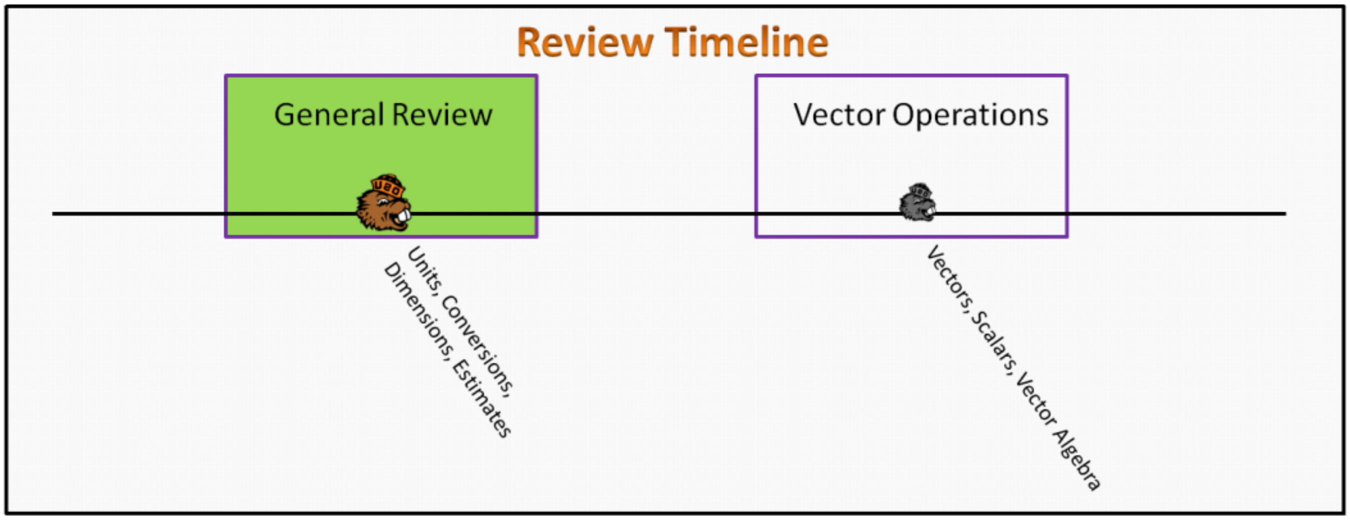
(GR) Ryan's Space

Monday, January 22, 2018 5:44 PM

General Review

Foundation Stage (GR.2)

lecture 1
Units, conversions, dimensions, estimates



Textbook Chapters

- **BoxSand** :: KC videos (Scientific notation ; significant figures ; Units ; Dimensional analysis ; Mathematization ; Basic functions ; proportional reasoning)
- **Giancoli** (Physics Principles with Applications 7th) :: 1-5 ; 1-6 ; 1-7 ; 1-8
- **Knight** (College Physics : A strategic approach 3rd) :: 1.4
- **Knight** (Physics for Scientists and Engineers 4th) :: 1.8

Warm up

GR.2-1:

Description: What is physics?

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: In your own words, what is physics? How is physics different than math? How is physics the same as math?

Selected Learning Objectives

Significant figures

1. Apply a working definition of answering with three significant figures.
2. Show that they must keep more than the number significant figures they intend to answer with during intermediate calculations.
3. Recognize that a complete algebraic solution reduces the error caused by significant figures of intermediate calculations.

Scientific Notation

4. Convert a number into scientific notation.
5. Perform mathematical operations using scientific notation.

6. Use commutative property of algebra, and scientific notation, to perform basic calculations to an order of magnitude in their head.

Dimensional Analysis

7. Identify the dimensions of physical quantities.
8. Perform dimensional analysis on derived expressions.

Units

9. Know the SI units for physical quantities.
10. Differentiate between dimensional analysis and unit analysis.
11. Differentiate between fundamental and derived units.
12. Convert between units.

Order of Magnitude Estimations

13. Differentiate between order of magnitude and "times as much".
14. Perform *back of the envelope* estimations.

Proportional Reasoning

15. Identify which quantities are changing and which are static, including numeric coefficients.
16. Perform proportional reasoning on single variables raised to the first power.
17. Perform proportional reasoning on single variables raised to a power other than one.
18. Perform proportional reasoning on relationships involving more than one variable changing.

Geometry & Trig

19. Differentiate between sin(x) and cos(x).
20. Recognize symmetries, like complementary angles, to simplify analysis.
21. SOHCAHTOA.
22. Apply the Laws of Sines and Cosines.

Algebra

23. Perform basic algebraic manipulations on equations to isolate a desired quantity or quantities.
24. Appreciate the value of keeping equations symbolic throughout the algebraic manipulation.
25. Solve N equations with N unknowns, by finding the value of an unknown from one equation and plugging that value into another equation.
26. Solve N equations with N unknowns where the set has to be solved simultaneously.

Mathematization

27. Translate between the descriptive (words) and the mathematical representation.

Basic Functions

28. Recognize the graphical features of these basic functions: $y = x$, $y = x^2$, $y = x^3$, $y = constant$, $y = 1/x$, $y = 1/x^2$, $y = \sqrt{x}$, $y = sin(x)$, $y = cos(x)$, for both positive and negative values of x.
29. Recognize the graphical features of these basic functions: $y = -x$, $y = -x^2$, $y = -x^3$, $y = -constant$, $y = -1/x$, $y = -1/x^2$, $y = -\sqrt{x}$, $y = -sin(x)$, $y = -cos(x)$, for both positive and negative values of x.

Sense making

30. Check the sign of their quantities makes sense.
31. Check the dimensionality and units of their quantities makes sense.
32. Check the order of magnitude of their quantities makes sense.
33. Check the behavior of a derived equation makes sense, e.g. proportional reasoning.
34. Check the behavior of a derived equation in limiting cases makes sense, e.g. as x goes to 90 degrees in sin(x).
35. Check derived equations, functions, or values, are self-consistent, e.g. check that the slope of a derived position plot matches the values of the given velocity plot.
36. Compare given or derived quantities with known values.

Key Terms

- Units
- Base units
- Derived unit
- Unit conversion
- Dimensions
- Prefix
- Order of magnitude
- Significant figures
- Dimensional analysis
- Unit analysis

- Mathematization
- Proportional reasoning

Key Equations

Base Quantity [Dimension]	SI base unit name	SI base unit symbol
length [L]	meter	m
mass [M]	kilogram	kg
time [T]	second	s
electric current [I]	ampere	A
thermodynamic temperature [K]	kelvin	K
amount of substance [mol]	mol	Mol
luminous intensity [cd]	candela	cd

prefix	abbreviation	value
yotta	Y	10^{24}
zetta	Z	10^{21}
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
hecto	h	10^{2}
deka	da	10^{1}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	µ	10^{-6}
nano	n	10^{-9}
pico	P	10^{-12}
femto	F	10^{-15}
atto	a	10^{-18}
zepto	z	10^{-21}
yocto	y	10^{-24}

Derived Quantity [Dimension]	SI derived unit symbol
area $[L]^2$	m^2
volume $[L]^3$	m^3
speed, velocity $[L]\cdot[T]^{-1}$	$m\cdot s^{-1}$
acceleration $[L]\cdot[T]^{-2}$	$m\cdot s^{-2}$
mass density $[M]\cdot[L]^{-3}$	$kg\cdot m^{-3}$
force $[L]\cdot[M]\cdot[T]^{-2}$	$m\cdot kg\cdot s^{-2}$
energy, work, heat $[L]^2\cdot[M]\cdot[T]^{-2}$	$m^2\cdot kg\cdot s^{-2}$

Key Concepts

- Scientific notation can help simplify a calculation done without a calculator.
- Units are not the same as dimensions.
- SI units for Mass, Length, and Time are: kilograms, meters, seconds respectively.
- If given a mathematical equation where you are told one or more quantities are changing, proportional reasoning can save time when trying to determine what happens to the unknown quantity.

Act I: Scientific Notation and Units

Questions

GR.2-2:

Description: Use basic rules of algebra with scientific notation. (4 minutes)

Learning Objectives: [1, 4, 5, 6]

d c t

Problem Statement: Given the equation, **distance** = **speed** * **time**, how far does light travel in 1.30 nanoseconds? The speed of light is about **c** = 3.00×10^8 m/s. Do not use a calculator.

(1) 3.90×10^{1} m
(2) 3.90×10^{-1} m
(3) 3.90×10^{-72} m
(4) 4.30×10^{1} m
(5) 4.30×10^{-1} m
(6) 4.30×10^{-72} m

$$d = ct$$

$$d = (3\times10^{8}\ m/s)(1.3\times10^{-9}\ s)$$

$$d = (3)(10^{8})(1.3)(10^{-9})\left(\frac{m}{s}\right)(s)$$

$$d = (3)(1.3)(10^{8})(10^{-9})\ m$$

$$d = (3.9)(10^{-1})m$$

GR.2-3:

Description: Unit conversion problem. (4 minutes)

Learning Objectives: [1, 2, 4, 12]

Problem Statement: Convert 55.0 miles per hour (mph or mi/hr) to meters per second.

(1) 88,500 m/s
(2) 1,480 m/s
(3) 24.6 m/s
(4) 0.410 m/s
(5) 42.0 m/s

Useful conversions

1 mi = 5280 ft

1 ft = 12 in

1 in = 2.54 cm

$$55\,\frac{MI}{HR} \times \frac{5280\,ft}{1\,MI} \times \frac{12\,in}{1\,ft} \times \frac{2.54\,cm}{1\,in} \times \frac{1\,m}{100\,cm} \times \frac{1\,HR}{60\,MIN} \times \frac{1\,MIN}{60\,SEC} \approx 24.5872\ m/s$$

GR.2-4:

Description: Unit conversion problem. (3 minutes)

Learning Objectives: [1, 4, 12]

Problem Statement: The area of Corvallis is about 37.0 km^2. What is the area of Corvallis in m^2? Do not use a calculator.

(1) 3.70×10^{4} m^2
(2) 3.70×10^{5} m^2
(3) 3.70×10^{6} m^2
(4) 3.70×10^{7} m^2
(5) 3.70×10^{8} m^2

$$37.0\ km^2 \times \left(\frac{1000\ m}{1\ km}\right)^2$$

$$(37.0)(1000)^2(km^2)\left(\frac{m^2}{\ }\right)$$

(4) 3.70 x 10^7 m^2
(5) 3.70 x 10^8 m^2

$$(37.0)(1000)^2(km^2)\left(\frac{m^2}{km^2}\right)$$
$$(3.7\times10^1)(1.0\times10^3)^2\ m^2$$
$$(3.7)(10^1)(1.0)^2(10^3)^2\ m^2$$
$$(3.7)(1)(10^1)(10^6)\ m^2$$
$$(3.7)(10^7)\ m^2$$

Act II: Dimensional Analysis

GR.2-5:

Description: Find the dimensions of a quantity. (3 minutes)

Learning Objectives: [7, 8, 10]

Problem Statement: Acceleration has the same dimensions as speed divided by time. What are the dimensions of acceleration?

(1) $[L]\cdot[T]^2$

(2) $\frac{[L]}{[T]^2}$

(3) $m\cdot s^2$

(4) $\frac{m}{s^2}$ UNITS ≠ DIMENSIONS

$$[ACCELERATION] = \frac{[SPEED]}{[TIME]}$$
$$[SPEED] = \frac{[L]}{[T]}$$
$$[TIME] = [T]$$
$$= \frac{\left(\frac{[L]}{[T]}\right)}{[T]}$$
$$[ACCELERATION] = \frac{[L]}{[T]^2}$$

GR.2-6:

Description: Use dimensional analysis to determine if an expression could be valid. (4 minutes)

Learning Objectives: [7, 8, 10]

Problem Statement: Use the dimensions for the quantity given in the table below to determine which of the following expressions could be correct.

Quantity	Dimensions
$\mathbf{p}$	$[M]\cdot[L]\cdot[T]^{-1}$

(1) $\mathbf{m} = \mathbf{p}\cdot\mathbf{v}$

(2) $\mathbf{v} = \mathbf{m}\cdot\mathbf{p}$

m	[M]
v	$[L] \cdot [T]^{-1}$

(3) $\frac{\mathbf{p}}{\mathbf{v}} = \mathbf{m}$ (4) $\frac{\mathbf{m}}{\mathbf{v}} = \mathbf{p}$

(1)

$$[M] \stackrel{?}{=} \frac{[M][L]}{[T]} \cdot \frac{[L]}{[T]}$$

$$[M] \stackrel{?}{=} \frac{[M][L]^2}{[T]^2}$$

X

(2)

$$\frac{[L]}{[T]} \stackrel{?}{=} [M] \cdot \frac{[M][L]}{[T]}$$

$$\frac{[L]}{[T]} \stackrel{?}{=} \frac{[M]^2[L]}{[T]}$$

X

(3)

$$\frac{[M][L]}{[T]} \cdot \frac{[T]}{[L]} \stackrel{?}{=} [M]$$

$$[M] \stackrel{?}{=} [M]$$

✓

(4)

$$\frac{[M][T]}{[L]} \stackrel{?}{=} \frac{[M][L]}{[T]}$$

X

Act III: Mathematization

GR.2-7:

Description: Construct a mathematical equation given a written story. (3 minutes + 6 minutes)

Learning Objectives: [23, 27, 35]

Problem Statement: Which of the following mathematical equations correctly describes the scenario?

(a) The number of duck fans (**d**) is 3 times the number of beaver fans (**b**).

(1) 3**d** = **b**
(2) 3**b** = **d**
(3) **d** = **b**/3
(4) **b** = **d**/3

EX: IF $b = 2$
THEN $d = 6$

(b) In Winnie the Pooh's dream there are three times as many heffalumps (**h**) as woozles (**w**).

(1) 3**h**/**w**
(2) 3**h** = **w**
(3) 3**h** + **w**
(4) **h** = 3**w**
(5) **h**/3 = **w**

EX: IF $w = 2$
THEN $h = 6$

Act IV: Proportional Reasoning

GR.2-8:

Description: Proportional reasoning problem. (5 minutes)

Learning Objectives: [15, 17, 18]

Problem Statement: A long straight wire carries a current **I** and generates a magnetic field $\vec{\mathbf{B}}$. If the current quadruples and the wires moves twice as far away, by what factor does the magnetic field change?

The equation for the magnetic field strength as a function of the distance (**r**) away from an effectively infinite wire is equal to the equation below, where **I** is the current and μ_0 is a constant.

$$|\vec{\mathbf{B}}| = \frac{\mu_0\, \mathbf{I}}{2\,\pi\,\mathbf{r}}$$

(1) 1
(2) 2
(3) 4
(4) 9
(5) 0.5
(6) 0.25

$B_i = \frac{\mu_0 I_i}{2\pi r_i}$

$B_f = \frac{\mu_0 I_f}{2\pi r_f}$

GIVEN

$I_f = 4 I_i$

$r_f = 2 r_i$

$B_f = \frac{\mu_0 4 I_i}{2\pi 2 r_i}$

$B_f = 2 \left(\frac{\mu_0 I_i}{2\pi r_i}\right)$

$B_f = 2 B_i$

QUICK

$B \propto \frac{I}{r}$

IF $I \rightarrow 4I$

AND $r \rightarrow 2r$

THEN $B \rightarrow \frac{4}{2}\frac{I}{r}$

$B \rightarrow 2B$

GR.2-9:

Description: Proportional reasoning problem. (5 minutes)

Learning Objectives: [15, 17, 18]

Problem Statement: A small spherical object has a net charge **q** and generates an electric field $\vec{\mathbf{E}}$. If the charge on the object quadruples and the object moves twice as far away, by what factor does the electric field change?

The equation for the electric field strength as a function of the distance (**r**) away from a point charge is equal to the equation below, where **q** is the charge and **k** is a constant.

$$|\vec{\mathbf{E}}| = \frac{\mathbf{k}\,\mathbf{q}}{\mathbf{r}^2}$$

(1) 1
(2) 2
(3) 4
(4) 9
(5) 0.5
(6) 0.25

$E \propto \frac{q}{r^2}$

IF $q \rightarrow 4q$

AND $r \rightarrow 2r$

(3) 4
(4) 9
(5) 0.5
(6) 0.25

$E \propto \frac{q}{r^2}$

IF $q \rightsquigarrow 4q$
AND $r \rightsquigarrow 2r$
THEN $E \rightsquigarrow \frac{4}{(2)^2} \frac{q}{r^2}$

$E \rightsquigarrow 1E$

Act V: Basic Functions

GR.2-10:

Description: Recognize the graphical features of basic functions. (4 minutes)

Learning Objectives: [28]

Problem Statement: Match the following equations to the sketch of the plot of the equation.

B (1) $\mathbf{y = 7}$

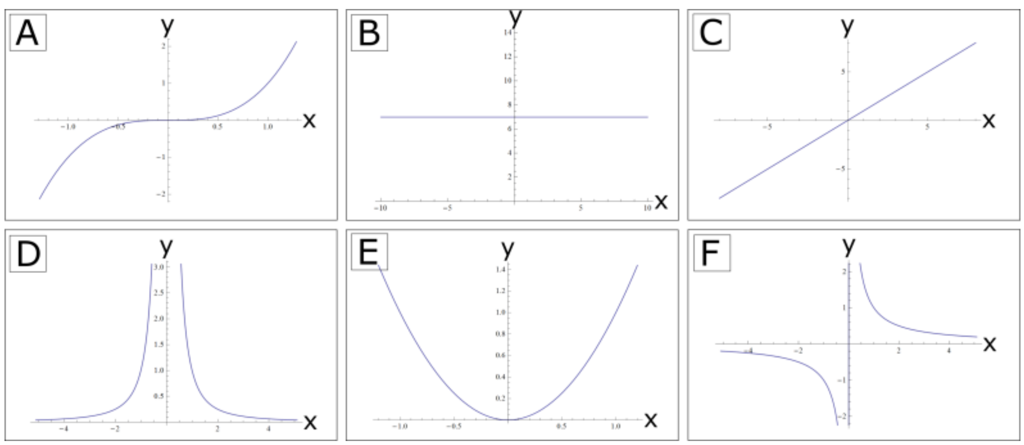
C (2) $\mathbf{y = x}$

E (3) $\mathbf{y = x^2}$

A (4) $\mathbf{y = x^3}$

F (5) $\mathbf{y = \frac{1}{x}}$

D (6) $\mathbf{y = \frac{1}{x^2}}$



Act VI: Finale

GR.2-11:

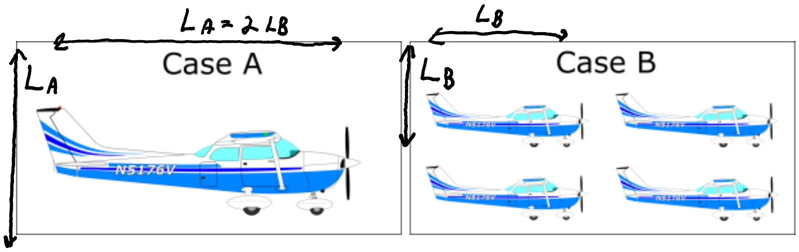
Description: Use proportional reasoning to relate two different cases. (10 minutes)

Learning Objectives: [7, 17, 23, 27]

Problem Statement: A wood worker has made four small airplanes and one large airplane. All airplanes are exactly the same shape, and all are made from the same kind of wood. The larger plane is twice as large in every dimension as one of the smaller planes. The planes are to be painted and then shipped as gifts.

planes are to be painted and then shipped as gifts.

(a) The amount of paint required to paint the planes is directly proportional to the surface area. The amount of paint required for the single plane in case **A** will be __________ the total amount of paint required for all four planes in case **B**.



(1) greater than
(2) equal to
(3) less than

AMMOUNT OF PAINT $\propto$ SA

SA $\propto [L]^2$

CASE A

$\text{AREA} = L_A^2$

$= (2L_B)^2$

$= 4L_B^2$

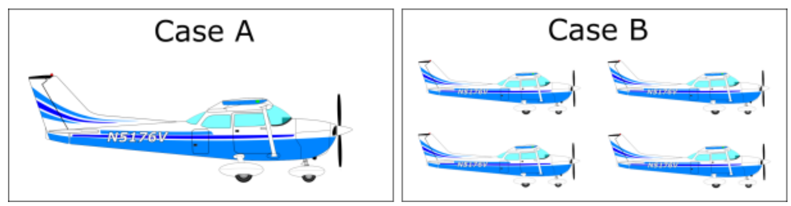
EQUAL

CASE B (4 PLANES)

$\text{AREA} = L_B^2 + L_B^2 + L_B^2 + L_B^2$

$= 4L_B^2$

(b) The shipping cost for the planes is proportional to the weight which is related directly to the volume. The weight of the single plane in case **A** will be ___________ the total weight of all four planes in case **B**.



(1) greater than
(2) equal to
(3) less than

SHIPPING COST $\propto$ WEIGHT $\propto$ VOLUME

VOLUME $\propto [L]^3$

CASE A

$\text{VOLUME} = L_A^3$

$= (2L_B)^3$

$= 8L_B^3$

A > B

CASE B (4 PLANES)

$\text{VOLUME} = L_B^3 + L_B^3 + L_B^3 + L_B^3$

$= 4L_B^3$

Conceptual questions for discussion

1. If two quantities have the same units, do they also have the same dimensions as each other?
2. If two quantities have the same dimensions, do they also have the same units as each other?
3. If two quantities have the same dimensions, do they necessarily represent the same physical quantity?
4. Dimensional analysis is very useful in helping us deduce possible functional relationships and also checking if a derived expression could be valid. However, there are a few limitations that restrict us from using dimensional analysis to derive functional relationships and expressions. What are some of these limitations?

Hints

GR.2-1: No hints.

GR.2-2: No hints.

GR.2-3: No hints.

GR.2-4: No hints.

GR.2-5: If stuck on dimensions, try a unit analysis first then convert final answer to dimensions. For example, SI units of time are seconds (s), so if you have "s" in your final answer, then replace "s" with [T] since seconds is a measure of time.

GR.2-6: $x^{-1} = 1/x$.

GR.2-7: After you pick an answer, plug in some values to see if you get the correct answer as per the written statement.

GR.2-8: Which quantities are changing, which are constant?

GR.2-9: Which quantities are changing, which are constant?

GR.2-10: Look at limiting cases, such as what happens when x goes to infinity or zero.

GR.2-11: Imagine making each plane into a square with side lengths $\mathbf{L}_{\mathrm{A}}$ so that the area of the plane in case **A** is $\mathbf{L}_{\mathrm{A}}^2$. How does the length of plane **A** compare to the length of plane **B**?