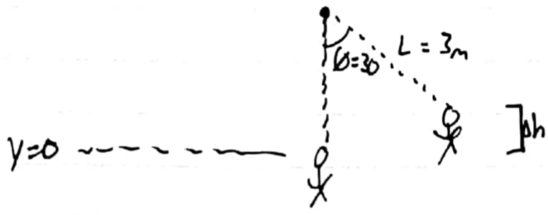
Pendulums to the Rescue! - Saving Indiana Jones Indiana Jones wraps his whip around a light fixture to swing to safety. What is the tension in the whip at the bottom of his swing? Estimate all values you may need.

Estimated values:

- angle max: $30°$
- mass: $80kg$



Conservation of Energy: at the top of the swing it's all potential energy, U^g_{max} & at the bottom of the swing it's all kinetic energy, K^E_{max}.

$$\implies U^g_{max} = K^E_{max} \tag{1}$$
$$mg\Delta h = \frac{1}{2}mv^2 \tag{2}$$
$$2g\Delta h = v^2 \tag{3}$$
$$\Delta h = L(1 - cos\theta) \tag{4}$$
$$\implies v^2 = 2gL(1 - cos\theta) \tag{5}$$

We now need to get to tension, so we need to have a free body diagram to know how to get to F^T.

Applying Newton's 2^{nd} Law, $\sum F_{net} = ma$ in the y-direction we can solve for F^T.
We know that there is an acceleration at the bottom of the motion, remembering from uniform circular motion $a = \frac{v^2}{r}$, where $r = L$, the length of the whip.

$$\sum F_{net,y} = ma_y \tag{6}$$
$$F^T - F^g = m\frac{v^2}{L} \tag{7}$$
$$F^T = m\frac{v^2}{L} + mg \tag{8}$$
$$F^T = m(\frac{v^2}{L} + g) \tag{9}$$
$$F^T = m(\frac{2gL(1 - cos\theta)}{L} + g) \tag{10}$$
$$F^T = mg(2(1 - cos\theta) + 1) \tag{11}$$

Plugging in the values we estimated.

$$F^T = (80kg)(9.8\frac{m}{s^2})(2(1 - cos30°) + 1) \tag{12}$$
$$F^T = 994N \tag{13}$$

This makes sense because it is greater than F^g ($mg = 784N$), which it should be because we know for circular motion the acceleration points inward, or in this case, upward.

[0]Select problems may be modified from Walsh, Naylor, or the Internet.

Damped Oscillators What is the general form of a damped oscillator?

$x(t) = \pm x(t)^{t=0}_{max} e^{-\frac{t}{\tau}} \cdot ^{sin}_{cos} (\omega t)$

What is the drag force, F^D?

$F^D = -bv$

What do you know about τ? Name? Units? Defining Equation?

Time constant, seconds, $\tau = \frac{2m}{b}$

Energy of Damped Oscillators A damped harmonic oscillator consists of a block ($m = 2.00kg$), a spring ($k = 10.0\frac{N}{m}$), and a damping force the is linearly proportional to the velocity. Initially, it is pulled to the left $25.0cm$; because of damping, the amplitude falls to $\frac{3}{4}$ of this initial value at the completion of 4 oscillations.

a. What is the value of time constant τ?

$$x(t) = \pm x_{max}^{t=0} e^{\frac{-t}{\tau}} \cdot \substack{sin \\ cos} (\omega t) \tag{14}$$

$$x_{max}(t) = x_{max}^{t=0} e^{\frac{-t}{\tau}} \tag{15}$$

$$\frac{3}{4} x_{max}^{t=0} = x_{max}^{t=0} e^{\frac{-4T}{\tau}} \tag{16}$$

$$\frac{3}{4} = e^{\frac{-4T}{\tau}} \tag{17}$$

$$ln(\frac{3}{4}) = \frac{-4T}{\tau} \tag{18}$$

$$\tau = \frac{-4T}{ln\frac{3}{4}} \tag{19}$$

$$T_{spring} = 2\pi\sqrt{\frac{m}{k}} \tag{20}$$

$$\tau = \frac{-4(2\pi\sqrt{\frac{m}{k}})}{ln\frac{3}{4}} \tag{21}$$

$$\implies \tau = \frac{-4(2\pi\sqrt{\frac{(2kg)}{(10\frac{N}{m})}})}{ln\frac{3}{4}} \tag{22}$$

$$\tau = 39s \tag{23}$$

b. What would the complete position equation look like for this oscillator?
$x(t) = -(25cm)e^{(-\frac{t}{39s})} cos(2.24\frac{rad}{s}t)$
Since $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{10\frac{N}{m}}{2kg}}$

c. How much energy has been "lost" during these 4 oscillations?

$$E_{lost} = E_{initial} - E_{final} \tag{24}$$

$$E_i = K_i^S + U_i^S \tag{25}$$

$$E_f = K_f^S + U_f^S \tag{26}$$

$$K^S = \frac{1}{2}mv^2 \tag{27}$$

$$U^S = \frac{1}{2}k\delta x^2 \tag{28}$$

$$v_i = v_f = 0 \tag{29}$$

Since we start at the max displacement and end at the new max displacement the velocity is zero in both cases.

$$U_i^S = \frac{1}{2}kx_{max,i}^2 \tag{30}$$

$$U_i^S = \frac{1}{2}(10\frac{N}{m})(0.25m)^2 \tag{31}$$

$$U_i^S = 0.3125J \tag{32}$$

$$U_f^S = \frac{1}{2}k\frac{3}{4}_{max,i}^2 \tag{33}$$

$$U_f^S = \frac{1}{2}(10\frac{N}{m})(\frac{3}{4}0.25m)^2 \tag{34}$$

$$U_f^S = 0.176J \tag{35}$$

$$\implies E_{lost} = U_i^S - U_f^S \tag{36}$$

$$E_{lost} = 0.3125J - 0.176J \tag{37}$$

$$E_{lost} = 0.137J \tag{38}$$

Driven Oscillators Brainstorm in your group what you know about driven oscillators. What does driven mean (in this context)? What kinds of questions could KC ask about driven oscillators? How does a driven oscillator differ from a damped oscillator?

$F^{Driving} = F_0 \cdot^{sin}_{cos} (\omega t)$

$Amplitude = \frac{\frac{F_0}{m}}{\sqrt{(\omega^2 - \omega_0^2)^2 - (\frac{b\omega}{m})^2}}$

A "damped" oscillation is the motion of an oscillating mass for which there is some frictional force (some force that always opposes the direction of motion); the only conceptual difference between a "damped" and "driven" oscillator is that, for a driven oscillator, the net external force is instead in the same direction of the motion of the object.

Traveling Waves Name and define, in your own words, the 3 types of waves. Give 1 example of each type.

- mechanical waves: physical movement to make the wave; sporting event waves
- matter waves: (sub)atomic particles making a wave; diffraction patterns
- electromagnetic waves: that are propagated by simultaneous periodic variations of electric and magnetic field intensity and that include radio waves, infrared, visible light, ultraviolet, X-rays, and gamma rays; radio waves

Name and define, in your own words, the 3 modes of waves. Give 1 example of a wave that travels via each mode.

- transverse: a wave vibrating at right angles to the direction of its propagation; water waves- like those for surfing
- longitudinal: a wave vibrating in the direction of propagation; sound waves
- combo of transverse and longitudinal: a wave vibrating in both the direction of propagation and at right angles to the direction of its propagation; waves out from a water droplet

What is the general form of a traveling wave?
$D(x.t) = \pm D_{max} \cdot_{cos}^{sin} (kx \pm \omega t)$
What is the wave speed in general? For a string?
$\nu = f\lambda$ $\nu_{string} = \sqrt{\frac{FT}{\mu}}$ $\mu = \frac{mass}{length}$
What do you know about k? Name? Units? Defining Equation?
wave number, $\frac{}{m}$, $k = \frac{2\pi}{\lambda}$

Traveling Through a Medium A traveling sound wave moves through a medium and the displacement can be described by the following function:

$$D(x,t) = (2.00\mu m)cos(1.75x - 858t)$$

where x is meters and t is in seconds.

$$\text{General form: } D(x,t) = \pm D_{max} \cdot_{cos}^{sin} (kx \pm \omega t)$$

Determine

a. Determine the amplitude. $2e^{-6}m$

b. Determine the wavelength. $k = \frac{2\pi}{\lambda} \implies \lambda = \frac{2\pi}{k} = \frac{2\pi}{1.75\frac{}{m}} = 3.59m$

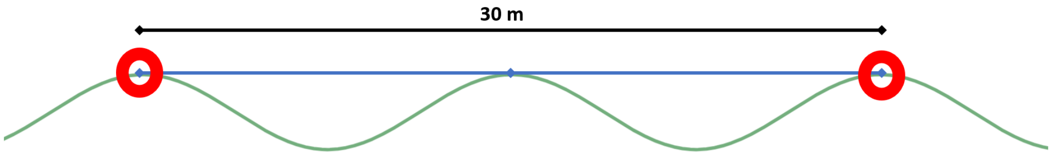
c. Determine the speed of this wave. $\nu = f\lambda = \frac{\omega}{2\pi}\frac{2\pi}{k} = \frac{\omega}{k} = \frac{858\frac{rad}{s}}{1.75\frac{}{m}} = 490.29\frac{m}{s}$

d. Determine the instantaneous displacement from equilibrium of the elements of the matter at the position $x = 0.050m$ and at $t = 3.00ms$.
$D(0.050m, 3.00ms) = ((2.00\mu m)cos(1.75\frac{}{m}(0.050m) - 858\frac{rad}{s}(0.003s)) = 1.998\mu m$

e. Determine the maximum speed of the **element's oscillatory motion**.
$v_{max} = x_{max}\omega = (2.00\mu m)(858\frac{rad}{s}) = 1716\frac{\mu m}{s} = 0.001716\frac{m}{s}$
remember that v_{max} for the oscillatory motion is not the same as the wave speed!

Waves-Oscillations.Traveling-Waves.**MS.KC.7:** Two buoys are bobbing up and down with a period of 3 s at a distance of 30 m from each other. It is noticed that when one is at its maximum position the other is also at its maximum position and there is one wave crest between the two buoys. Which of the following statements regarding this situation are true?

(a) The speed of the water waves is 5 m/s.
(b) The speed of the water waves is 10 m/s.
(c) The frequency of the water waves is 0.33 Hz.
(d) The wavelength of the water waves is 30 m.
(e) The wavelength of the water waves is 15 m.



A, C, & E are right. See below for reasoning

$$\nu = f\lambda = \frac{\lambda}{T} = \frac{15m}{3s} = 5\frac{m}{s} \implies (a) \quad (39)$$

$$f = \frac{1}{T} = \frac{1}{3s} = 0.33Hz \implies (c) \quad (40)$$

$$(41)$$

We know that the buoys are $30m$ away from each other, but there is also 1 crest between them, thus they are 2 wavelengths away from each other.

$30m = 2\lambda \implies \lambda = 15m \implies (e)$