Magnetic Forces

Magnetic Forces

Charged particles experience an electric force when in an electric field regardless of whether they are moving or not moving

There is another force that charged particles can experience even in the absence of an electric field but only when they are motion

A Magnetic Force

Magnetic Interactions

are the result of relative motion

Quick Note on Magnetic Fields

Like the electric field, the magnetic field is a *Vector*, having both direction and magnitude

We denote the magnetic field with the symbol $\vec{B}$

The unit for the magnetic field is the *tesla*

$$1\,\textbf{tesla} = 1\,\mathbf{T} = 1\,N/A\cdot m$$

There is another unit that is also used and that is the *gauss*

$$1\,\textbf{gauss} = 10^{-4}\,\mathbf{T}$$

Unlike Electric Fields which begin and end on charges, Magnetic Fields have neither a beginning nor an end

Magnetic Forces

Given a charge *q* moving with a velocity *v* in a magnetic field, it is found that there is a force on the charge

This force is

- **proportional to the charge *q***
- **proportional to the speed *v***
- **perpendicular to both *v* and B**
- **proportional to sinϕ where ϕ is the angle between *v* and B**

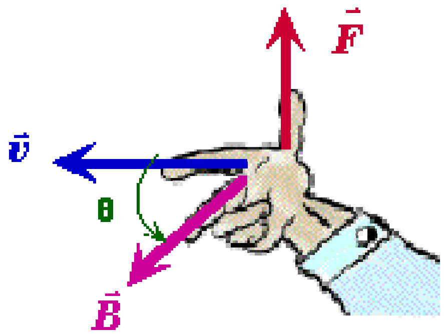
This can be summarized as $\vec{F} = q\vec{v} \times \vec{B}$

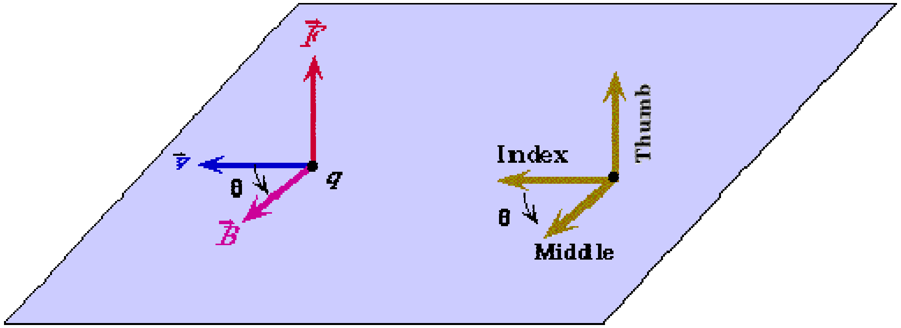
This is the *cross product* of the velocity vector of the charged particle and the magnetic field vector

Right Hand Rule

To get the resultant direction for the force do the following:

1. Point your index finger (and your middle finger) along the direction of motion of the charge v
2. Rotate your middle finger away from your index finger by the angle θ between v and B
3. Hold your thumb perpendicular to the plane formed by both your index finger and middle finger
4. Your thumb will then point in the direction of the force F if the charge q is positive
5. For q < 0, the direction of the force is opposite your thumb

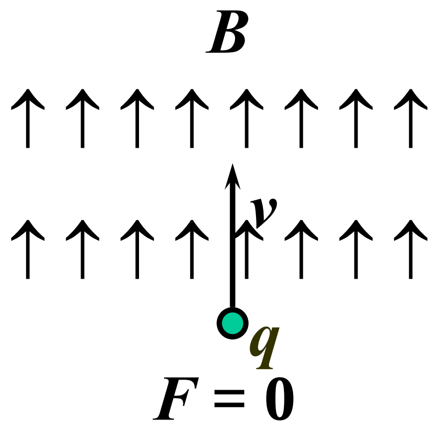




Magnetic Forces

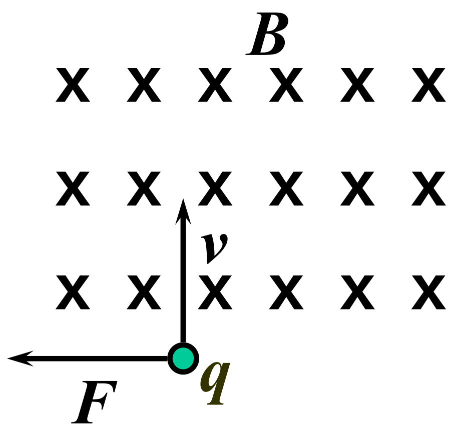
There is no force if *v* and B are either parallel or antiparallel

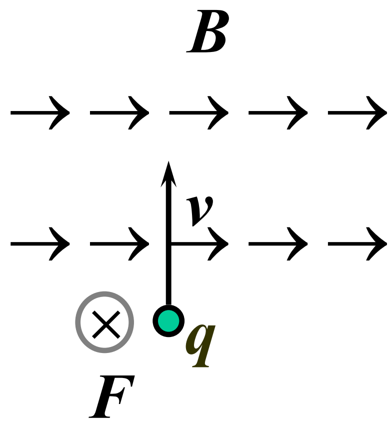
Sin(0) = Sin(180) = 0



The force is maximum when *v* and B are perpendicular to each other

Sin(90) = 1

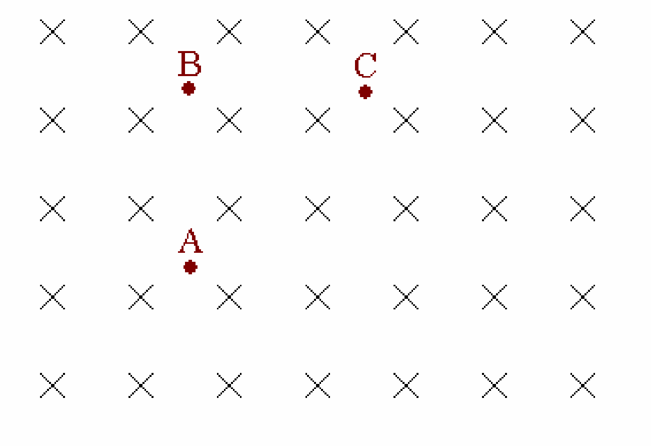




The force on a *negative charge* is in the *opposite direction*

Example

Three points are arranged in a uniform magnetic field. The magnetic field points into the screen.



1) A positively charged particle is located at point A and is stationary. The direction of the magnetic force on the particle is:

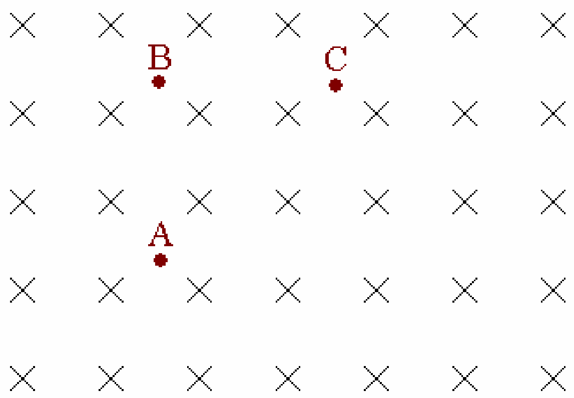
a) Right b) Left c) Into the screen

d) Out of the screen e) Zero

The magnetic force is given by $\vec{F} = q\vec{v} \times \vec{B}$

But v is zero. Therefore the force is also zero.

Example



Three points are arranged in a uniform magnetic field. The magnetic field points into the screen.

2) The positive charge moves from point A toward B. The direction of the magnetic force on the particle is:

a) Right b) Left c) Into the screen

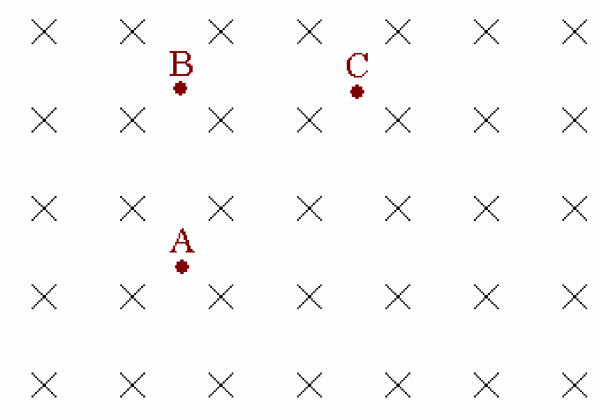
d) Out of the screen e) Zero

The magnetic force is given by $\vec{F} = q\vec{v} \times \vec{B}$

The cross product of the velocity with the magnetic field is to the left and since the charge is positive the force is then to the left

Example

Three points are arranged in a uniform magnetic field. The magnetic field points into the screen.



3) The positive charge moves from point A toward C. The direction of the magnetic force on the particle is:

a) up and right

b) up and left

c) down and right

d) down and left

The magnetic force is given by $\vec{F} = q\vec{v} \times \vec{B}$

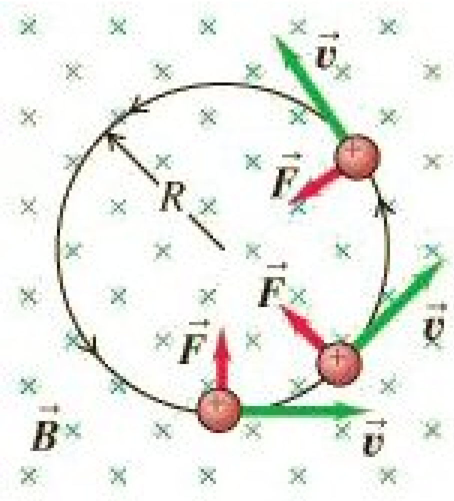
The cross product of the velocity with the magnetic field is to the upper left and since the charge is positive the force is then to the upper left

Motion due to a Magnetic Force

When a charged particle moves in a magnetic field it experiences a force that is perpendicular to the velocity

Since the force is perpendicular to the velocity, the charged particle experiences an acceleration that is perpendicular to the velocity

The magnitude of the velocity does not change, but the direction of the velocity does producing circular motion



The magnetic force does **no** work on the particle

Motion due to a Magnetic Force

The magnetic force produces circular motion with the centripetal acceleration being given by

$$v^2/R$$

where R is the radius of the orbit

Using Newton's second law we have $F = qvB = m\dfrac{v^2}{R}$

The radius of the orbit is then given by $R = \dfrac{m\,v}{q\,B}$

The angular speed ω is given by $\omega = \dfrac{v}{R} = \dfrac{q\,B}{m}$

Motion due to a Magnetic Force

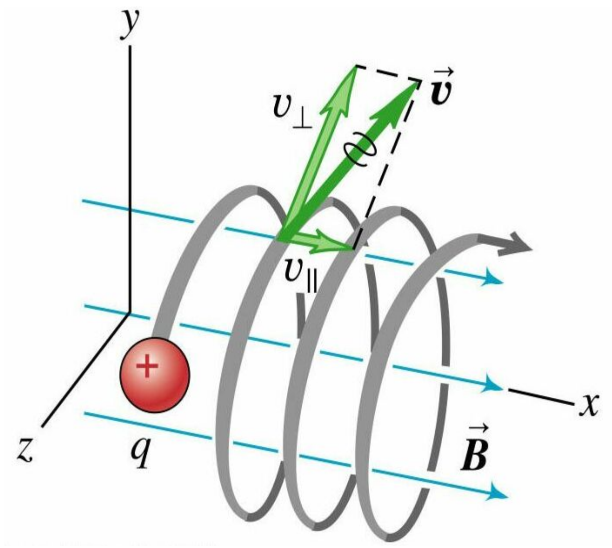
What is the motion like if the velocity is not perpendicular to B?

We break the velocity into components along the magnetic field and perpendicular to the magnetic field

The component of the velocity perpendicular to the magnetic field will still produce circular motion

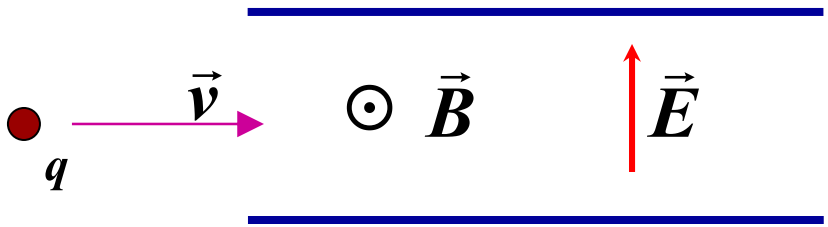
The component of the velocity parallel to the field produces no force and this motion is unaffected

The combination of these two motions results in a helical type motion



Velocity Selector

An interesting device can be built that uses both magnetic and electric fields that are perpendicular to each other

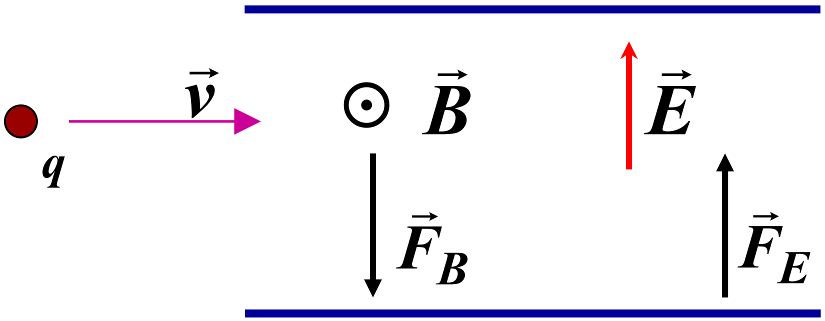


A charged particle entering this device with a velocity $\vec{v}$ will experience both

an electric force $\vec{F}_E = q\vec{E}$

and a magnetic force $\vec{F}_B = q\vec{v} \times \vec{B}$

Velocity Selector



If the particle is positively charged then the magnetic force on the particle will be downwards

and the electric force will be upwards

If the velocity of the charged particle is just right then the net force on the charged particle will be zero

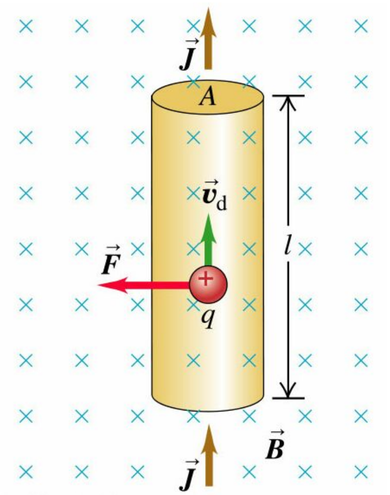
$$qvB = qE \Rightarrow v = \frac{E}{B}$$

Magnetic Forces

We know that a single moving charge experiences a force when it moves in a magnetic field

What is the net effect if we have multiple charges moving together, as a current in a wire?

We start with a wire of length l and cross section area A in a magnetic field of strength B with the charges having a drift velocity of v_d



The total number of charges in this section is then nAl where n is the charge density

The force on a single charge moving with drift velocity v_d is given by $F = qv_d B$

So the total force on this segment is

$$F = n q v_d A l B$$

Magnetic Force on a Current Carrying Wire

We have so far that $F = n\,q\,v_d\,A\,l\,B$

But we also have that $J = n\,q\,v_d$ and $I = J\,A$

Combining these, we then have that $F = I\,l\,B$

The force on the wire is related to the current in the wire and the length of the wire in the magnetic field

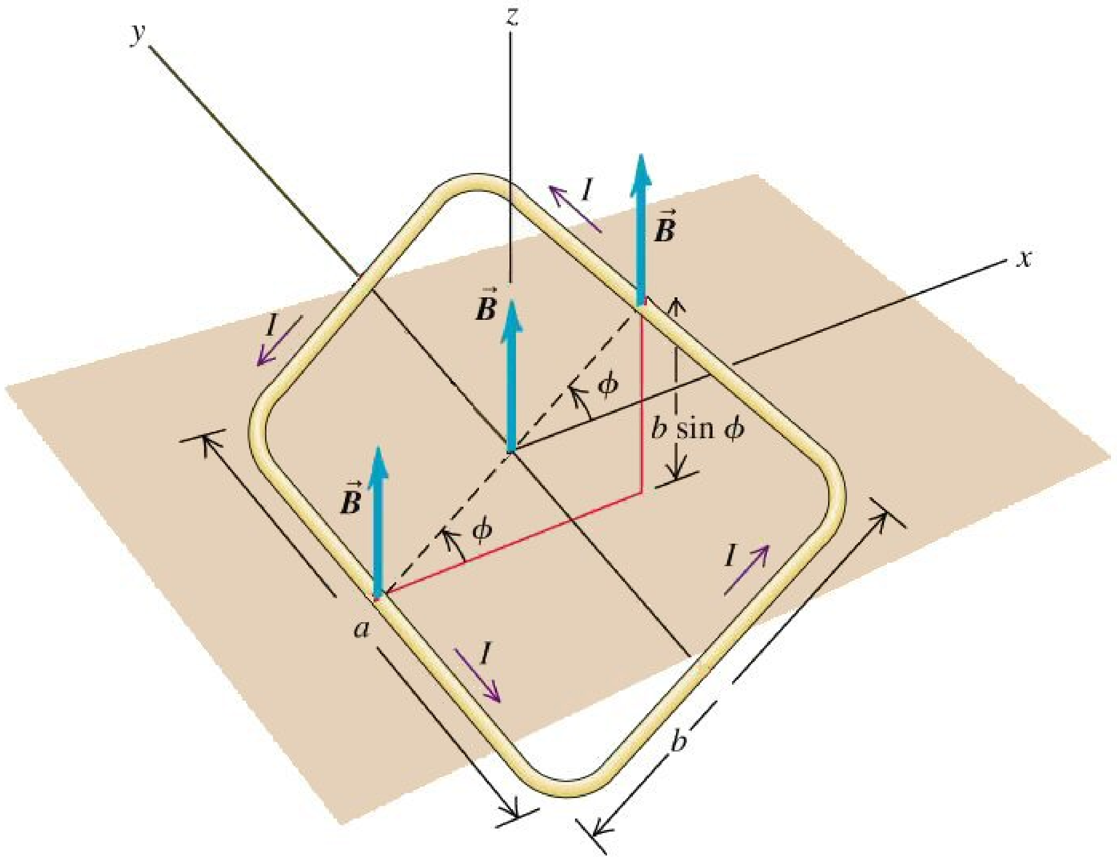
If the field and the wire are not perpendicular to each the full relationship is

$$\vec{F} = I\,\vec{l} \times \vec{B}$$

The direction of *l* is the direction of the current

Current Loop in a Magnetic Field

Suppose that instead of a current element, we have a closed loop in a magnetic field

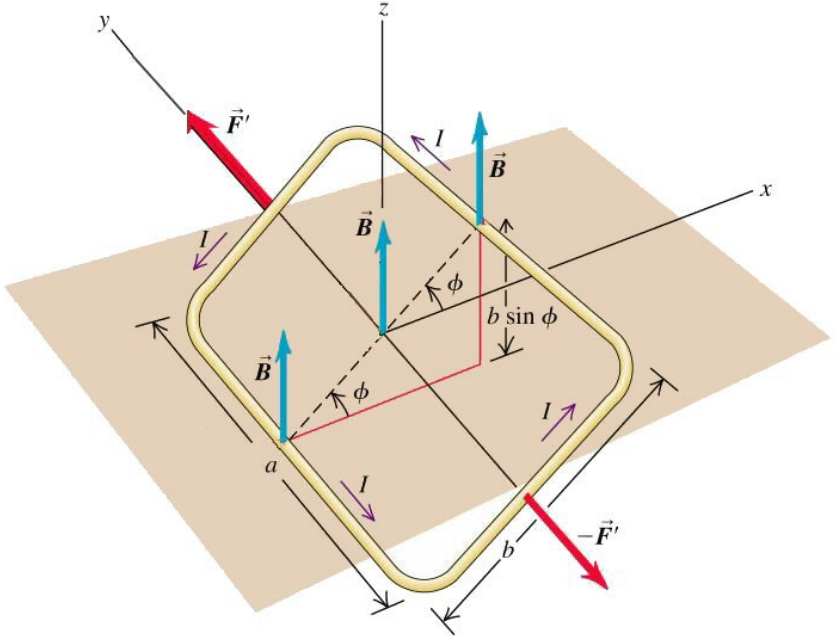


We ask what happens to this loop

Current Loop in a Magnetic Field

Each segment experiences a magnetic force since there is a current in each segment

As with the velocity, it is only the component of the wire that is perpendicular to B that matters



Each of the two shorter sides experiences a force given by

$$F' = I\,b\,B\cos\phi$$

in the directions shown

Since the magnitudes are the same, the net force in the y-direction is $\sum F_y = 0$

No translational motion in the y-direction

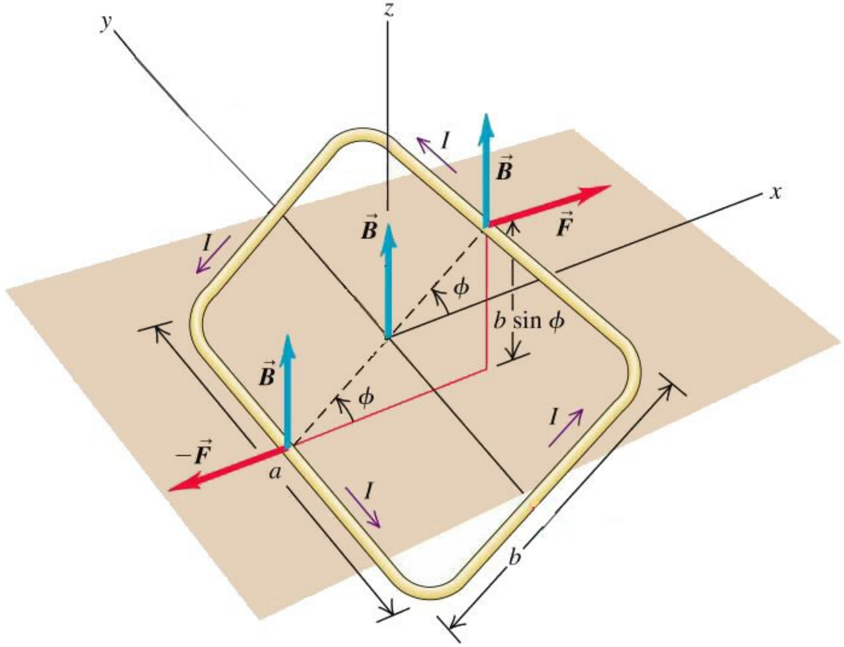
Current Loop in a Magnetic Field

Now for the two longer sides of length *a*

Each of these two sides experiences a force given by

$$F = I\,a\,B$$

in the directions shown



But since the forces are of the same magnitude but in opposite directions we have

$$\sum F_x = 0$$

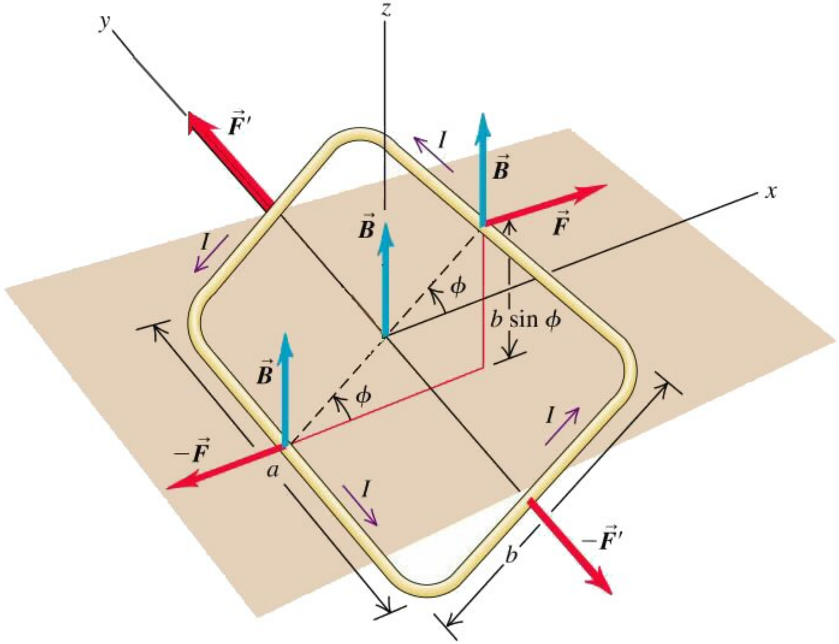
No translational motion in the x-direction

Current Loop in a Magnetic Field

There is no translational motion in either the x- or y-directions

While the two forces in the y-direction are colinear, the two forces in the x-direction are **not**

Therefore there is a *torque* about the y-axis



The lever arm for each force is

$$\frac{b}{2}\sin\phi$$

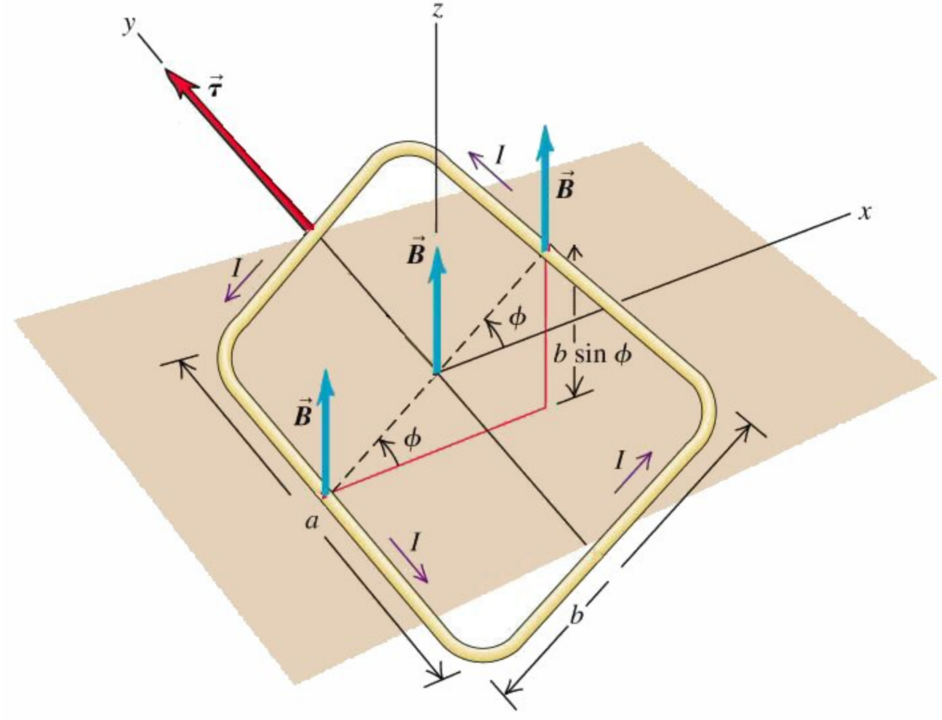
The net torque about the y-axis is

$$\tau = 2F\left(\frac{b}{2}\right)\sin\phi = I\,B\,a\,b\sin\phi$$

Current Loop in a Magnetic Field

This torque is along the positive y-axis and is given by

$$\tau = \boldsymbol{I}\,\boldsymbol{B}\,\boldsymbol{A}\sin\phi$$



The product *IA* is referred to as the ***magnetic moment***

$$\mu = \boldsymbol{I}\,\boldsymbol{A}$$

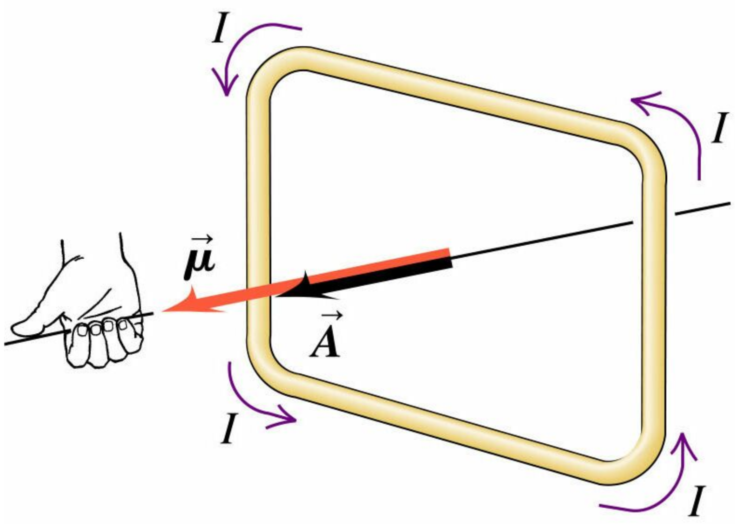
We rewrite the torque as

$$\tau = \mu\,\boldsymbol{B}\sin\phi$$

Magnetic Moment

We defined the magnetic moment to be $\mu = I\,A$

It also is a *vector* whose direction is given by the direction of the area of the loop



The direction of the area is defined by the sense of the current

We can now write the torque as $\vec{\tau} = \vec{\mu} \times \vec{B}$

Potential Energy of a Current Loop

As the loop rotates because of the torque, the magnetic field does work on the loop

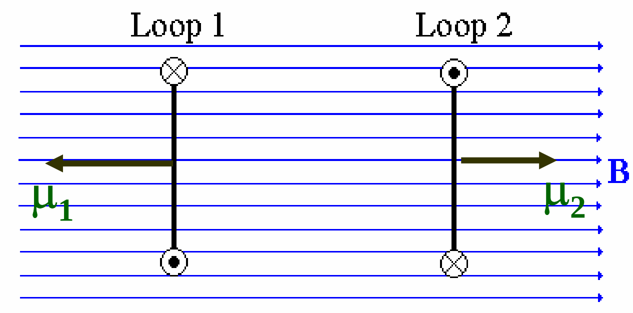
We can talk about the potential energy of the loop and this potential energy is given by

$$U = -\vec{\mu} \cdot \vec{B}$$

The potential energy is the least when μ and B are parallel and largest when μ and B are antiparallel

Example

Two current carrying loops are oriented in a uniform magnetic field. The loops are nearly identical, except the direction of current is reversed.



1) What direction is the torque on loop 1?

a) clockwise b) counter-clockwise c) zero

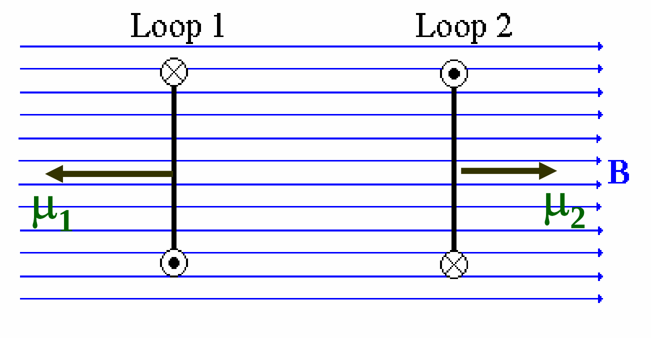
The magnetic moment for Loop 1, μ_1, points to the left, while that for Loop 2, μ_2, points to the right

The torque is given by $\vec{\tau} = \vec{\mu} \times \vec{B}$

But since μ_1 and B are antiparallel, the cross product is zero, therefore the torque is zero!

Example

Two current carrying loops are oriented in a uniform magnetic field. The loops are nearly identical, except the direction of current is reversed.



2) How does the magnitude of the torques on the two loops compare?

a) $\tau_1 > \tau_2$ b) $\tau_1 = \tau_2$ c) $\tau_1 < \tau_2$

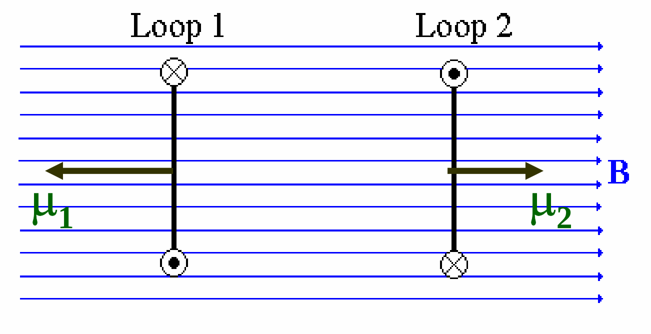
Loop 1: Since μ_1 points to the left the angle between μ_1 and B is equal to 180° therefore $\tau_1 = 0$.

Loop 2: Since μ_2 points to the right the angle between μ_2 and B is equal to 0° therefore $\tau_2 = 0$.

So the two torques are equal!

Example

Two current carrying loops are oriented in a uniform magnetic field. The loops are nearly identical, except the direction of current is reversed.



3) Which loop occupies a potential energy minimum, and is therefore stable?

a) Loop 1 b) Loop 2 c) the same

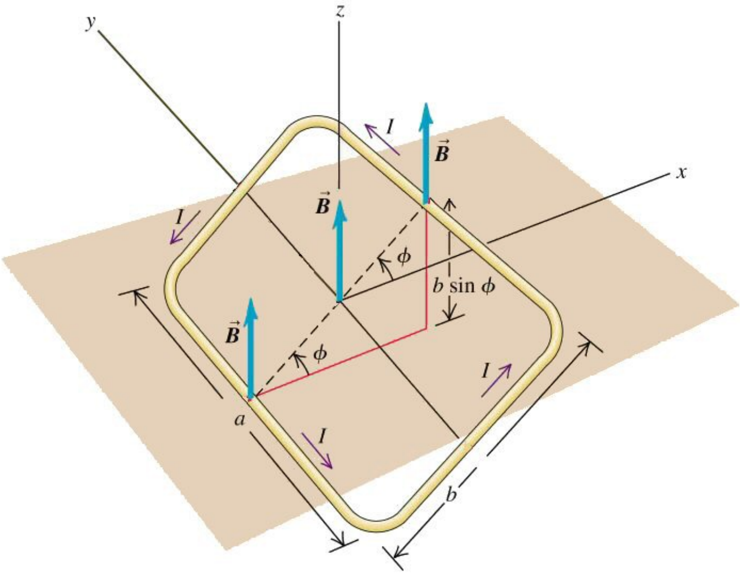
The potential energy is given by $U = -\vec{\mu} \bullet \vec{B}$

For Loop 1 the potential energy is then $U_1 = +\mu_1 B$

While for Loop 2 the potential energy is then $U_2 = -\mu B$

The potential energy for Loop 2 is less than that for Loop 1

Motion of Current Loop



The current loop in its motion will oscillate about the point of minimum potential energy

If the loop starts from the point of minimum potential energy and is then displaced slightly from its position, it will "return", i.e. it will oscillate about this point

This initial point is a point of ***Stable equilibrium***

If the loop starts from the point of maximum potential energy and is then displaced, it will not return, but will then oscillate about the point of minimum potential energy

This initial point is a point of ***Unstable equilibrium***

More Than One Loop

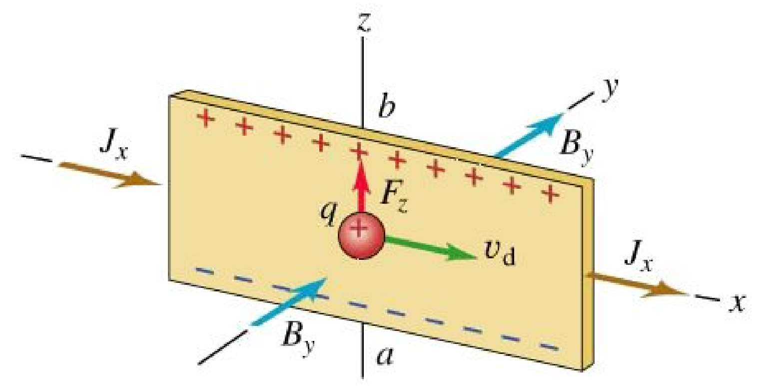
If the current element has more than one loop, all that is necessary is to multiply the previous results by the number of loops that are in the current element

Hall Effect

There is another effect that occurs when a wire carrying a current is immersed in a magnetic field

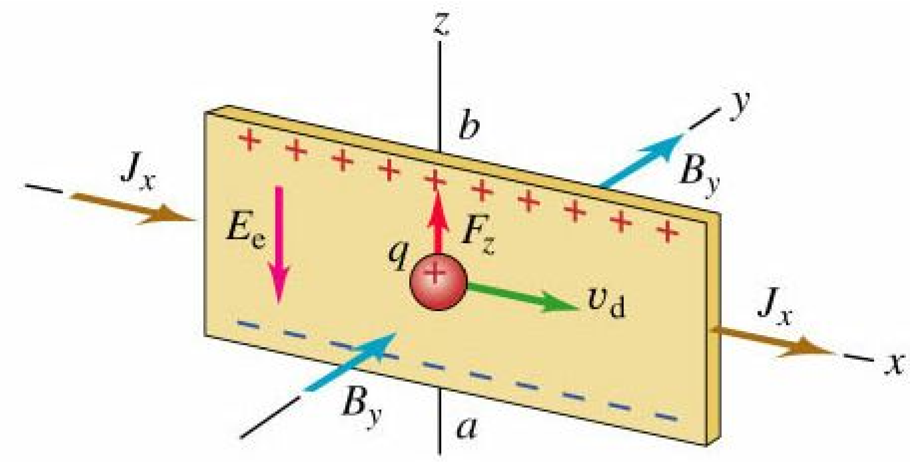
Assume that it is the positive charges that are in motion

These positive charges will experience a force that will cause them to also move in the direction of the force towards the edge of the conductor, leaving an apparent negative charge at the opposite edge



Hall Effect

The fact that the there is an apparent charge separation produces an electric field across the conductor



Eventually the electric field will be strong enough so that subsequent charges feel an equivalent force in the opposite direction

$$q\,E_e = q\,v_d\,B \quad \text{or} \quad E_e = v_d\,B$$

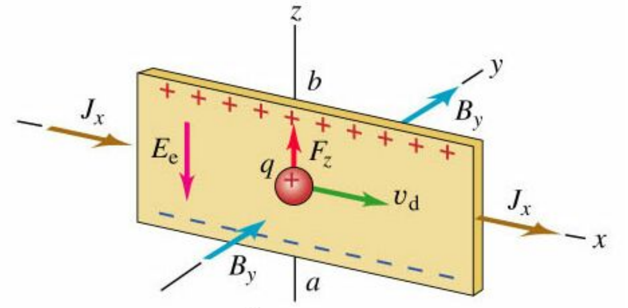
Since there is an electric field, there is a potential difference across the conductor which is given by

$$V = E_e\,d = v_d\,B\,d$$

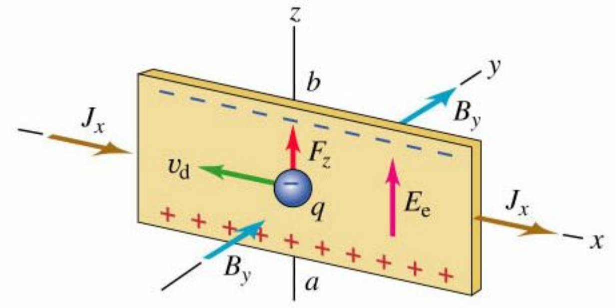
Hall Effect

The Hall Effect allows us to determine the sign of the charges that actually make up the current

If the positive charges in fact constitute the current, then potential will be higher at the upper edge



If the negative charges in fact constitute the current, then potential will be higher at the lower edge



Experiment shows that the second case is true

The charge carriers are in fact the negative electrons