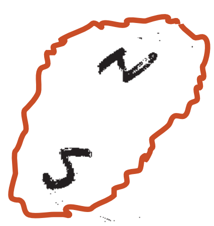
# MAGNETISM

**2000 years ago :**
**(Natural magnets)**

**Greeks were aware that "magnetite" stones attract pieces of iron**

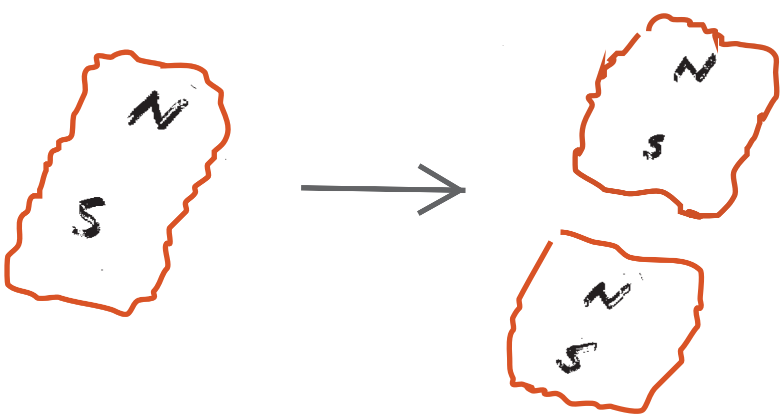
**1269, Pierre de Maricourt:**





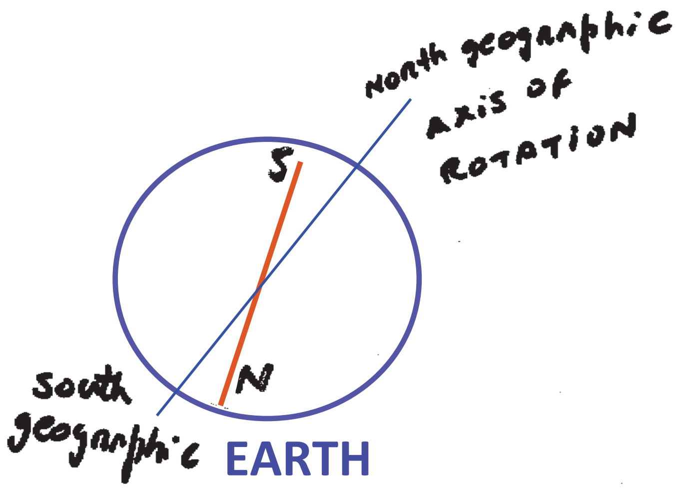
**Every magnet has 2 poles**

**Magnetic poles always occur in pairs**




**So far there is not conclisive evidence of the existence of isolated magnetic poles (MONOPOLE)**

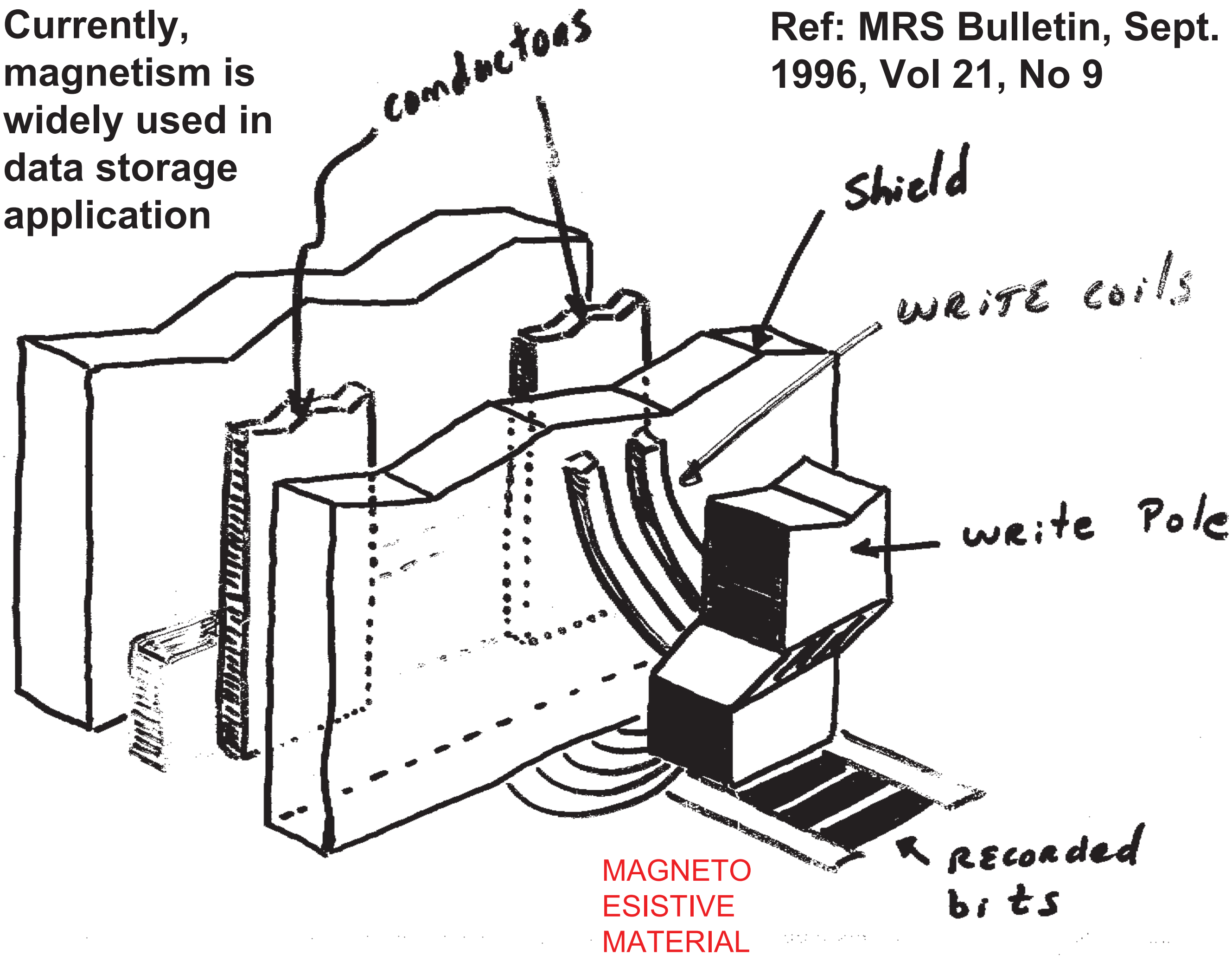
1600, Wialliam Gilbert




The EARTH itself is a natural magnet

Currently, magnetism is widely used in data storage application

Ref: MRS Bulletin, Sept. 1996, Vol 21, No 9

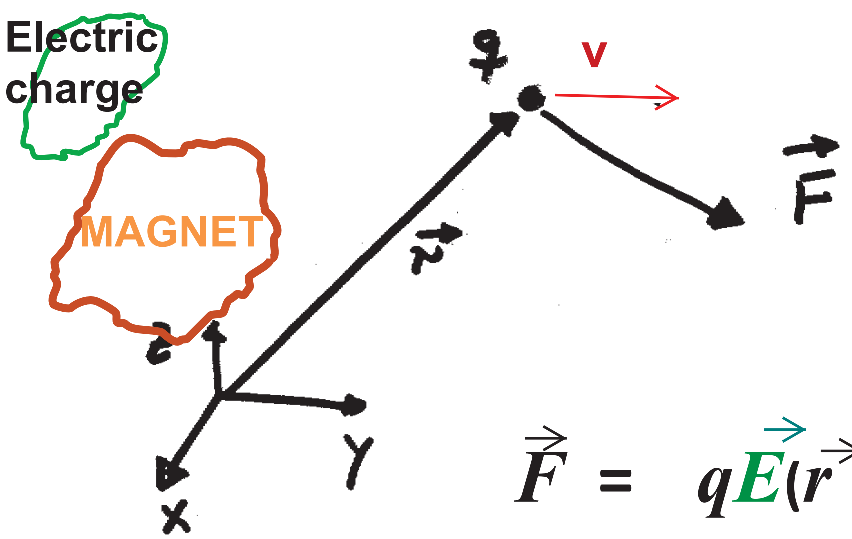


# THE MAGNETIC FIELD: $\vec{B}$

The force on an electric charge $q$ depends

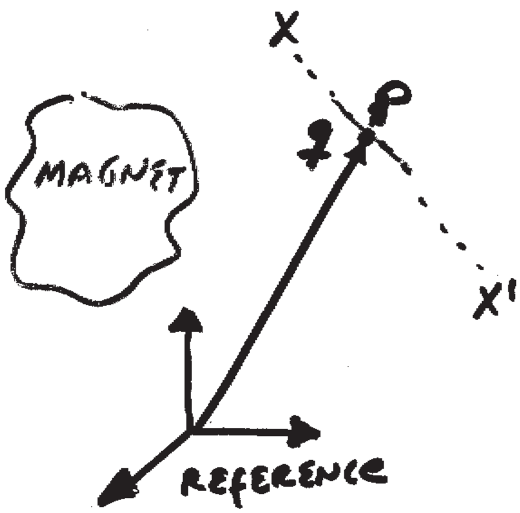
↳ Not only on WHERE IT is: $(\vec{r})$

↳ but also on how is it moving $(\vec{v})$




$$\vec{F} = q\vec{E}(\vec{r}) + \underbrace{q\,\vec{v} \times \vec{B}(\vec{r})}_{\text{Force that depends on the velocity of the charge}}$$

# Definition of the magnetic field $\vec{B}$




For each arbitrary position "P" it occurs the following:

a) When a charges passes through "P", no matter its velocity, it experiences a force perpendicular to the line XX', except when the velocity is parallel to XX'.

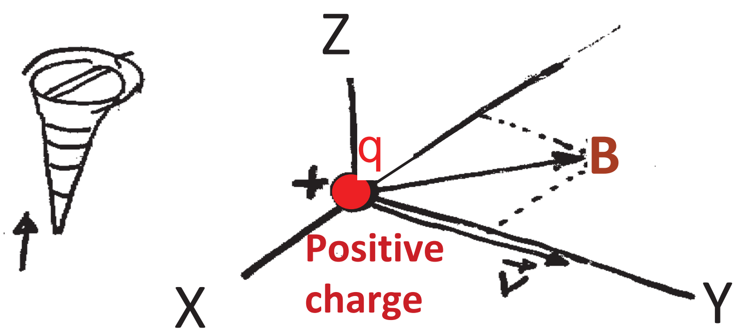
b) But when q is stationary at "P" it experiences no force

Conclusion:

XX' defines the direction of the magnetic field at the point P.

$$\vec{F}_B = q\,\vec{v} \times \vec{B}$$ Definition of the magnetic field B

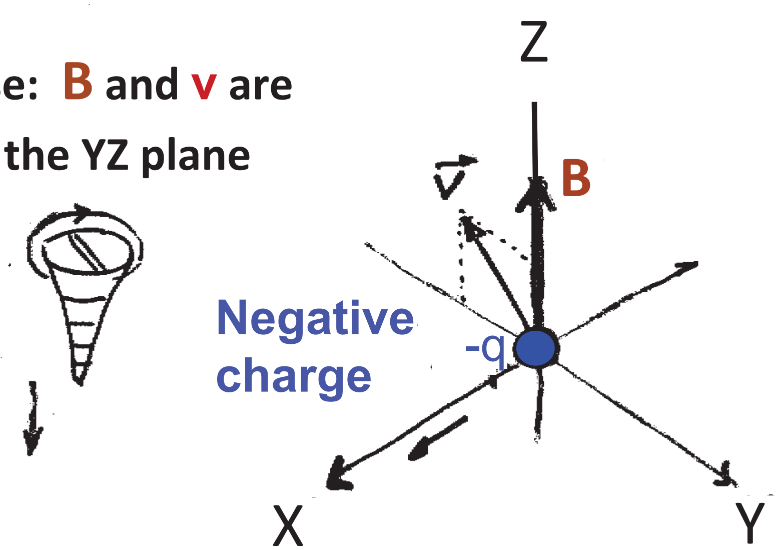
Case: **B** and **v** are in the XY plane




Answer:

Force = F (0, 0,1 )

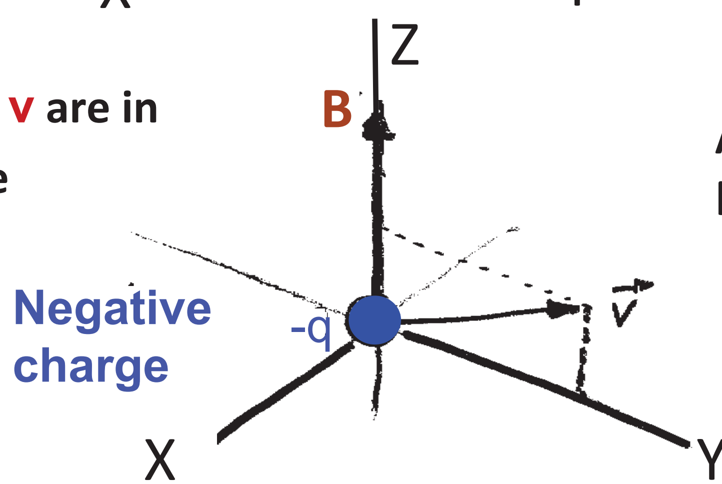
Case: **B** and **v** are in the YZ plane




Answer:

Force = F (1, 0,0 )

Case: **B** and **v** are in the YZ plane

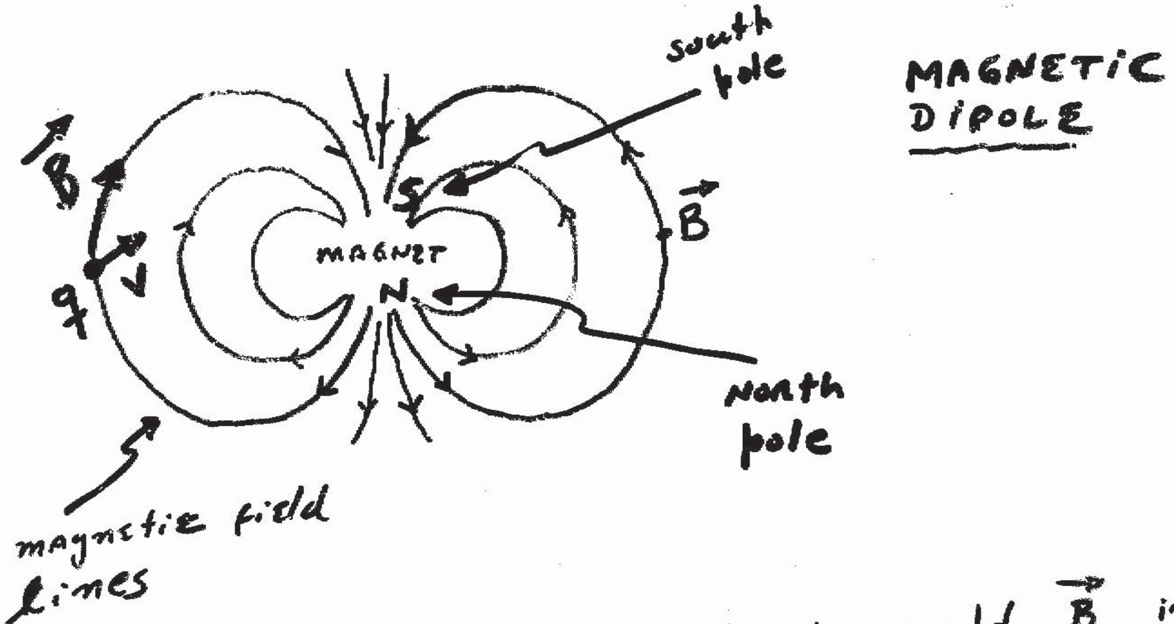



Answer:

Force = F (-1, 0, 0 )

$$\vec{F} = q\,\vec{v} \times \vec{B}$$

# Magnetic field lines




A magnet produces a magnetic field $\vec{B}$ in its surrounding region.

A charge q, moving with velocity $\vec{v}$ passing by a region where it exists a magnetic field $\vec{B}$, experiences a force $\vec{F} = q\, \vec{v} \times \vec{B}$

indicates vector product

F (Newtons) v (m/s)

q (Coulomb) B (Tesla)

**Units**

- **In this chapter, we will analyze those situation in which the magnetic B is given (without worrying about how it is generated).**

- **In the next chapter we will learn how to calculate the magnetic filed produced by currents flowing along a line or along a ring.**

**We start with the simplest case to analyze: the motion of a point charge moving across a uniform magnetic field.**

## Motion of a point charges q in a uniform magnetic field

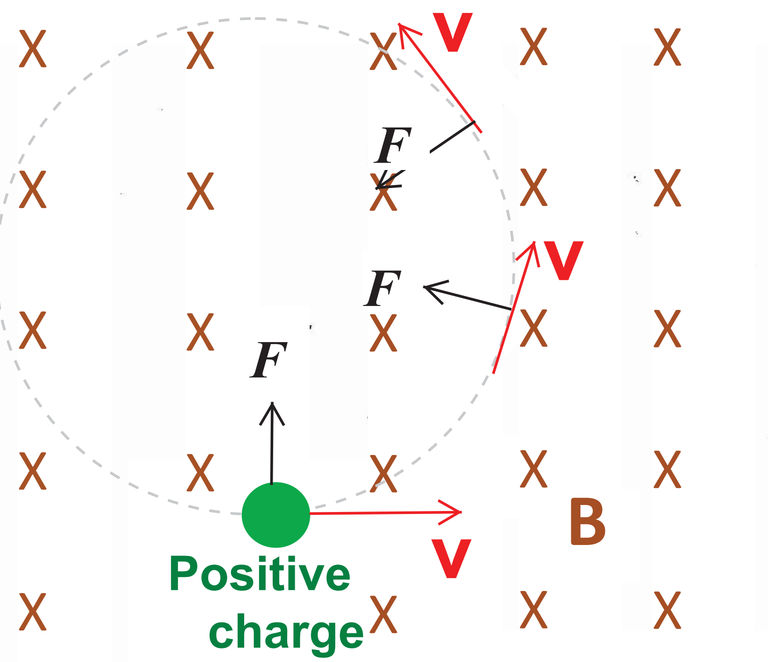
$$\vec{F} = q\, \vec{v} \times \vec{B}$$

$\vec{F}$ Perpendicular to $\vec{v}$

Hence,

$\vec{F}$ changes the direction of $\vec{v}$ but not its magnitude

That is, the kinetic energy of the charge does not change. Accordingly,




The X indicates that B is perpendicular to the plane of this page, oriented into the page.

**Magnetic fields do not do work on the charged particles**

$F = q\,v\,B$ (1)

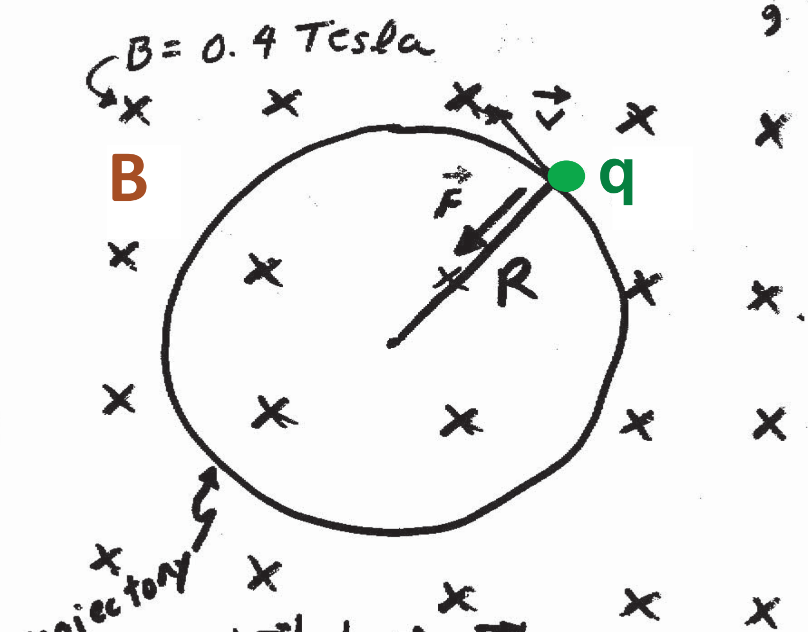
q is a proton

$q = 1.6 \times 10^{-19}\,C$

$R = 0.21\,m$ (radius)

$B = 0.4$ Tesla

$v = ?$




$$|\vec{F}| = |q\,\vec{v} \times \vec{B}| = q\,v\,B \underbrace{\sin 90°}_{=1}$$

**If q is undergoing circular motion, the magnetic force must be responsible for the centripetal acceleration**

$F = m\dfrac{v^2}{R}$ (2)

From (1) and (2) we obtain

$$q\,v\,B = m\frac{v^2}{R} \Rightarrow \boxed{v = \frac{q}{m} B R}$$

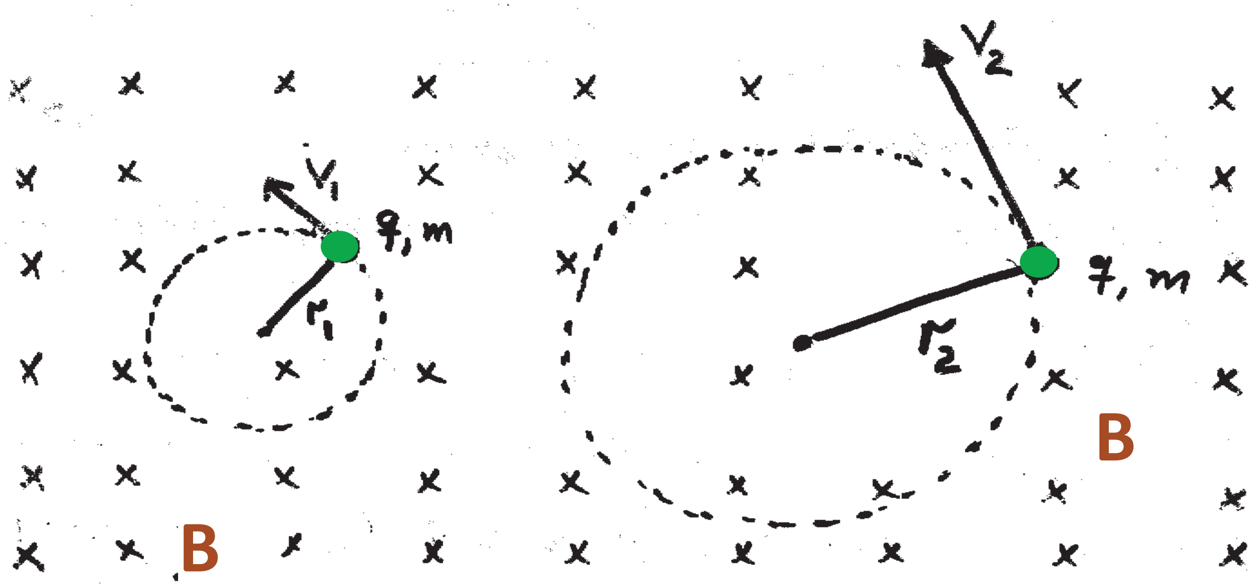
$v = 8 \times 10^6$ m/s

use $m = 1.67 \times 10^{-27}$ Kg

the result

$$r = \frac{1}{(q/m)\ B} v$$

indicates that, for a given charged particle and a fixed value of the magnetic field, particles moving at higher speed describe circular paths of bigger radii




$$r_1 < r_2$$

$$v_1 < v_2$$

How long does the proton take to complete ONE revolution (from the previous example)

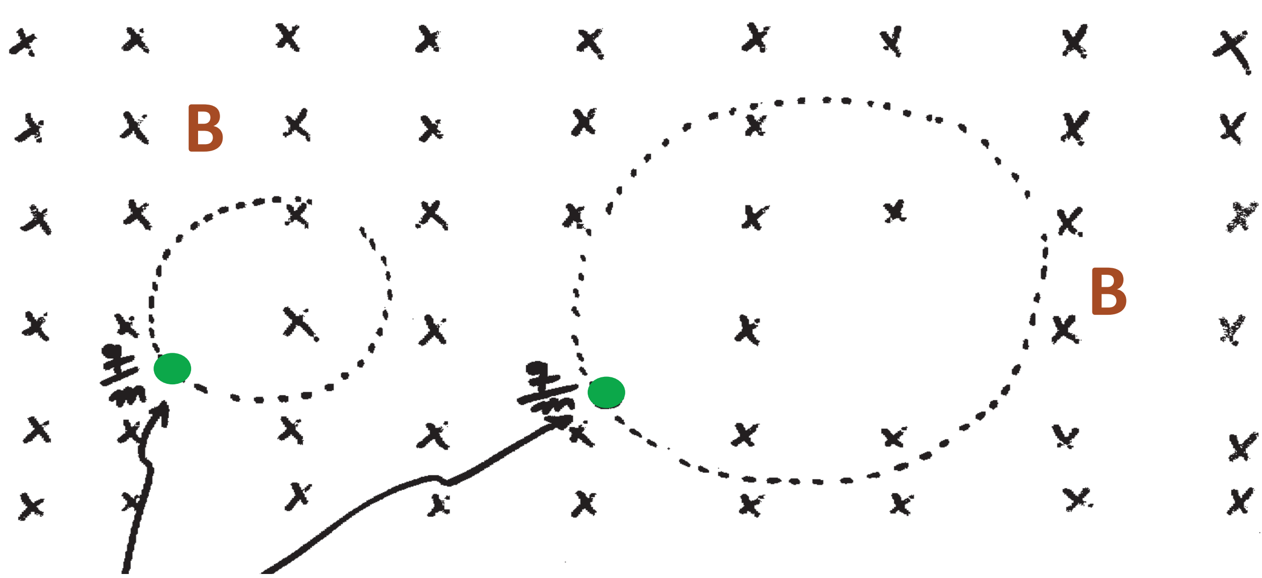
$$T = \frac{\text{length}}{\text{velocity}} = \frac{2\pi R}{v}$$

period

$$= \frac{2\pi R}{\frac{q}{m} B R} = \boxed{2\pi \frac{1}{\frac{q}{m} B} = T}$$

**Period of the circular orbit**

Notice: $T$ is independent of the radius.




**Both complete one orbit in the same amount of time T**

How many turns does the proton make in 1 sec?
In other words what is the frequency $f$?

$$f = \frac{1}{T}$$

$$\boxed{f = \frac{1}{2\pi} \frac{q}{m} B}$$ frequency

People typically use angular frequency $\omega$:

$$\omega = \frac{\text{radians}}{\text{second}} = \frac{\#\text{ revolutions}}{\text{sec}} \times 2\pi \text{ rad}$$

$$\boxed{\omega = \frac{q}{m} B}$$

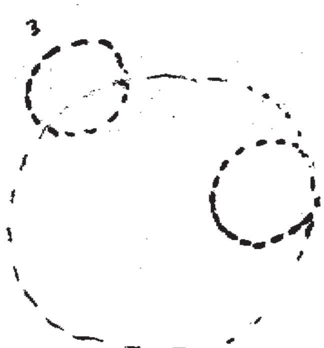
Notice: For a given q/m

$$\frac{v}{R} = \frac{q}{m} B$$

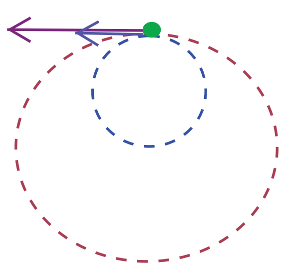
For a constant magnetic field:
The higher the speed, the larger the radius

$$\frac{1}{T} = f = \frac{1}{2\pi} \frac{q}{m} B$$

For a constant magnetic field,
T and f are independent of the speed



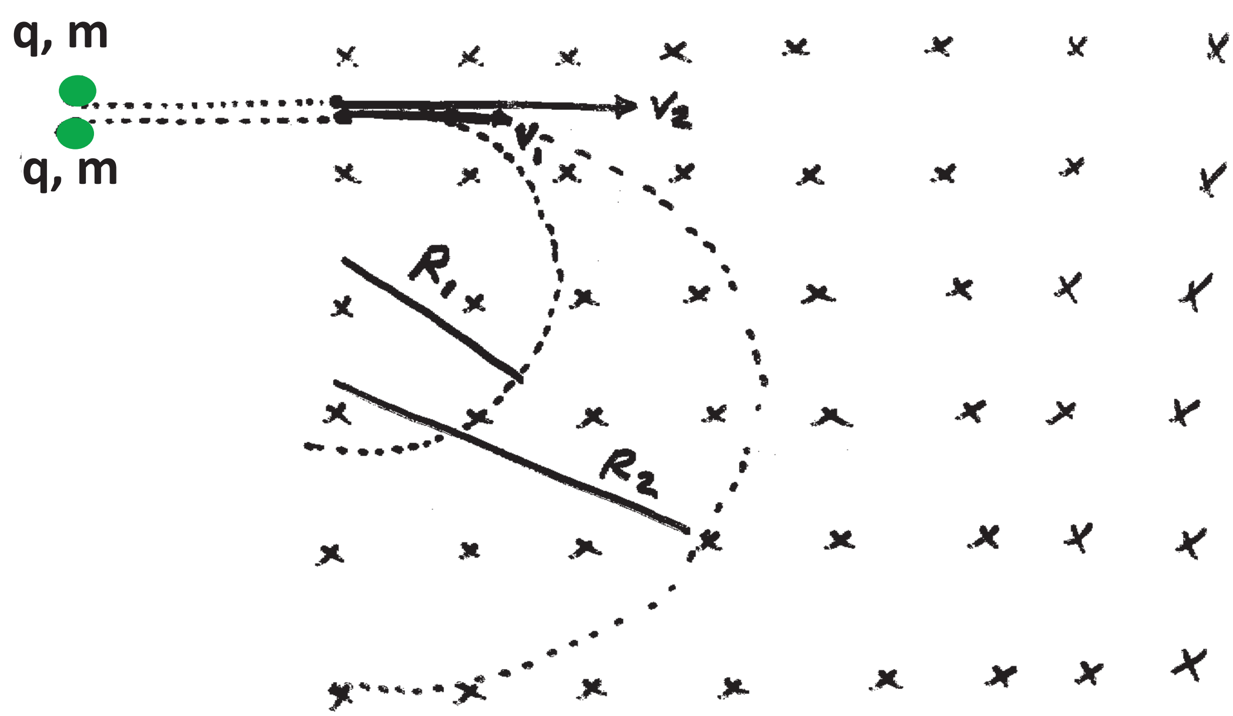
Particles with higher speed move in large circles,
slower particles move in smaller circles



All particles with the same q/m
take the same time to complete one revolution

## For constant magnetic field (and same q/m):

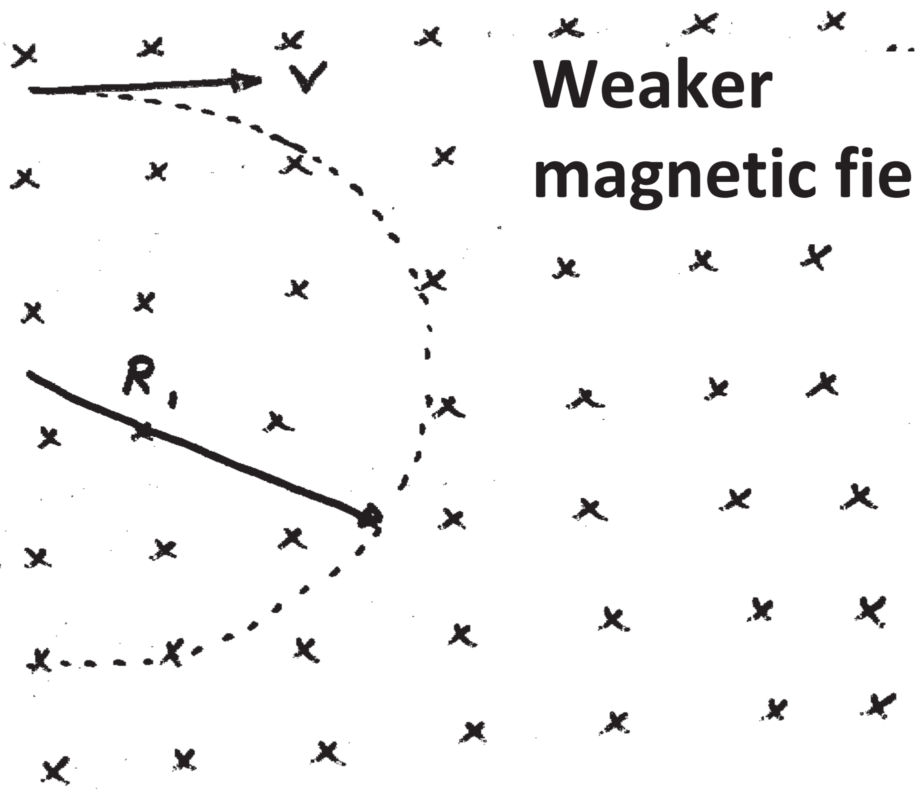
$$v/R = (q/m)\ B$$




- B does not control $\vec{v}$
- changing B does not change $|\vec{v}|$
  (the speed $v = |\vec{v}|$ remains the same)

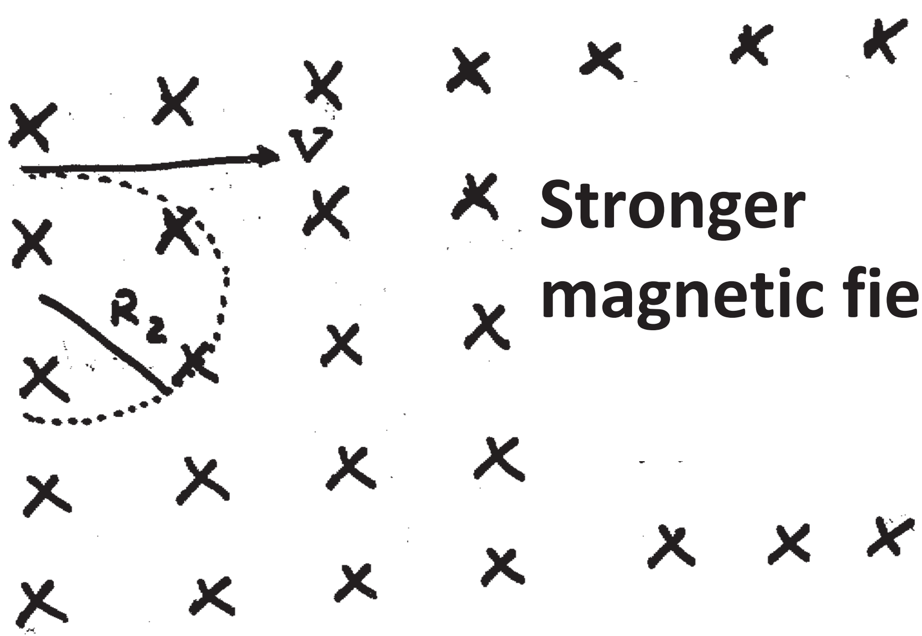
## For constant speed (and same q/m):

$$v = (q/m)\ R\ B$$



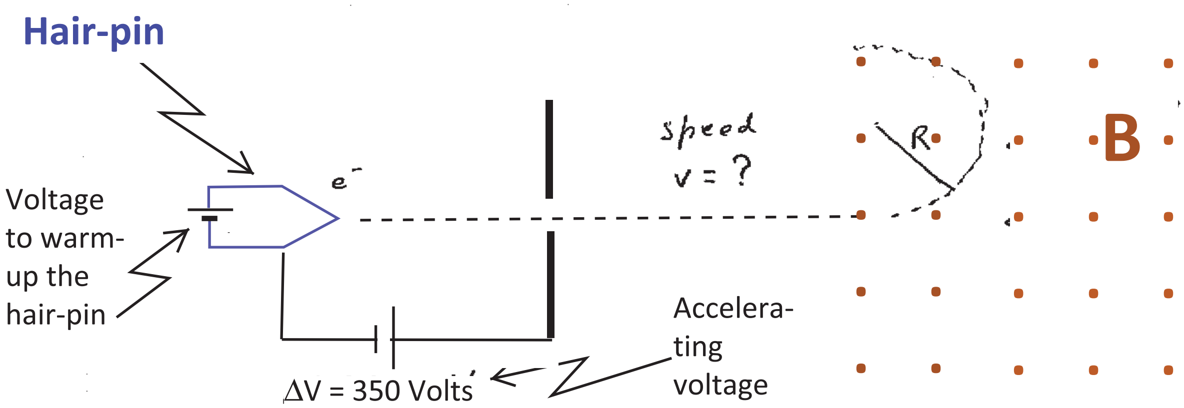

Weaker magnetic field $B_1$

$B_1 < B_2$ implies $R_1 > R_2$




Stronger magnetic field $B_2$

**Example.** Electrons are extracted from a metallic hair-pin source, and accelerated by a 350 Volts. Subsequently the electrons enter a region where there exists a magnetic field. Calculate the radius of the electrons trajectory when inside the region of magnetic field.




$B = 200\ mT$
$= 200 \times 10^{-3} T$
$= 0.2 T$

Work done by the electrical force acting on the electron = Change in the electron's kinetic energy

$$e\Delta V = \frac{1}{2} m v^2$$

$$v^2 = \frac{2 e \Delta V}{m} = \frac{2 \times 1.6 \times 10^{-19} C \times 350\ volts}{9.1 \times 10^{-31}\ Kg} =$$

$$= \frac{2 \times 1.6 \times 3.5 \times 10^{14}}{9.1} = 1.23 \times 10^{14} \frac{m}{s}$$

$$\boxed{v = 1.11 \times 10^{7}\ m/s}$$

$$\frac{m v^2}{R} = e v B$$

$$\frac{m v}{e R} = B \quad \text{or} \quad \boxed{\frac{m v}{e B} = R}$$

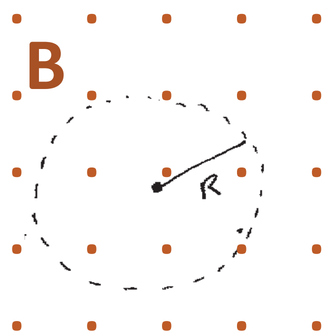
$$R = \frac{9.1 \times 10^{-31} Kg \times 1.11 \times 10^{7} m/s}{1.6 \times 10^{-19} C \times 0.2}$$

$$= \frac{9.1 \times 1.1}{1.6 \times 0.2} \times 10^{-5}\ m$$

$$= 31.55 \times 10^{-5} m$$

$$\boxed{R = 315.5 \times 10^{-6}\ m}$$

**Example.** Electrons undergo circular motion (radius R= 0.25 m) inside a region where there exists a uniform magnetic field. The kinetic energy of the electrons is $K$= 1.2 x 10$^3$ electron-volts. Calculate the magnitude of the magnetic field. Calculate also the frequency of the motion.




$R = 25\ \text{cm} = 0.25\ \text{m}$

$K = 1.2\ \text{keV} = 1.2 \times 10^{3}\ \text{eV} = 1.2 \times 10^{3} \times 1.6 \times 10^{-19}\ \text{J}$

$1\ \text{eV} = 1.6 \times 10^{-19}\ \text{Joules}$

$\boxed{K = 1.92 \times 10^{-16}\ \text{Joules}}$

**a)** $\frac{1}{2} m v^2 = K$

$$v^2 = \frac{2K}{m} = \frac{2 \times 1.92 \times 10^{-16}\ \text{Joules}}{9.1 \times 10^{-31}\ \text{Kg}} = 0.42 \times 10^{15} = 4.2 \times 10^{14}$$

$\boxed{v = 2.05 \times 10^{7}\ \text{m/s}}$

**b)**
$$B = \frac{mv}{eR} = \frac{9.1 \times 10^{-31}\ \text{Kg} \times 2.05 \times 10^{7}\ \text{m/s}}{1.6 \times 10^{-19}\ \text{C} \times 0.25} = \frac{9.1 \times 2.05}{1.6 \times 0.25} \times 10^{-5}\ \text{T}$$

$\boxed{B = 4.6 \times 10^{-4}\ \text{Tesla} = 4.6\ \text{Gauss}}$

Equivalence

$1\ \text{Tesla} = 10^{4}\ \text{Gauss}$

**c)** frequency of circling

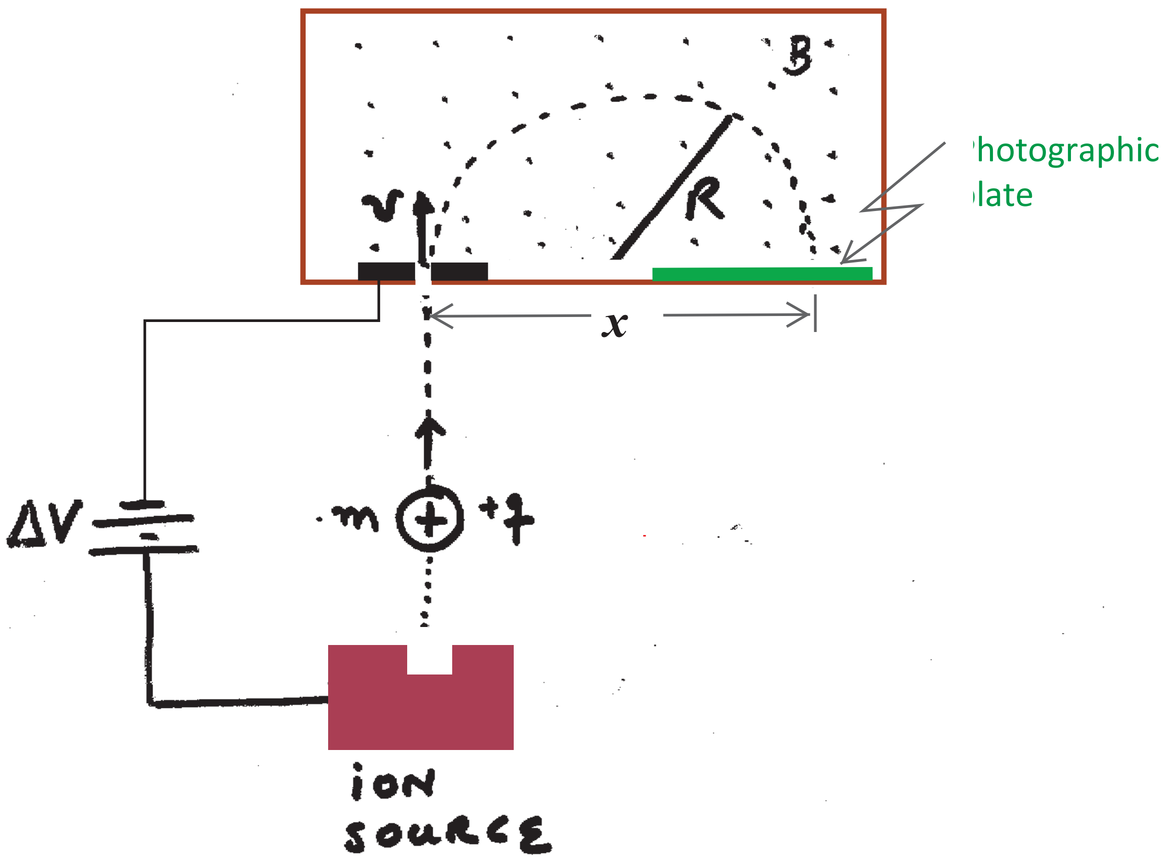
$$f = \frac{1}{2\pi} \frac{e}{m} B = \frac{1}{2\pi} \frac{1.6 \times 10^{-19}\ \text{C}}{9.1 \times 10^{-31}\ \text{Kg}} 4.6 \times 10^{-4}\ \text{T} = 0.129 \times 10^{8}$$

$f = 12.9 \times 10^{6}\ \text{Hz}$ or $\boxed{f \approx 13\ \text{MHz}}$

**d)**
$$T = \frac{1}{f} = \frac{1}{12.9 \times 10^{6}\ \text{Hz}} = 77 \times 10^{-9}\ \text{sec}$$

$= \boxed{77\ \text{ns} = T\ \text{period}}$

# The Mass Spectrometer




$$q\,\Delta V = \frac{1}{2} m v^2 \qquad \frac{m v^2}{R} = q v B$$

$$v = \sqrt{\frac{2 q \Delta V}{m}} \qquad R = \frac{m v}{q B} = \frac{x}{2}$$

$$x = 2\frac{m}{qB}v$$

$$= 2\frac{m}{qB}\sqrt{\frac{2q\,\Delta V}{m}} = \sqrt{\frac{8m\,\Delta V}{q\,B^2}}$$

$$\boxed{\frac{m}{q} = \frac{B^2}{8\,\Delta V}x^2}$$

Isotopes of different mass $m$ (same $q$) will strike the photographic plate at different values of $x$.

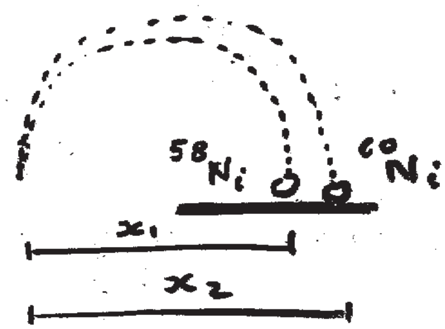
<u>Example</u> A $^{58}Ni$ ion of charge $+e$ and mass $m = 9.62 \times 10^{-26}$ kg is accelerated through a potential difference of 3 kV and deflected in a magnetic field of 0.12 T

a) Find the radius of curvature of the orbit of the ion.

$$\frac{9.62 \times 10^{-26}\,\text{kg}}{1.6 \times 10^{-19}\,\text{C}} = \frac{(0.12\,\text{T})^2}{8 \times 3 \times 10^3\,\text{V}}x^2 \Rightarrow \begin{array}{l} x = 1.0020\ \text{m} \\ R = 0.5010\ \text{m} \end{array}$$

b) Find the difference in the radii of curvature of $^{58}Ni$ and $^{60}Ni$

$$\frac{m_1}{m_2} = \frac{x_1^2}{x_2^2}$$



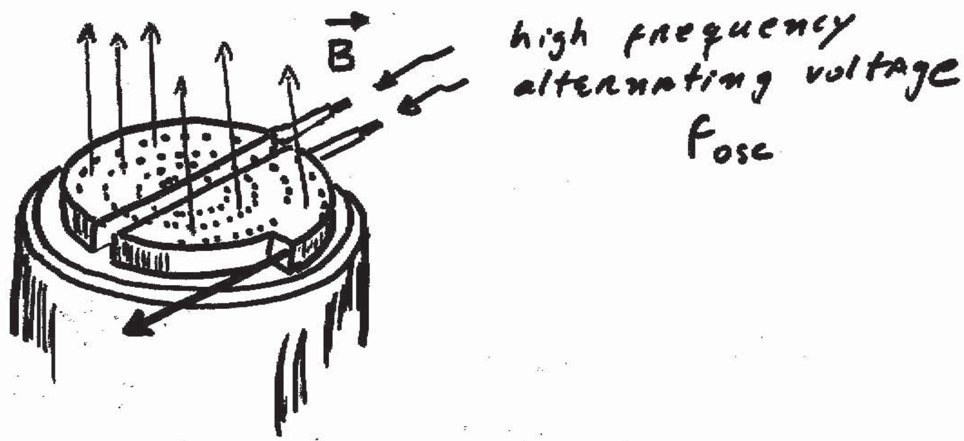

we take $\frac{m_1}{m_2} = \frac{58}{60}$

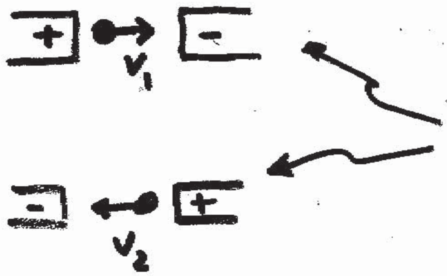
$$\Rightarrow \frac{x_1}{x_2} = 0.983 = \frac{R_1}{R_2}$$

Since $R_1 = 0.5010\ m$

then $R_2 = 0.5095 m \Rightarrow$ $R_2 - R_1 = 8.56\ mm$

# The Cyclotron




the polarity of the electrodes is changed in a synchronized way with the motion of the particle

We already know that, given a particle of mass $m$ and charge $q$, it will circle inside a uniform magnetic field with a frequency $f = \frac{1}{2\pi} \frac{q}{m} B$

regardless of the particle's speed

So, in a cyclotron the alternating voltage (see previous figure) is tuned until $f_{osc} = \frac{1}{2\pi} \frac{q}{m} B$

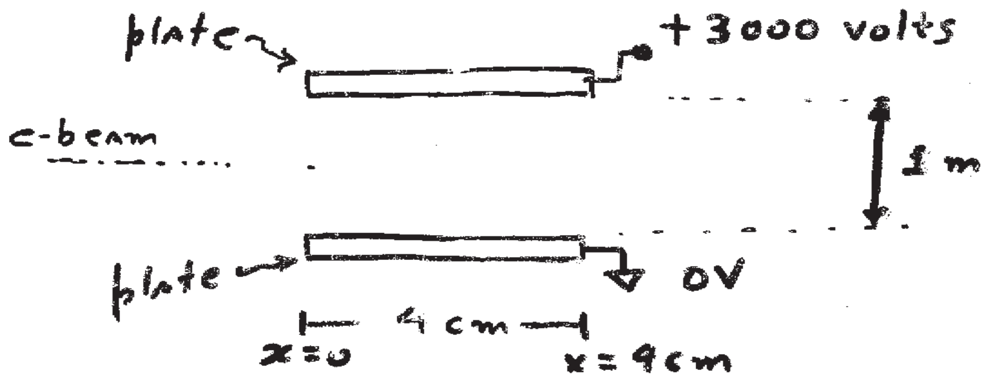
<u>Example</u> A cyclotron uses a magnetic field $B = 1.5\,T$ and it is going to be used to accelerate protons.

a) What should be the frequency of the alternating voltage?

$$f_{osc} = \frac{1}{2\pi} \frac{1.6 \times 10^{-19} C \times 1.5\,T}{1.67 \times 10^{-27} Kg} = 23\ MHz$$

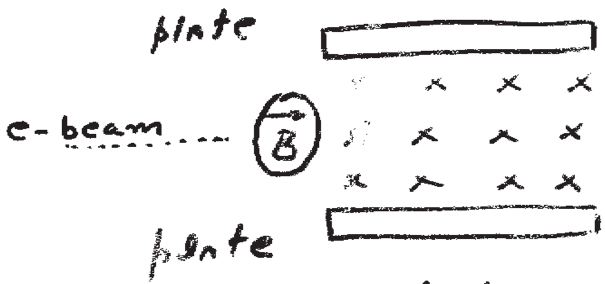
b) If the radius of the D-shape electrodes is 0.5 m, what is the kinetic energy of the protons when they emerge?

EXERCISE: DRAW schematically the trajectory followed by the indicated electron beam

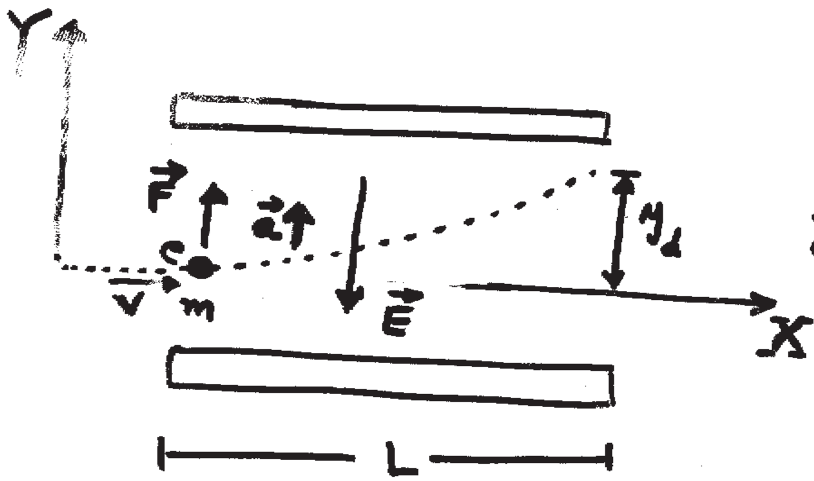



Find the position of the beam when it exists the plates (at x = 4 cm)

EXERCISE: DRAW schematically the trajectory followed by the electron beam




(NO electric field are applied in this case)




$$\vec{F} = e\,\vec{E}$$

$$\vec{a} = \frac{\vec{F}}{m} = \frac{e}{m}\vec{E}$$

$$y = \frac{1}{2} a t^2$$

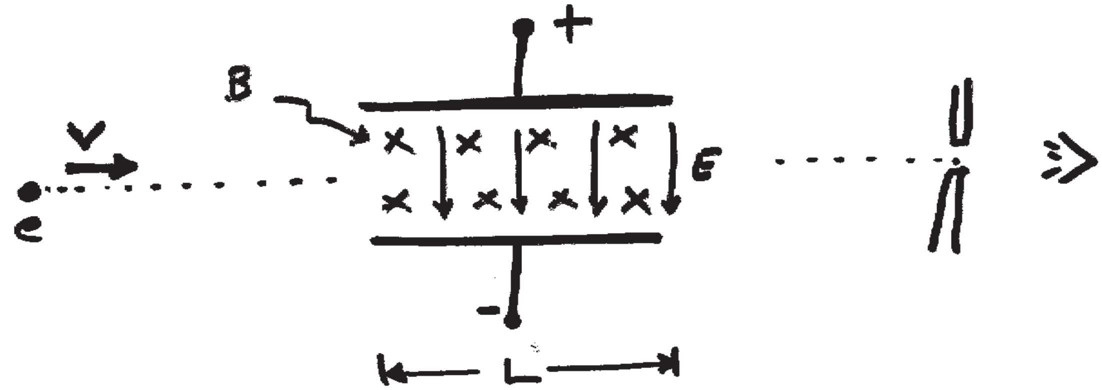
How much does the electron deflect after passing the plates? $y_d = ?$

Answer: We obtain $y_d$ when $t = \frac{L}{v}$

$$y_d = \frac{1}{2} a \left(\frac{L}{v}\right)^2$$

$$\boxed{y_d = \frac{1}{2}\frac{e}{m} E \frac{L^2}{v^2}}$$

# Discovery of the electron




- the strength of the magnetic field is increased and adjusted until the incident particle does not experience any vertical deflection.

No deflection implies:

$$eE = evB$$

$$v = \frac{E}{B} \qquad (1)$$

- When the magnetic field is turned off the particle is deflected vertically by a distance $y_d$ whose relationship with $v$ is

$$v^2 = \left(\frac{e}{m}\right) \frac{E}{2} \frac{L^2}{y_d} \qquad (2)$$

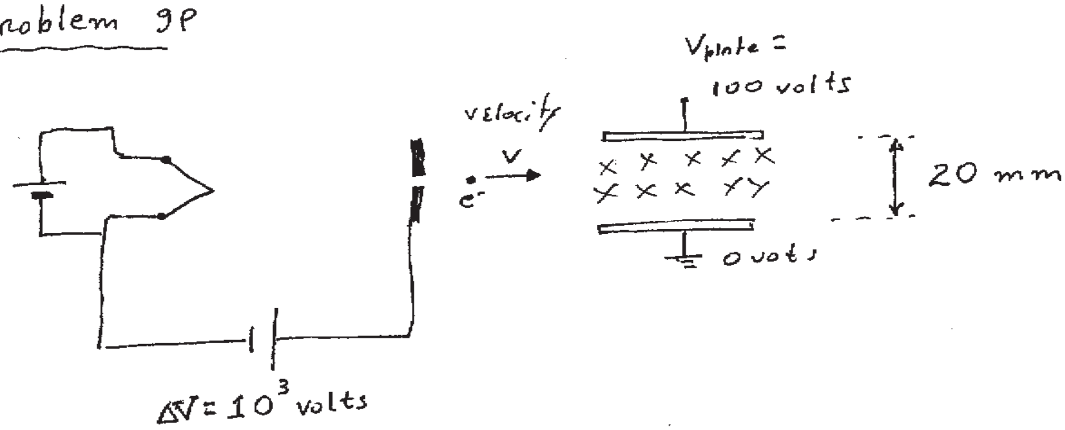
From experiments ① and ② we obtain

$$\frac{E^2}{B^2} = \left(\frac{e}{m}\right)\frac{E}{2}\,\frac{L^2}{y_d}$$

$$\Rightarrow \boxed{\frac{e}{m} = \frac{B^2}{E}\,\frac{L^2}{2y_d}}$$

J. J. Thomson
1897

Problem 9P




Let's find first the velocity:

change in electric potential = change in kinetic energy

$$e \times \Delta V = \frac{1}{2} m v^2$$

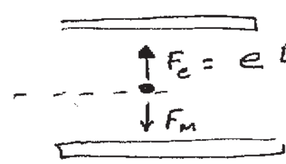
$$1.6 \times 10^{-19} C \times 10^3 \text{ volts} = \ldots$$

$$\boxed{v = \sqrt{\frac{2 e \Delta V}{m}}}$$

$$v = \sqrt{\frac{2 \times 1.6 \times 10^{-19} C \times 10^3 \text{ volts}}{9.1 \times 10^{-31} \text{ Kg}}}$$

$$= \sqrt{\frac{32}{9.1} \times 10^{14}}$$

$$= 1.87 \times 10^7 \text{ m/s}$$




$$F_e = eE = e \frac{V_{plate}}{d}$$ electric force

$$F_m = e v B$$

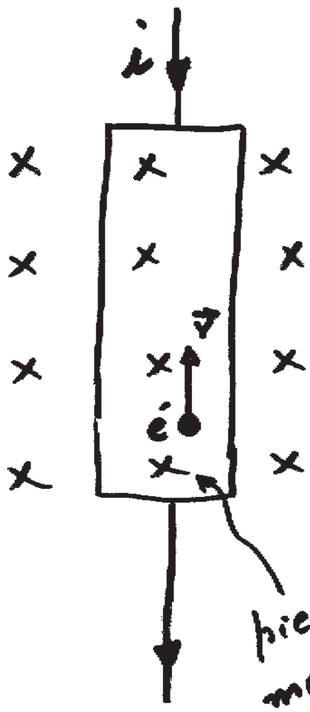
electron travels along a straight line $\Rightarrow$ $F_e = F_m$

$$\cancel{e} \frac{V_{plate}}{d} = \cancel{e} v B$$

$$\Rightarrow \boxed{B = \frac{V_{plate}}{d\, v}} = \frac{100}{20 \times 10^{-3} \times 1.87 \times 10^7 \text{ m/s}}$$

$$= 2.67 \times 10^{-4} \text{ T}$$

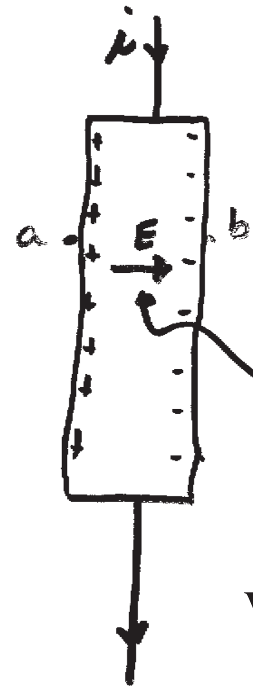
# The Hall Effect




piece of metal

$\vec{F}_{mag} =$

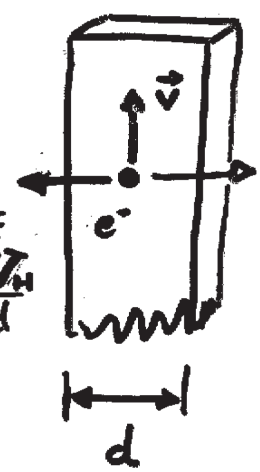
$= -e\vec{v} \times \vec{B}$




a new electric field has been established.

$V_a > V_b$

Let's call $V_a - V_b = V_H$ ← Hall voltage



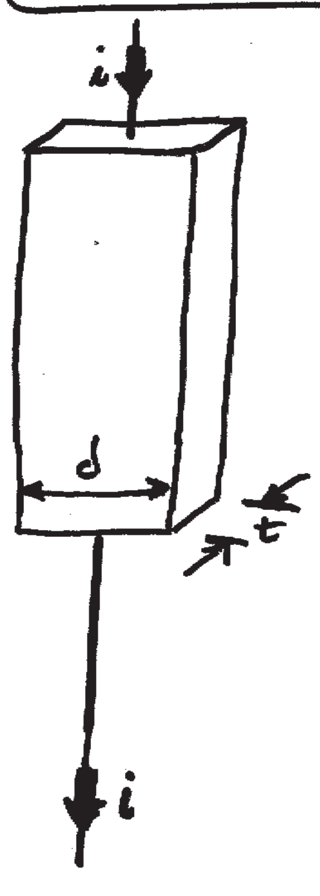

$F_{elect} = eE = e\frac{V_H}{d}$

$F_{mag} = evB$

$$F_{elect} = F_{mag}$$

$$e\frac{V_H}{d} = e v B$$

$$\boxed{v = \frac{V_H}{d B}}$$ drift velocity




# of charge carriers per unit volume

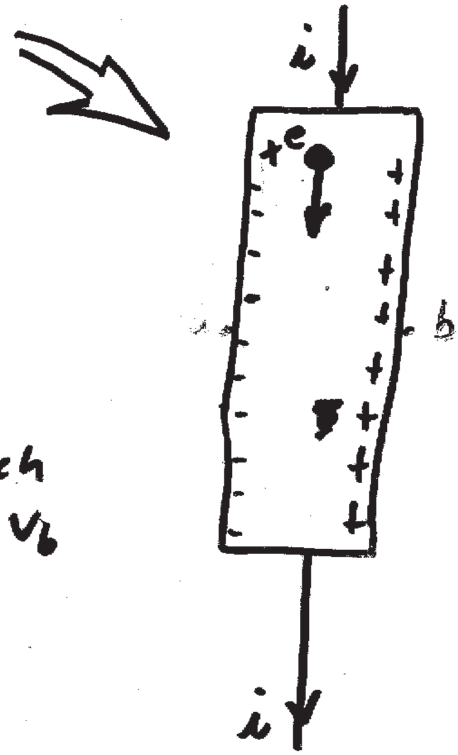
$$\frac{i}{td} = j = n e v$$

$$\frac{i}{td} = ne\frac{V_H}{d B}$$

$$\boxed{V_H = \frac{i}{net} B}$$

The Hall voltage $V_H$ provides a method to measure magnetic fields.

Had we assumed the current $i$ was established by positive charges in motion, we would have obtain the following situation:




$V_a < V_b$

measurable

Since we can measure

$V = V_b - V_a$

we should have been able to determine which voltage, $V_a$ or $V_b$ is higher.

- the situation

  $V_a < V_b$

  has not been observed in METALS, which confirm that the charge carriers in metals are negative charges.

- However, both situations $V_a < V_b$ and $V_a > V_b$ are observed in SEMICONDUCTORS.