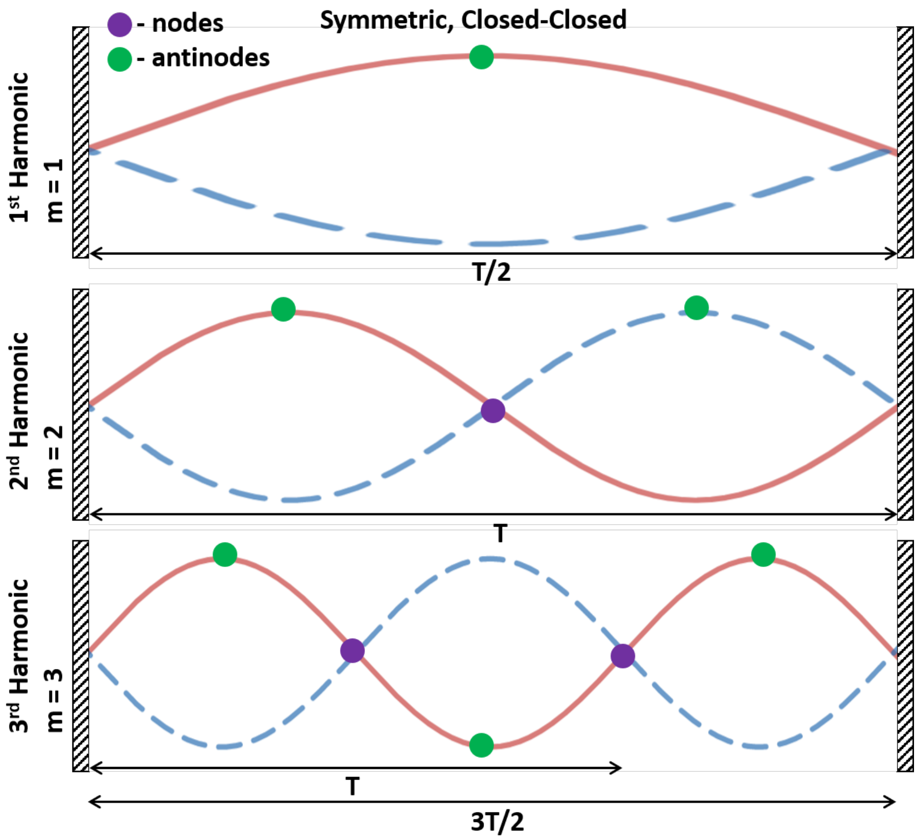
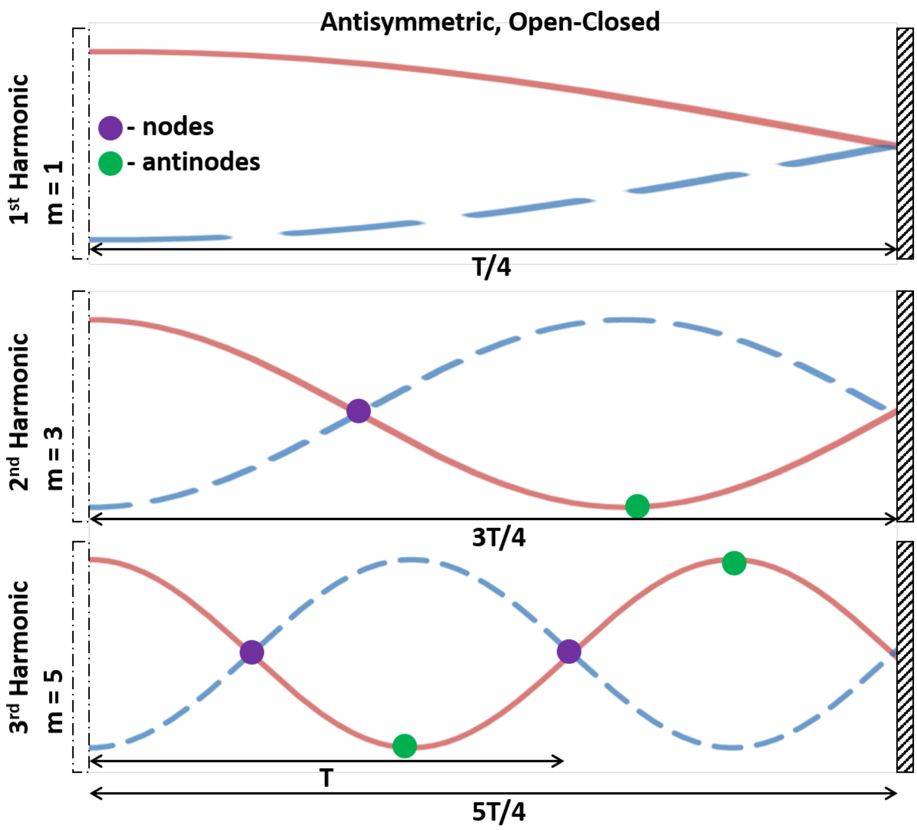
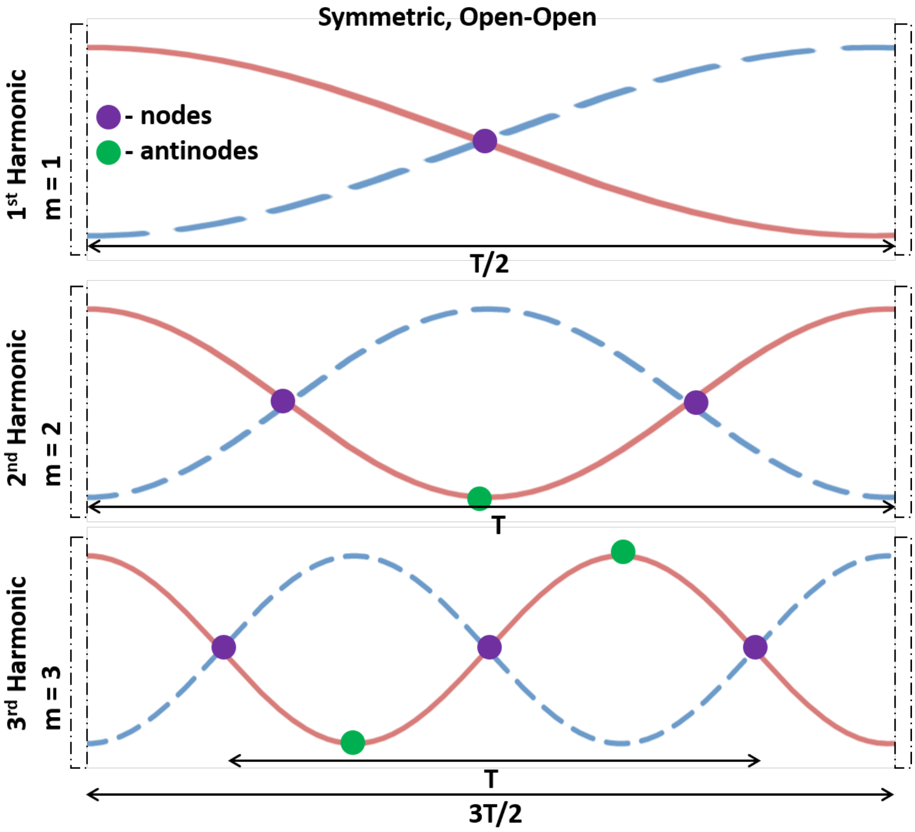
Harmonics Draw the first three harmonics for both the symmetric and antisymmetric set up of a traveling wave. Label & indicate all relevant information, including; harmonic number, m value, and all nodes (antinodes). Also mark the length of 1 period on your picture.





[0]Select problems may be modified from Walsh, Naylor, or the Internet.



Doppler Shift Suppose a train that has a 150Hz horn is moving at 35.0m/s in still air. What frequencies are observed by a stationary person at the side of the tracks as the train approaches and after it passes?
What frequency is observed by the train's engineer traveling on the train?

$$f_0 = f_s\left(\frac{v \pm v_0}{v \mp v_s}\right) \tag{1}$$

$$v_s = 35\frac{m}{s} \tag{2}$$

$$v_0 = person = 0\frac{m}{s} \tag{3}$$

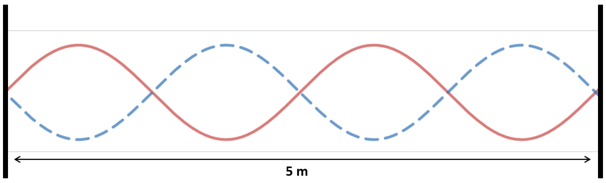
$$v = v_{air} = 343\frac{m}{s} \tag{4}$$

As the train approaches we expect the pitch to increase and we know that $\Delta r \uparrow \implies -v_s$
$f_0 = 150Hz\left(\frac{343\frac{m}{s}+0\frac{m}{s}}{343\frac{m}{s}-35\frac{m}{s}}\right) = 167Hz$
This makes sense because it is greater than f_s which gives us the higher pitch that we expected.
As the train leaves we expect the pitch to decrease and we know that $\Delta r \downarrow \implies +v_s$
$f_0 = 150Hz\left(\frac{343\frac{m}{s}+0\frac{m}{s}}{343\frac{m}{s}+35\frac{m}{s}}\right) = 136Hz$
This makes sense because it is less than f_s which gives us the lower pitch that we expected.
The engineer on the train is not moving with respect to the train; in other words $v_{engineer,0} = v_{train,s}$ therefore the frequency is unchanged. $f = 150Hz$

String Properties The string is 5.0 meters long and is vibrating at the fourth harmonic. The string vibrates up and down with 48 cycles in 20 seconds. For this wave, determine the:

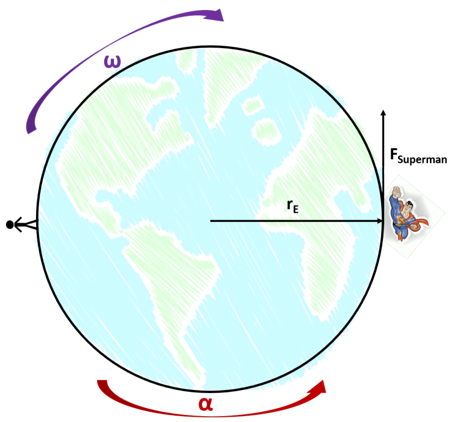
- frequency,
- period,
- wavelength, and
- speed.

If the string has a mass of 10g, what is the tension in the string?



Can be assumed to be a closed-closed system, therefore it is a symmetric set up. It's in the 4^{th} harmonic and since it is symmetric that implies that $m = 4$. We also know that the equations $\lambda_m = \frac{2L}{m}$ and $f_m = m\frac{v}{2L}$ are true for this system since it is symmetric.
To find the frequency: $f = \frac{cycles}{second} = \frac{48cycles}{20sec} = 2.4Hz$
For the period: $T = \frac{1}{f} = \frac{1}{2.4Hz} = 0.417s$
For the wavelength: $\lambda_m = \frac{2L}{m} \implies \lambda_4 = \frac{2L}{4} = \frac{2 \cdot 5m}{4} = 2.5m$
For the velocity: $v = \lambda f = 2.5m \cdot 2.4Hz = 6\frac{m}{s}$ For the force tension (F_T): $\nu = \sqrt{\frac{F_T}{\mu}}$ For this system $\nu = v$ (that you already found) $\implies F_T = v^2\mu$ Remembering $\mu = \frac{m}{L}$ then $F_T = \frac{mv^2}{L} = \frac{0.06kg}{5m}(6\frac{m}{s})^2 = 0.072N$

Superman Superman, foolishly attempting to turn back time, applies a constant force tangent to the Earth at the equator. He mistakenly believes that if he changes the direction the Earth rotates, the clocks will somehow run backwards. He should have taken physics. $M_E = 5.97x10^{24}kg$, $R_E = 6.37x10^6m$.
(a) If it takes him 2 mins to stop the rotation of the Earth, how much force did he apply?
(b) If he continues pushing with that same force, the Earth begins rotating in the opposite direction. How long will it take for some people to feel weightless?
(c) Where on the Earth will people first start feeling weightless?



Knowns:

$$\omega_i = \frac{-2\pi}{T} = \frac{-2\pi rad}{1day} = \frac{-2\pi rad}{86,400s} = -7.272 * 10^{-5}\frac{rad}{s} \quad (5)$$

$$\Delta t = 2min = 120sec \quad (6)$$

$$\omega_f = 0\frac{rad}{s} \quad (7)$$

Unknowns:

$$\Delta\theta \tag{8}$$

$$\alpha \tag{9}$$

(a) To find the force Superman applied (F_s) we need to apply Newton's second law to the rotation (sum the torques). $\sum \tau = I\alpha$ We know that the only torque is from Superman and will be given by $\tau_S = |r_E||F_S|sin\theta_s$. For this situation $\theta_s = 90\deg \implies sin\theta_s = 1$ thus $\tau_s = |r_E||F_S|$. The moment of inertia, I is just that of a sphere which can be looked up: $I_{sphere} = \frac{2}{5}mr^2$. So we just need to find α.

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t} = \frac{0 - \omega_i}{\Delta t} = 6.06 * 10^{-7}\frac{rad}{s^2} \tag{10}$$

Now we can set up the sum of the torques plugging in what we know.

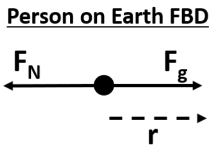
$$|r_E||F_S| = \frac{2}{5}m_E r_E^2 \alpha \tag{11}$$

$$|F_S| = \frac{2}{5}m_E r_E \alpha \tag{12}$$

$$|F_S| = \frac{2}{5}(5.97x10^{24}kg)(6.37x10^6 m)(6.06 * 10^{-7}\frac{rad}{s^2}) \tag{13}$$

$$|F_S| = 9.22 * 10^{24}N \tag{14}$$

(b) To start, a free body diagram of what the stick figure person on the Earth feels.



Then, apply Newton's second law radially. $\sum F_r = ma_r$ remembering that $a_r = \frac{v^2}{r}$

$F_g - F_N = m\frac{v^2}{r_E}$

Apparent weightlessness occurs when $F_N = 0$ because you no longer feel a force acting up on you.

$\implies F_g = m\frac{v^2}{r_E}$ canceling the mass and using the equation $v = \omega r$ we get $g = \omega^2 r_E$ from this we can get the angular velocity (ω) that the Earth is rotating at when people begin to feel weightless.

$\omega = \sqrt{\frac{g}{r_E}}$ using angular kinematics, with the starting point of when the Earth wasn't spinning $\omega_i = o$ we can find the time it took.

$$\omega_f = \omega_i + \alpha\Delta t \tag{15}$$

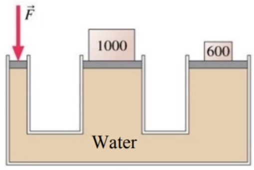
$$\sqrt{\frac{g}{r_E}} = \alpha\Delta t \tag{16}$$

$$\Delta t = \frac{1}{\alpha}\sqrt{\frac{g}{r_E}} \tag{17}$$

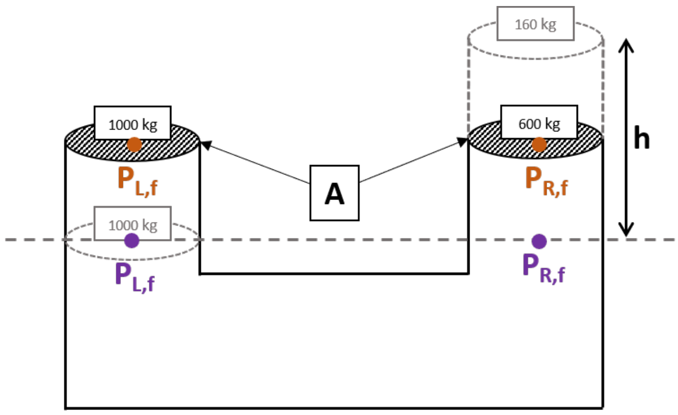
$$\implies \Delta t = 2097s = 34min \tag{18}$$

(c) At the equator where a_r is the largest.

Pistons Two different masses, 1000 kg and 600 kg, are initially placed on top of equal 1.20-m-diameter low friction pistons. The pistons are connected to a third piston where a force F is applied such that it doesn't move. Water is the hydraulic fluid. Do the pistons move? If so, how much, if not, why?



Since the force applied to the left most piston is applied such that it doesn't move, meaning that the force will automatically adjust to keep the piston in one place, that piston might as well be a wall. So when we redraw the picture we're going to just leave off that piston.



The most fundamental equation for pressure, $P = \frac{F}{A}$ lets us know that these pistons are not currently in equilibrium- meaning that they will move.
We know the areas of the two pistons are the same. We also know that the force on each piston is equal to their mass times gravity.
Since $A_L = A_R$ and $F_L \neq F_R$ then $P_L \neq P_R$. Using that proportional reasoning we can determine that the $1000kg$ mass will go up and the $600kg$ mass will go down until $P_L = P_R$ at the same height in the fluid.
To help articulate that the figure above shows where the pistons start and where they would end.
The purple points represent where the final pressure for each side should be equal. When compared with the gray, final location of the pistons it can be seen that the left point has just the pressure of its piston and the right point has the pressure of its piston plus the fluid that is on top of it.
This can be represented in equation form by:
$P_{L,f} = \frac{1000kg \cdot g}{A}$ $\qquad P_{R,f} = \frac{600kg \cdot g}{A} + \rho g h$
By setting these equal- since they are at the same depth (Pascal's principle), on the dotted line, we can find the height, h.

$$\frac{1000kg \cdot g}{\pi r^2} = \frac{600kg \cdot g}{\pi r^2} + \rho g h \tag{19}$$
$$\frac{1000kg}{\pi r^2} = \frac{600kg}{\pi r^2} + \rho h \tag{20}$$
$$\frac{1000kg}{\pi \rho r^2} - \frac{600kg}{\pi \rho r^2} = h \tag{21}$$
$$\implies h = 0.354m \tag{22}$$

Double checking at the picture we can see that h is the distance between the final locations of the two pistons, not how far each one traveled. To get how far each one traveled we divide h by 2. Thus each piston traveled $0.177m$ with the $600kg$ one going up and the $1000kg$ one going down.