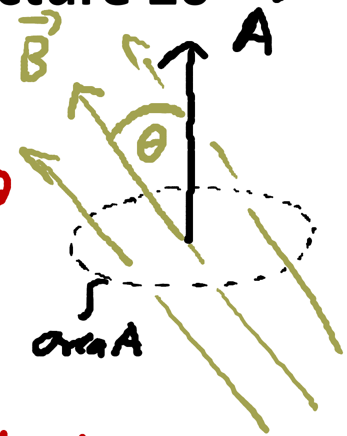
Recap

- *Magnetic flux:*

for uniform $\vec{B}$-field and flat area

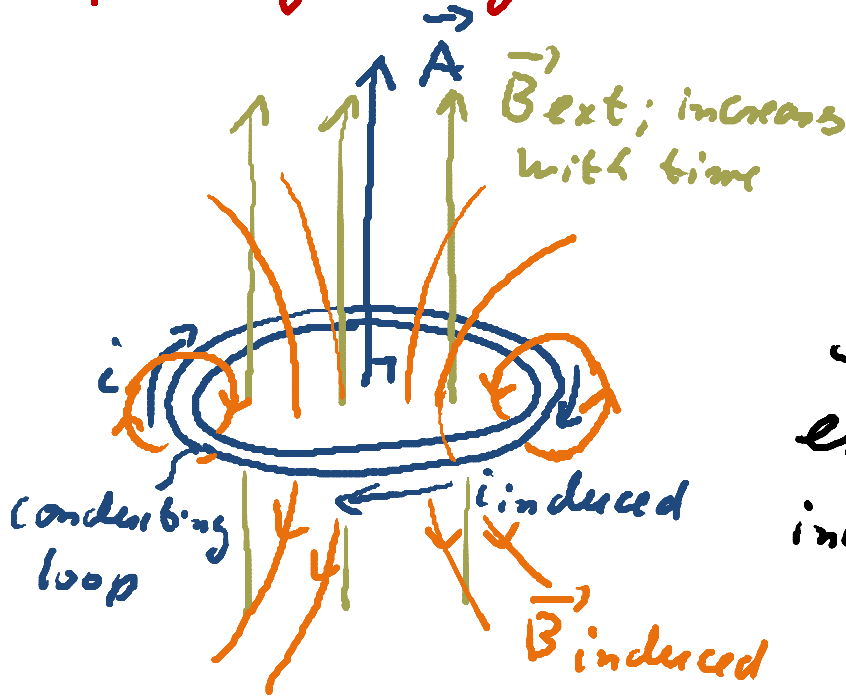
$$\Phi_B = \vec{B}\cdot\vec{A} = BA\cos\theta$$

$$[\Phi_B] = Tm^2 = 1\,Wb$$



- *Magnetic induction:*

change in magnetic flux passing through a conducting loop $\Rightarrow$ produces/induces an emf and thus a current in the loop



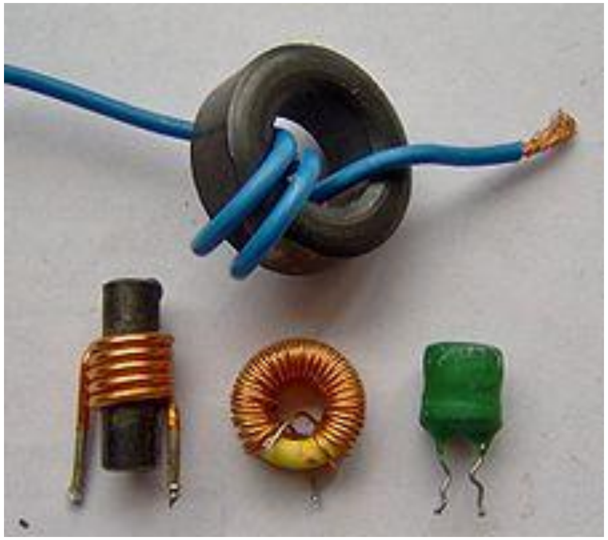
!

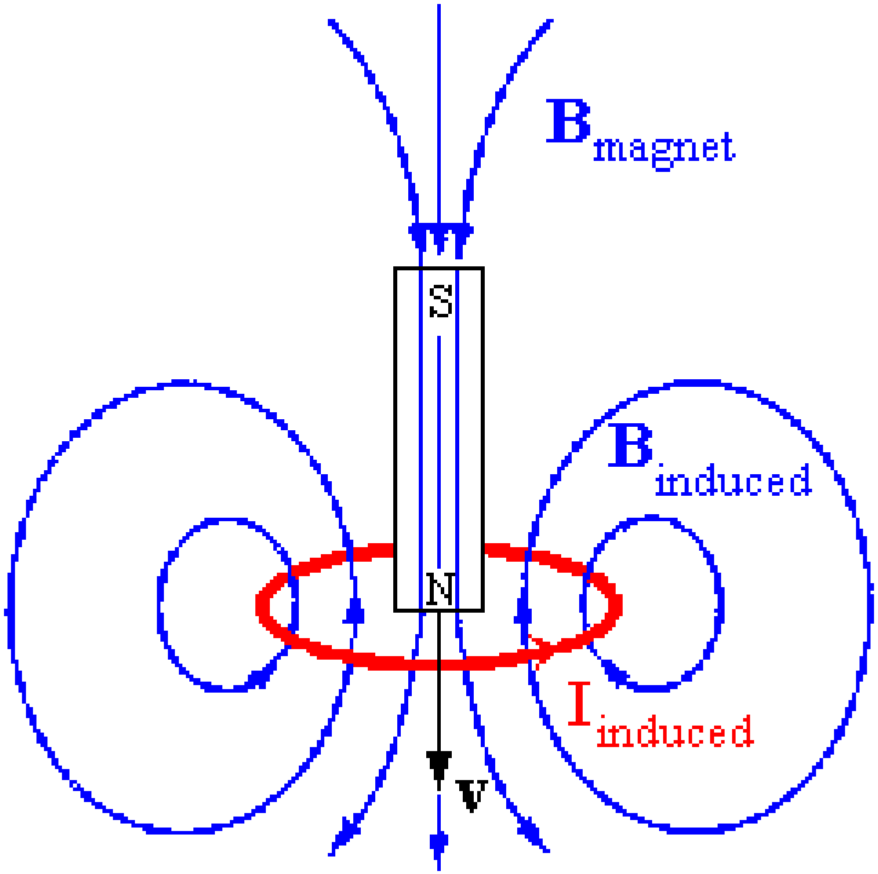
$$\underbrace{\mathcal{E}}_{\substack{\text{emf}\\\text{induced}}} = -\underbrace{N}_{\substack{\text{loop}\\\text{with}\\N\\\text{turns}}} \underbrace{\frac{d\Phi_{B,ext}}{dt}}_{\substack{\text{rate of change of}\\\text{magnetic flux}\\\text{through the loop}}}$$

Faraday's Law

Today:

- More on magnetic induction:
  - Lenz's law
- Inductors and their inductance





Magnetic Induction:

flux Φ_B through conducting loop by external field $\vec{B}_{ext}$ changes with time

$\Downarrow$

induces electric field $\vec{E}_{ind}$ in conducting loop

$\Downarrow$

electric field does work on charge carriers

$$|emf| = |\mathcal{E}| = \frac{\Delta W_{\text{on charge } \Delta q \text{ while charge goes around loop}}}{\Delta q}$$

Direction: (Lenz's Law) Induced emf acts to oppose the flux change that produced it!

$\Downarrow$

induced current: $\boxed{i_{ind} = \frac{\mathcal{E}}{R}}$ with R = resistance of the loop

$\Downarrow$

induced current produces a magnetic field around itself $\vec{B}_{ind}$.

Lenz's law:

To determine the direction of the induced current in the loop, use:

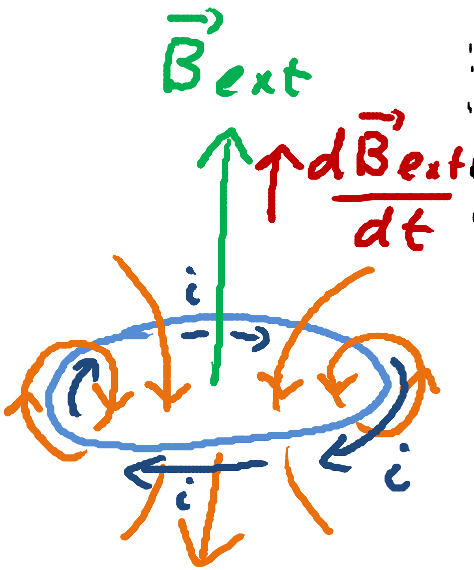
1. An induced current has a direction such that the magnetic field due to the induced current opposes the change in the magnetic flux that induces the current.

Same as saying::

2. **An induced emf acts to oppose the change that produces it.**

Examples:

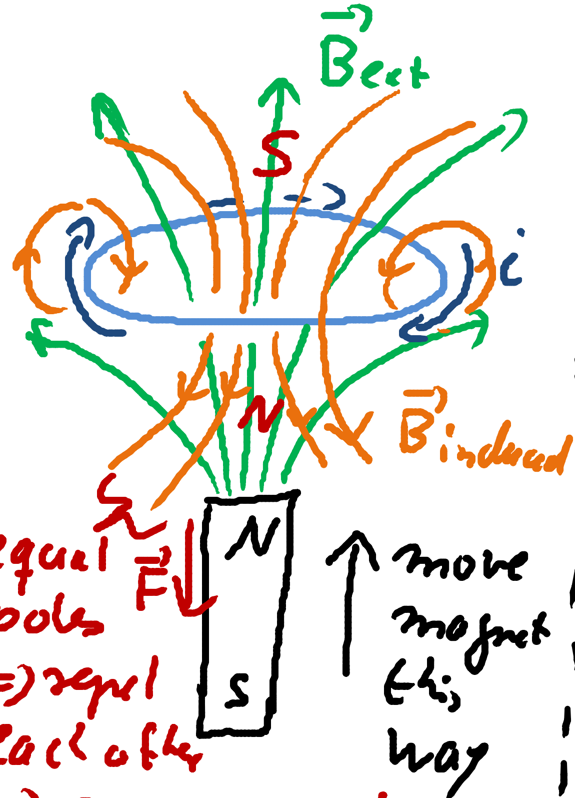
Increasing $\vec{B}_{ext}$:



$\vec{B}_{induced}$ (by i induced)

inside loop

Increasing $\vec{B}_{ext}$:

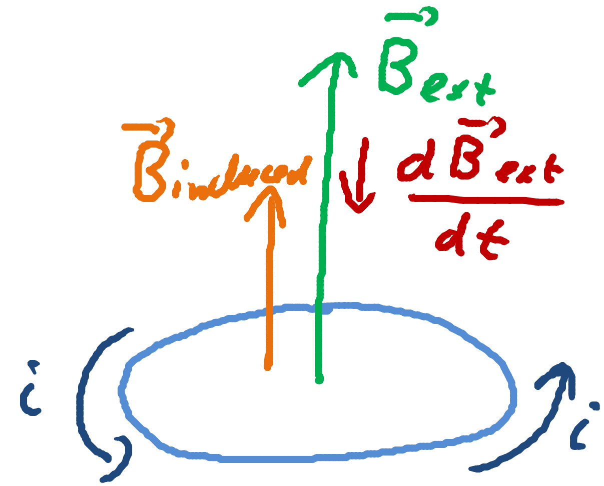


equal poles

$\Rightarrow$ repel each other

$\Rightarrow$ oppose motion of magnet!

Decreasing $\vec{B}_{ext}$:



Note: $\vec{B}_{induced}$ opposes change in external field!

Lenz's law:

Another way to determine the direction of the induced current in the loop:

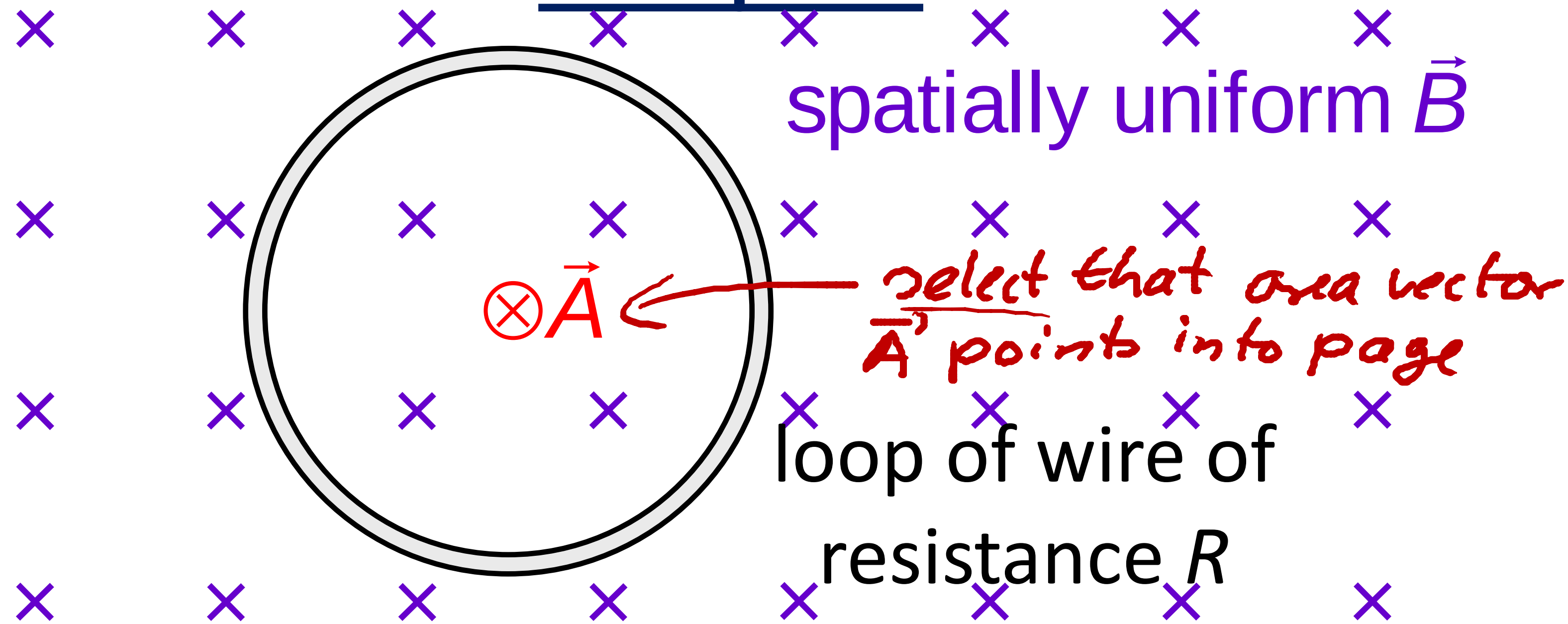
- **Select the positive direction of the area vector for the given loop (this vector is always normal to the loop!)**

- **Determine the direction of positive (+) emf in the loop according to a right-hand rule (point thump in positive direction of the area vector; finger then point in positive direction of the emf).**

- **Calculate the induced emf with Faraday's law. The sign (+ or –) of the induced emf calculated then tells the direction of the induced emf.**

3 Examples

Example 1:



Suppose that B is changing with time t according to $B(t) = kt + B_0$, where k and B_0 are positive constants.

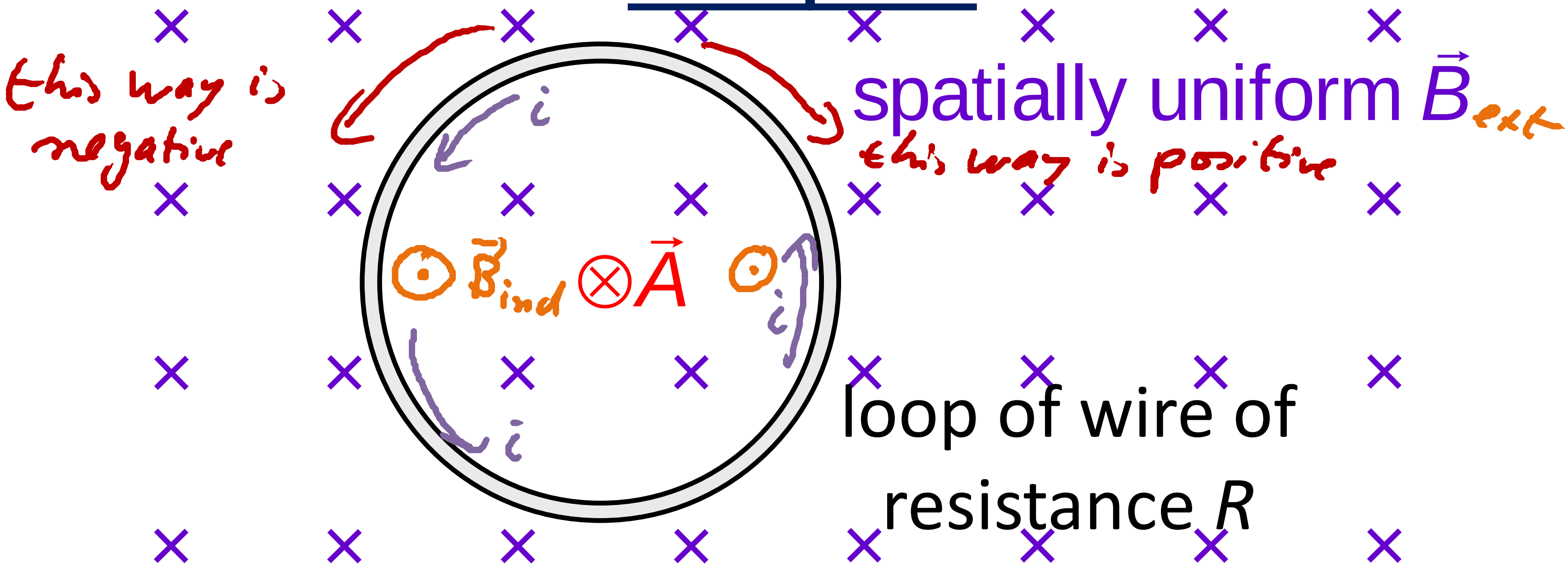
$\Phi_B = \vec{B}\cdot\vec{A} = BA\cos 0° = BA = (kt + B_0)A \Rightarrow \dfrac{d\Phi_B}{dt} = kA$

$\Rightarrow \mathcal{E} = -d\Phi_B/dt = -\underline{\underline{kA}} < 0$

What is the emf $\mathcal{E}$ induced in the loop?

~~A. 0~~ ~~B. $-(kt + B_0)A$~~ ~~C. $-kAt$~~ D. kA E. $-kA$

Example 1:



Suppose that B is changing with time t according to $B(t) = kt + B_0$, where k and B_0 are positive constants.

found $\mathcal{E} = -kA < 0$ here $\Rightarrow$ current flows in "− direction"

note: $\vec{B}_{ind}$ opposes change in $\vec{B}_{ext}$

What is the direction of the induced current?

A. Clockwise

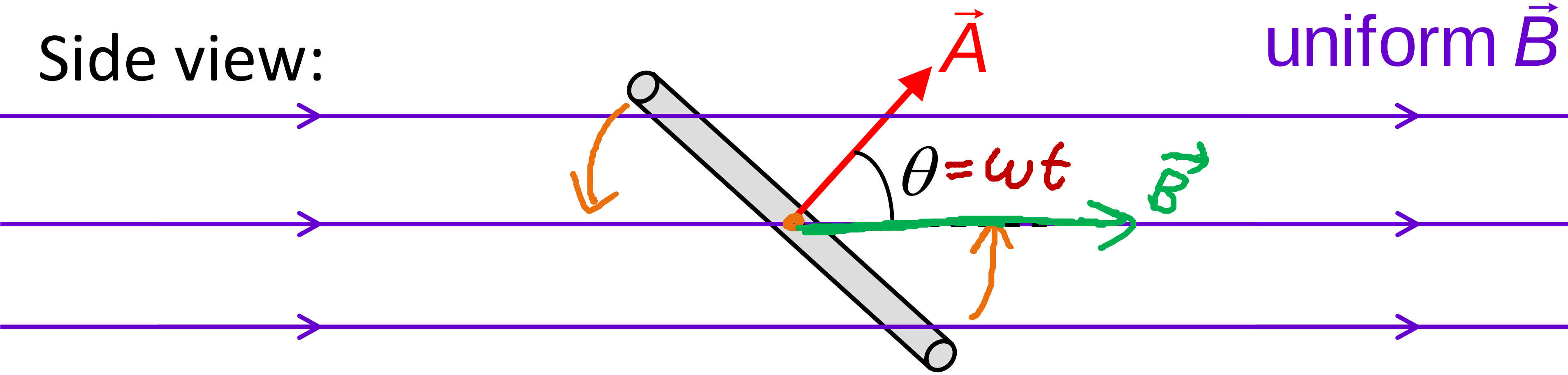
B. Counterclockwise

C. There is no induced current

D. Can't tell

Example 2: Wire loop rotating counterclockwise with constant angular speed ω in a uniform magnetic field:

Side view:



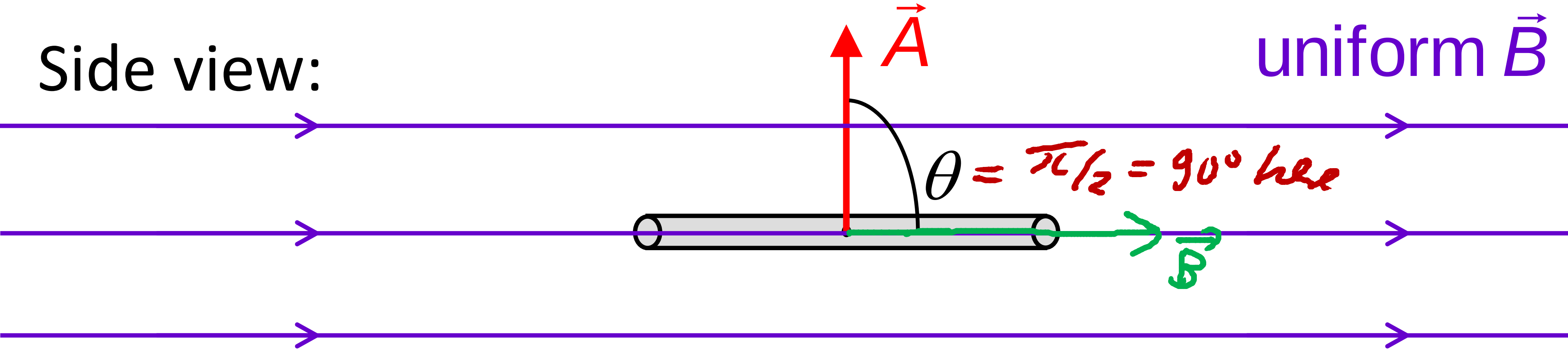
At time $t = 0$, $\theta = 0$. What is the magnetic flux Φ_B through the loop at some time $t > 0$?

$$\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta = BA\cos(\omega t), \text{ since } \theta = \omega t$$
$$= \Phi_B(t)$$

A. $BA\sin(\omega t)$ B. $BA\cos(\omega t)$ C. *0*

Example 2: Wire loop rotating counterclockwise with constant angular speed ω in a uniform magnetic field:

Side view:



At the instant shown above, what is the emf $\mathcal{E}$ induced in the loop?

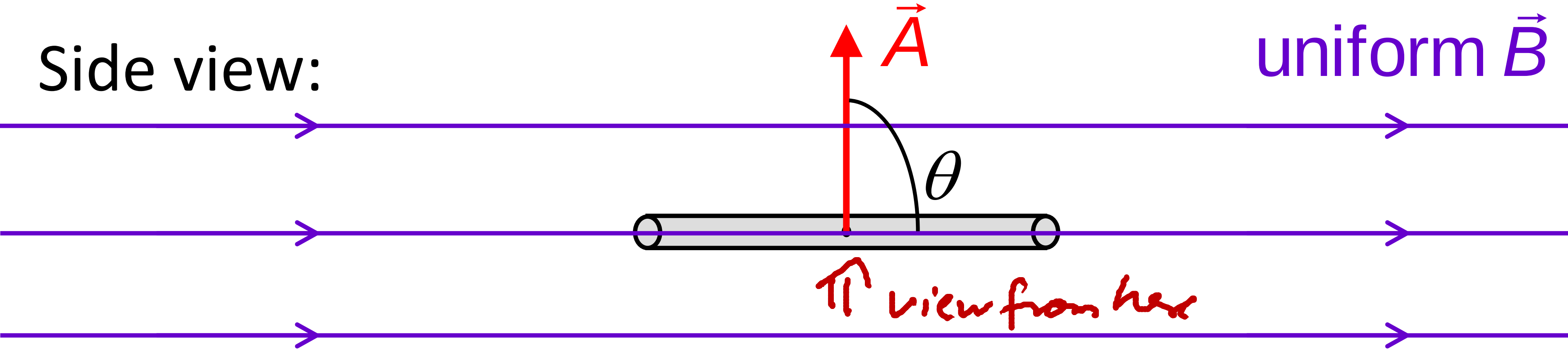
found $\Phi_B(t) = BA\cos(\omega t) \Rightarrow \dfrac{d\Phi_B}{dt} = -\omega BA\sin(\omega t)$

$\Rightarrow \mathcal{E} = -\dfrac{d\Phi_B}{dt} = \omega BA \underbrace{\sin(\omega t)}_{=1 \text{ here}} \Rightarrow$ for $\theta = \omega t = \dfrac{\pi}{2} \Rightarrow \boxed{\mathcal{E} = \omega BA}$

~~A. 0~~ | B. ωBA | ~~C. $-\omega BA$~~ | D. Can't tell?

Example 2: Wire loop rotating counterclockwise with constant angular speed ω in a uniform magnetic field:

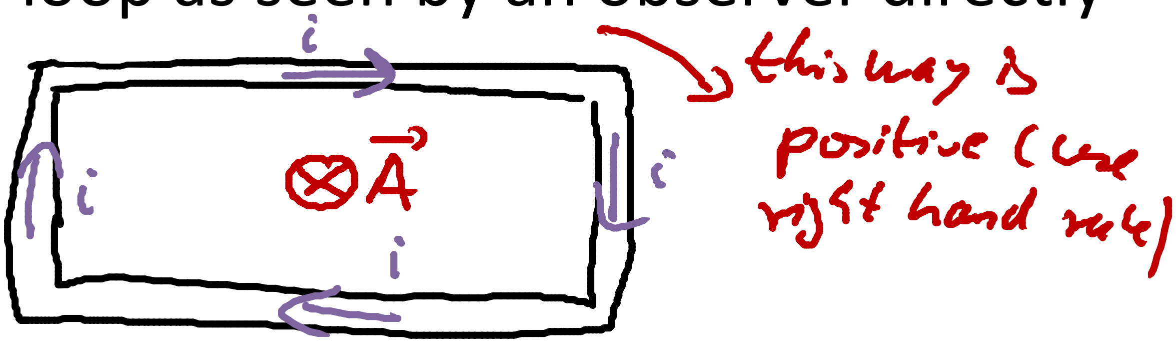
Side view:



At the instant shown above, what is the direction of the induced current in the loop as seen by an observer directly below the loop?

from above:

$\mathcal{E} > 0$ for $\theta = \frac{\pi}{2}$



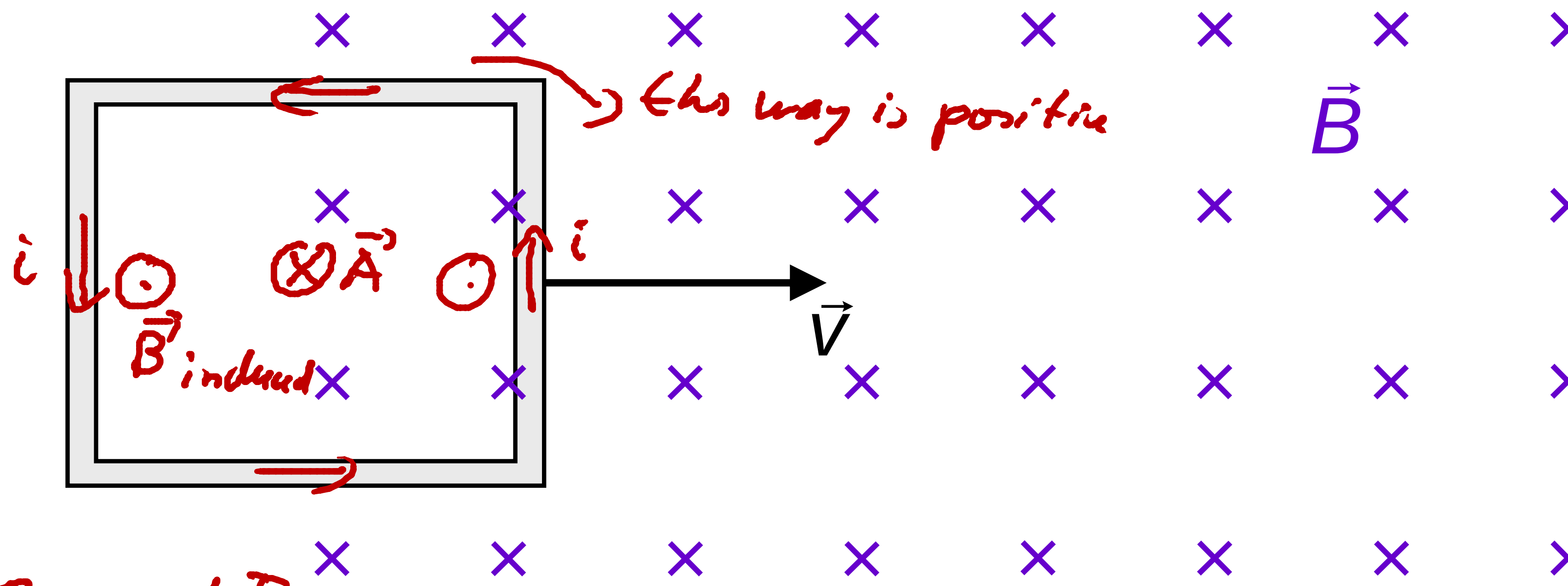
A. Clockwise

B. Counterclockwise

C. There is no induced current

D. Can't tell

Example 3:



$$\mathcal{E} = -d\Phi_B/dt \underline{< 0}$$

At the instant shown above, what is the **direction of the induced current** in the metal loop?

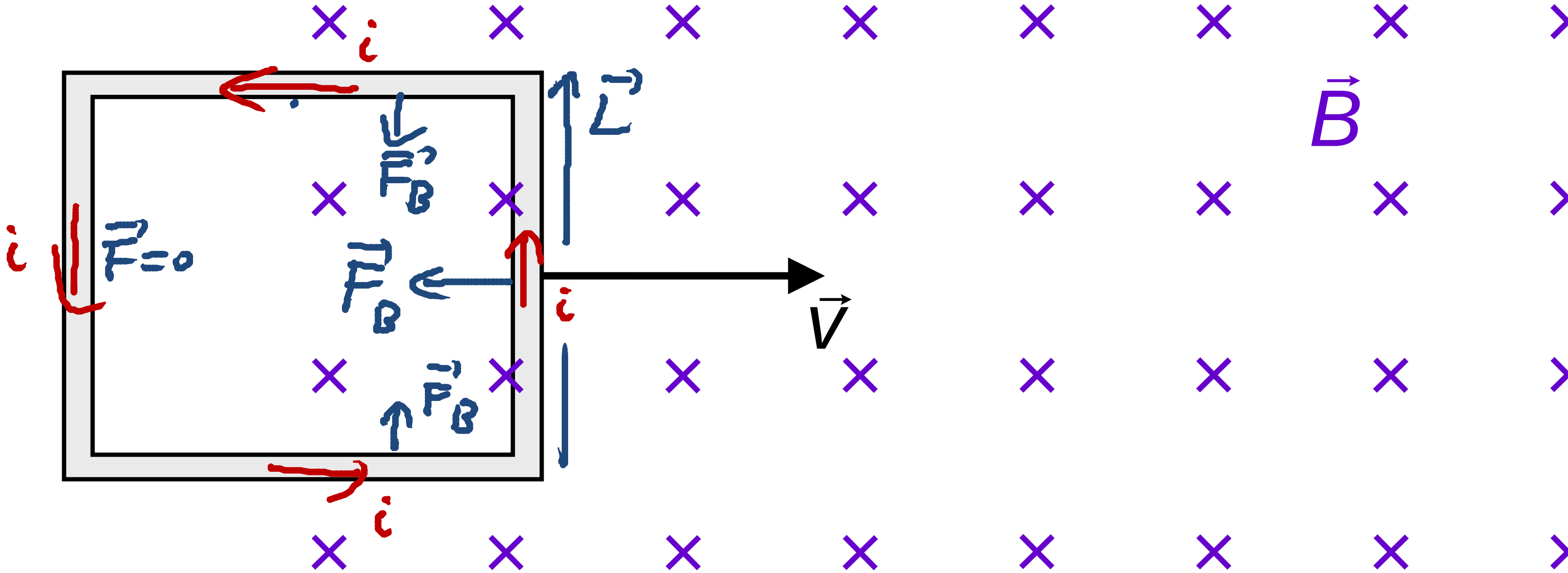
A. Clockwise

B. Counterclockwise

C. There is no induced current

D. Can't tell

Example 3:



force on wire in $\vec{B}$-field: $\vec{F}_{wire} = i\,\vec{L}\times\vec{B}$

← points along direction of current

At the instant shown above, what is the **direction of the net magnetic force** on the metal loop?

A. ← opposes! B. → C. ↑ D. ↓

~~E.~~ The net magnetic force on the loop is zero.

Inductors and Inductance L:

↝ Recall: Capacitors and Capacitance

- Capacitor: produces electric field between two plates
  - Symbol: 
  - Described by capacitance: $C = \frac{Q}{\Delta V_c}$ } depends on geometry of capacitor
  - Energy of electric field in capacitor: $U = \frac{1}{2} C \Delta V_c^2$

↝ Now:

- Inductors: produce magnetic field around current carrying wire
- Symbol: 
- Described by inductance L