Magnetic Forces and Magnetic Fields

1 – Magnets

Magnets are metallic objects, mostly made out of iron, which attract other iron containing objects (nails) etc.

Magnets orient themselves in roughly a north - south direction if they are allowed to rotate freely (**compass**).

Assume that a magnet has bar form. Objects are attracted most strongly to the ends of the magnet called **poles**.

There are two poles:

- north pole and
- south pole

Magnetic poles exert attractive or repulsive forces on each other similar to electric forces between charged objects.

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Like poles repel each other
and unlike poles attract each other

Important difference to electric charges:

Electric charges can be isolated (proton, electron), but magnetic poles cannot be isolated ⇒ **magnetic poles always occur in pairs!**

By placing iron containing objects close to a magnet, these objects become magnetized, *ie.* they develop magnetic poles.

To describe the interaction of magnets and magnetized materials, it is convenient to introduce the concept of the **magnetic field**, analogous to the electric field.

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2 – Magnetic Fields

Experiments demonstrate that a stationary (non-moving) particle does not interact with a static magnetic field.

However, *when moving through a magnetic field a charged particle experiences a force.*

Properties:

- The force has its maximum value when the charge moves perpendicular to the magnetic field lines.
- The force is **zero** when the particle moves along the field lines.

The magnetic force exerted on a test charge q_0, moving with velocity $\vec{\mathbf{v}}$ can be used to describe the properties of the magnetic field, $\vec{\mathbf{B}}$.

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From experiment we know:

- The force is proportional to the strength of the external magnetic field, B.
- It is proportional to the sine of the angle θ between the direction of $\vec{\mathbf{v}}$ and the direction of $\vec{\mathbf{B}}$.
- It is proportional to the charge q_0.
- It is proportional to the magnitude of the velocity, v.

$$\boxed{F = q_0 v B \sin\theta} \qquad (1)$$

The magnitude of the magnetic field is then defined as

$$\boxed{B = \frac{F}{q_0 v \sin\theta}} \qquad (2)$$

SI unit of $\vec{\mathbf{B}}$: Tesla $1\;T = 1\;\frac{N}{C\;m/s} = 1\;\frac{N}{A\;m}$. In practice one often uses the **gauss** as an unit: $1\;T = 10^4\;G$

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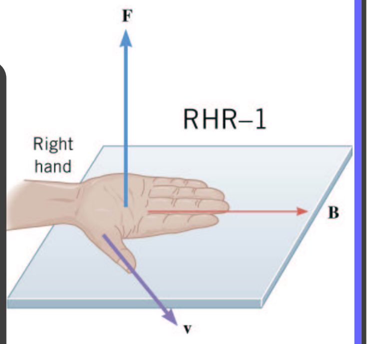
Direction of the magnetic force:

Experiments show that the direction of the magnetic force is always perpendicular to both $\vec{v}$ and $\vec{B}$. The direction can be determined by the right hand rule:

Hold your right hand open with
your fingers pointing in the direction of $\vec{B}$
and your thumb pointing in the direction of $\vec{v}$
$\vec{F}$, on a positive charge, is then
directed out of the palm of your hand.



For a negative charge reverse the direction of $\vec{F}$.

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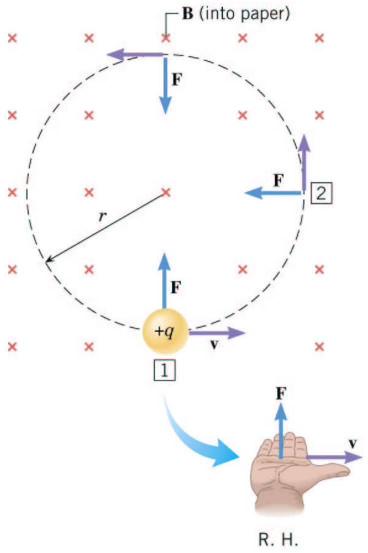
3 – Motion of a Charged Particle in a Magnetic Field

Consider a positively charged particle moving in a uniform magnetic field so that the direction of the particle's velocity is perpendicular to the field.



Notation:
If $\vec{B}$ is directed into the page,
a series of crosses (arrow tails) is used.
If $\vec{B}$ is directed out of the page,
a series of dots (arrowheads) is used.

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The magnetic force always acts in a direction perpendicular to the motion of the charge.

$\Rightarrow$ the magnetic force does no work

$\Rightarrow$ the kinematic energy does not change

$\Rightarrow$ only the direction of the motion changes and the speed stays the same.

The magnetic force (right-hand rule!) is always directed toward the center of a circular path → the magnetic force is effectively a centripetal force:

$$\vec{\mathbf{F}}_{mag.} = \vec{\mathbf{F}}_c$$

$$F_{mag.} = q_0 v B \text{ and } F_c = \frac{mv^2}{r}$$

which gives for the radius r of the path

$$\boxed{r = \frac{mv}{q_0 B}} \quad (3)$$

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If the initial direction of the velocity of the charged particle is not perpendicular to the magnetic field, the path of the particle is a spiral along the magnetic field lines.

Mass spectrometer:

1. Atoms or molecules are vaporized and ionized by removing one electron so that their net charge is $+e$.

2. The ions are accelerated in an electric potential difference V: $1/2mv^2 = eV$ when they enter a magnetic field.

3. Only ions which are forced on a circular path by the magnetic force with radius r given by $r = \frac{mv}{q_0 B} = \sqrt{2Vm/(eB^2)}$ reach the detector.

4. The mass of these ions is then determined as

$$\boxed{m = \frac{er^2B^2}{2V}} \quad (4)$$

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4 – Magnetic Force on a current-carrying Conductor

An electric current is a collection of many charged particles in motion $\rightarrow$ a current-carrying wire experiences a force when placed in a magnetic field.

Force on an individual charge carrier:

$$F = qv_dB\sin\theta$$

where v_d is the drift velocity of the charge and θ the angle between the current and $\vec{\mathbf{B}}$.

Force on wire: multiply by number of charge carriers per unit volume, n, and the volume $V = A\ell$ (A is the cross section of the wire and ℓ its length).

$$F = (qv_dB\sin\theta)(nA\ell)$$

But $I = nqv_dA$ and therefore

$$\boxed{F = BI\ell\sin\theta} \qquad (5)$$

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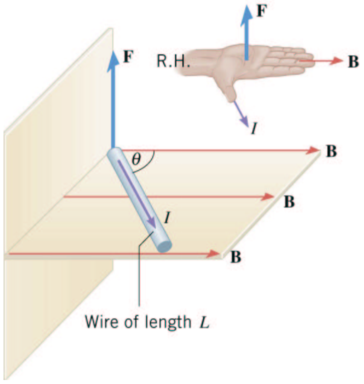
The direction of the force can be determined using the right-hand rule with the thumb pointing in the direction of the current.



Application: Loudspeaker in sound systems.

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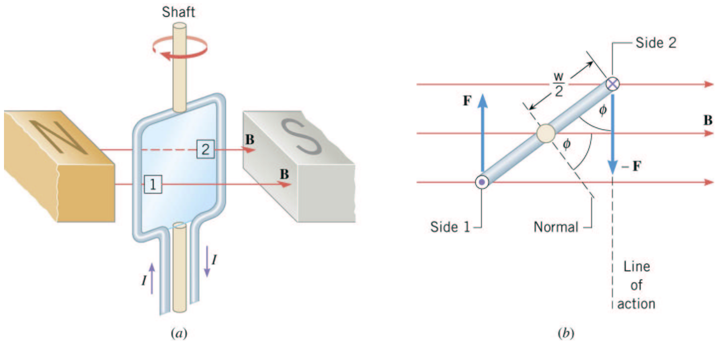
5 – Torque on a Current Loop

- Consider a rectangular loop carrying a current I in the presence of an external magnetic field in the plane of the loop:



(a) (b)

- The force on the two sides parallel to the magnetic field is **zero**.
- The magnitude of the forces on the two sides perpendicular to the magnetic field (with length b) is

$$F_1 = F_2 = BIb$$

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- This leads to a net torque (a is the distance from the axis of rotation)

$$\tau = F_1 \frac{a}{2} + F_2 \frac{a}{2} = BIab = BIA$$

where $A = ab$ is the area of the loop.

- If $\vec{\mathrm{B}}$ makes an angle Φ with a line perpendicular to the plane of the loop one finds

$$\boxed{\tau = BIA \sin \Phi} \qquad (6)$$

- For a loop with N turns:

$$\tau = NBIA \sin \Phi$$

- **applications:** galvanometer, generator

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6 – The Galvanometer and its Applications

A **galvanometer** is a device used in the construction of ammeters and voltmeters.

Ammeter: a device which measures electric currents.

It makes use of the fact that a torque acts on a current loop in presence of a magnetic field. The larger the current, the larger the torque $\rightarrow$ the larger the deflection.

Internal resistance of a galvanometer $\approx$ 60 Ω. This makes it hard to measure the current in a circuit where the resistance of the circuit is $\ll$ 60 Ω.

Example: A circuit with 3 V battery and a 3 Ω resistor. From Ohm's law: $I = 1\ A$. Including the galvanometer: the resistance is now 60 $\Omega + 3\Omega = 63\Omega$ and $I = 3\ V/63\ \Omega = 0.048\ A$.

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In addition: a galvanometer gives full deflection for currents of $< 1\ mA$. To make it work for larger currents, a *shunt resistor* is used. A shunt resistor is a resistor R_p which is placed in parallel to the galvanometer so that only a current of less than 1 mA passes through the galvanometer.

$$R_p = 0.06\ A\Omega/I$$

The equivalent resistance of the galvanometer is then $< R_p$.

A galvanometer can also be used to measure voltages: For $I < 1\ mA$ and $R = 60\ \Omega$, voltages less than 0.06 V can be measured. To measure larger voltages an additional resistor R_s is placed in *series* with the galvanometer. This allows to measure voltages up to $1\ mA \times (R_s + 60\ \Omega)$.

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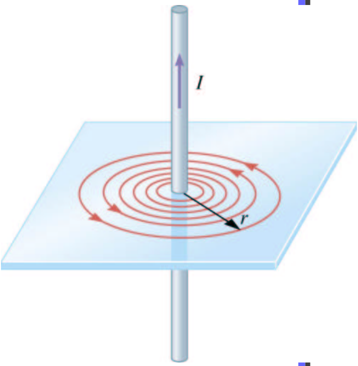
7 – Magnetic Field of a Long Straight Wire

In 1819, Hans Oersted found that an electric current in a wire deflected a nearby compass needle.

Conclusion: A current - carrying conductor produces a magnetic field:

The magnetic field lines around a wire
form concentric circles.



If the wire is grasped in the right hand with the thumb in the direction of the current, the fingers will curl in the direction of B.

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By varying the current and distance from the wire, one finds that $\vec{\mathbf{B}}$ is proportional to the current and inversely proportional to the distance from the wire:

$$\boxed{B = \frac{\mu_0 I}{2\pi r}} \tag{7}$$

μ_0, called the **permeability of free space** is defined to be

$$\boxed{\mu_0 = 4\pi \times 10^{-7}\, T \cdot m/A} \tag{8}$$

8 – Magnetic Force Between Two Parallel Conductors

A magnetic force acts on a current-carrying conductor when the conductor is placed in an external magnetic field. Since a current in a conductor creates its own magnetic field, two current carrying wires placed close together exert magnetic forces on each other.

Consider two straight parallel wires separated by a distance d, carrying currents I_1 and I_2 in the same direction.

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Wire 2, carrying I_2 causes a magnetic field $\mathbf{B}_2$ at wire 1:

$$B_2 = \frac{\mu_0 I_2}{2\pi d}$$

The magnetic force on wire 1 (length: ℓ) due to $\mathbf{B}_2$ is:

$$F_1 = B_2 I_1 \ell = \left(\frac{\mu_0 I_2}{2\pi d}\right) I_1 \ell = \frac{\mu_0 I_1 I_2 \ell}{2\pi d}$$

- The direction of F_1 is toward wire 2, *ie* if I_1 and I_2 flow in the same direction, the two wires attract each other.
- If the direction of I_1 is opposite to the direction of I_2, the force between the wires is repulsive.
- The force between two parallel wires carrying a current is used to define the SI unit of current (Ampere).

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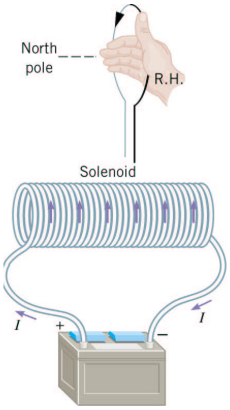
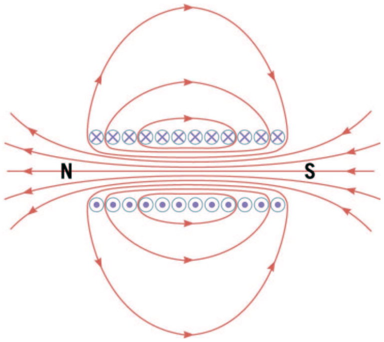
9 – Magnetic Field of a Current Loop and of a Solenoid

- The magnetic field of a circular wire carrying a current is very similar to that of a bar magnet:





- A **solenoid** or **electromagnet** is a coil of several closely spaced loops.
- They act as magnets only when they carry a current.
- When the loops are spaced closely together, and the length of the

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solenoid is much larger than its radius, the magnetic field inside is strong and **uniform**, and weak outside.

- The magnetic field inside a solenoid is given by

$$\boxed{B = \mu_0 n I} \tag{9}$$

where $n = N/\ell$ is the number of turns per unit length, and I is the current flowing through the solenoid.

Applications: Magnetic resonance imaging, TV