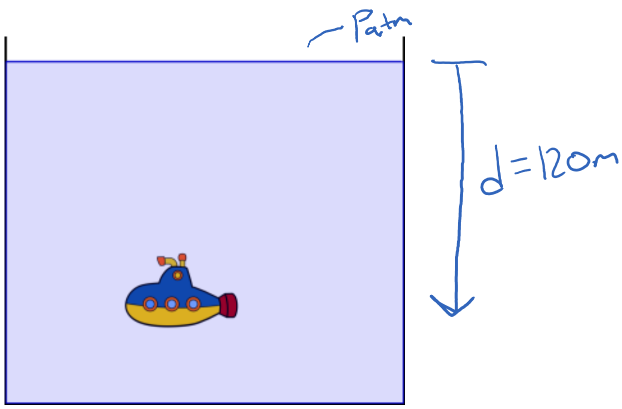
Act III: Hydrostatic Pressure at a Depth

FS.2.L1-8:

Description: Identify which statements best represents an adiabatic curve on PV diagram (3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: When a submarine dives down to a depth of 120 m, to how large a total pressure is the exterior of the submarine subjected to? The density of seawater is 1030 kg/m^3.



$$P = P_0 + \rho g d$$

$$= 101{,}325\ \text{Pa} + (1030\ \tfrac{\text{kg}}{\text{m}^3})(9.81\ \tfrac{\text{m}}{\text{s}^2})(120\ \text{m})$$

$$P = 1.31 \times 10^6\ \text{Pa}$$

$$= 12 \times P_{atm}!$$

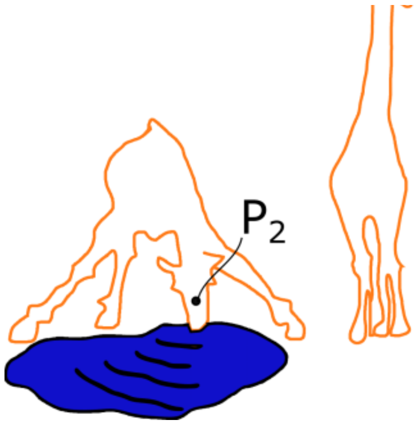
FS.2.L1-9:

Description: Identify proportionality for adiabatic process. (3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Calculate the difference in pressure that the blood vessels in a giraffe's head have to accommodate as the head is lowered from a full upright position to the ground level for a drink. The height of an average giraffe is about 6 m. The density of blood is about 1050 kg/m3.

$$P_2 = P_1 + \rho g d$$



$$P_2 = P_1 + \rho g d$$

$$P_2 - P_1 = \rho g d = 61{,}800 \text{ Pa}$$

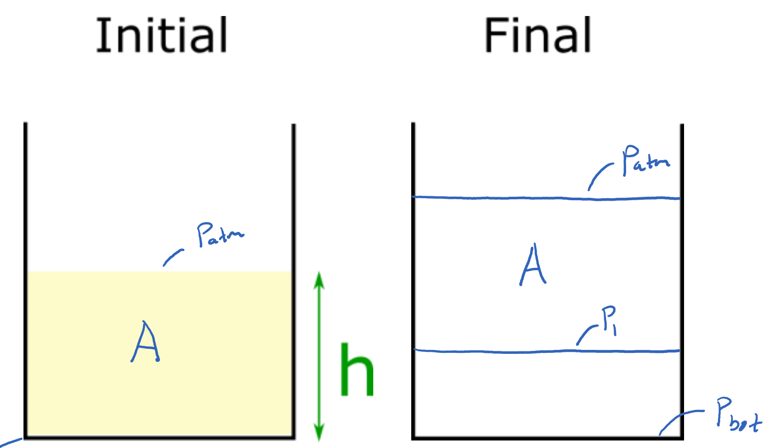
FS.2.L1-10:

Description: Determine signs of first law quantities. (5 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A beaker is filled with a fluid **A** to a depth of **h** and the pressure at the bottom of the fluid is found to be equal to $P_1 = \rho_A g h$. Fluid **B**, which is 3 times as dense as fluid **A**, is also added. If only half as much as **A** of fluid **B** is added, what is the new pressure at the bottom of the beaker?

(1) 1/3 P_1
(2) 1/2 P_1
(3) 2/3 P_1
(4) P_1
(5) 3/2 P_1
(6) 5/2 P_1
(7) 2 P_1
(8) 3 P_1



$$P_{bot} = P_{atm} + \rho_A g h + \rho_B g \frac{h}{2}$$

$$P_1 = \rho_A g h \Rightarrow P_{atm} = 0!$$

$$P_1 = \rho_A g h$$

$$\Rightarrow P_{bot} = \rho_A g h + 3\rho_A g \frac{h}{2}$$

$$= \frac{5}{2} \rho_A g h$$

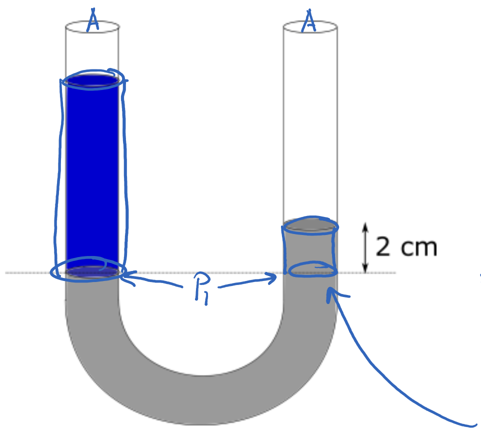
$$= \frac{5}{2} P_1$$

FS.2.L1-11:

Description: Sketch an isobaric process. (2 minutes)

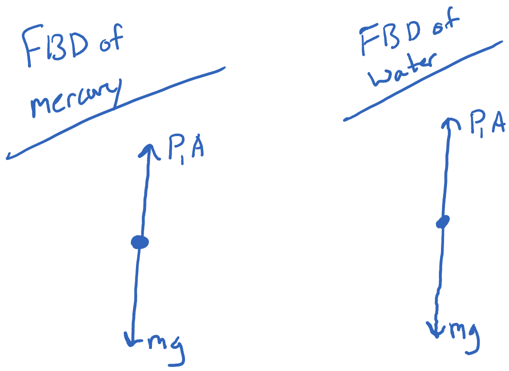
Learning Objectives: [1, 12, 13]

Problem Statement: A U-Tube which is open to the atmosphere on both ends is initially filled with mercury, then an unknown amount of water is poured into the left hand side of the tube. The water and mercury don't mix and the equilibrium state is shown in the image below. The mercury is pushed up 2 cm on the right hand side. To what height above the given reference line does the water extend up to? The density of mercury is about 13600 kg/m3.



$P = P_o + \rho g d \Rightarrow$ Fluid at same height has same pressure (if static & same sys!)

$\Rightarrow$ F at bottom of each outlined volume is $F = P_1 A$



$\Rightarrow m_m g = m_w g$

$m_m = m_w$

$\rho_m V_m = \rho_w V_w$

$\rho_m A h_m = \rho_w A h_w$

$\Rightarrow h_w = h_m \frac{\rho_m}{\rho_w} = (0.02)\frac{13600}{1000}$

$= 0.02(13.6)$

$\boxed{h_w = 27.2 \text{ cm}}$

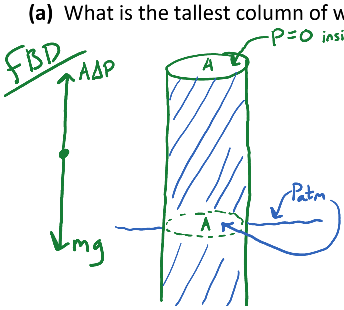
FS.2.L1-12:

Description: Proportional reasoning with ideal gas law. (4 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Steve is selling a vibranium straw that is 5.0 m long for a price of $20. Steve's good pal is also selling a straw that is made out of uru metal for $20 but is 20 m long. You wish to buy a straw to be used for drinking from a cup of water in the vertical direction on the surface of the Earth. Neither of these straws can be cut into smaller pieces.

(a) What is the tallest column of water you can get up a straw near the surface of the earth?

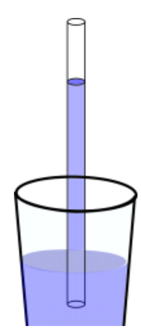


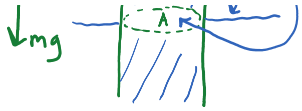
$A(P_{atm} - 0) = mg$

$A P_{atm} = \rho_w V g$

$A P_{atm} = \rho_w A h g$

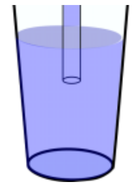
$h_{max} = \frac{P_{atm}}{\rho_w g} = \frac{101{,}325}{(1000)(9.81)} = \boxed{10.3 \text{ m}}$





$P_{atm} = \rho_w h g$

$$h_{max} = \frac{P_{atm}}{\rho_w g} = \frac{101{,}325}{(1000)(9.81)} = \boxed{10.3\ m}$$



(b) Which straw would you buy and why?

the 5.0m straw

the other wouldn't work

Act IV: Pascal's Principle

FS.2.L1-13:

Description: Determine signs of first law quantities. (5 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Two identical uninflated balloons are connected to a T-shaped tube as shown below.

(a) When air is blown into the open end of the tube,

(1) the balloon on the left inflates more at equilibrium.
(2) the balloon on the right inflates more at equilibrium.
(3) both balloons inflate equally at equilibrium.

(b) What will be the difference between the two balloons?

the right balloon will inflate more quickly

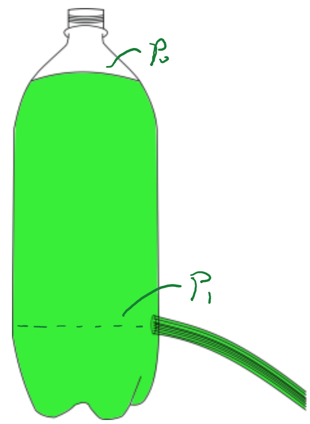
FS.2.L1-14:

Description: Determine signs of first law quantities. (5 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: When a hole is made in the side of a container holding water, water flows out and follows a parabolic trajectory as shown in the image below. If the container is dropped in free fall, the water flow

(1) diminishes.
(2) stops altogether.
(3) goes out in a straight line.
(4) curves upwards.



$$P_1 = P_0 + \rho g d$$

if in free fall the effective "g" is zero

(if you are falling at g, and you "drop" a tennis ball next to you, what happens? it stays right next to you!)

Conceptual questions for discussion

1. Coming soon.

Hints

FS.2.L1-1: No hints.

FS.2.L1-2: No hints.

FS.2.L1-3: No hints.

FS.2.L1-4: No hints.

FS.2.L1-5: No hints.

FS.2.L1-6: No hints.

FS.2.L1-7: No hints.

FS.2.L1-8: No hints.

FS.2.L1-9: No hints.

FS.2.L1-10: No hints.

FS.2.L1-11: No hints.

FS.2.L1-12: No hints.

FS.2.L1-13: No hints.

FS.2.L1-14: No hints.