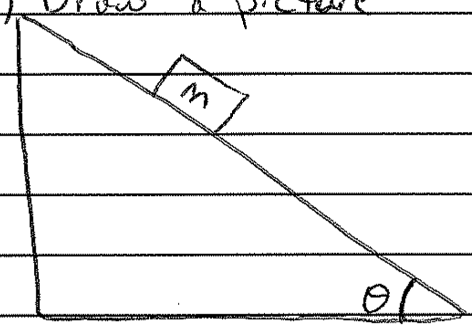
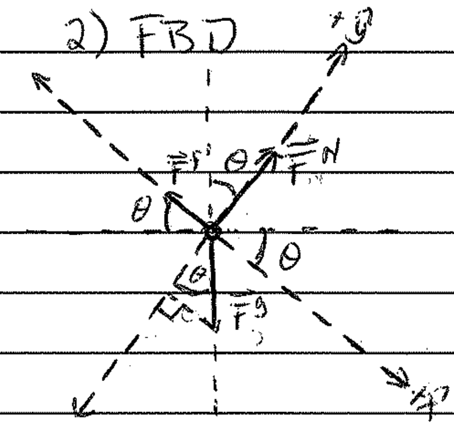
Incline planes

⇒ Consider a stationary mass M on an incline which makes angle θ with the horizontal (include friction?)

1) Draw a picture

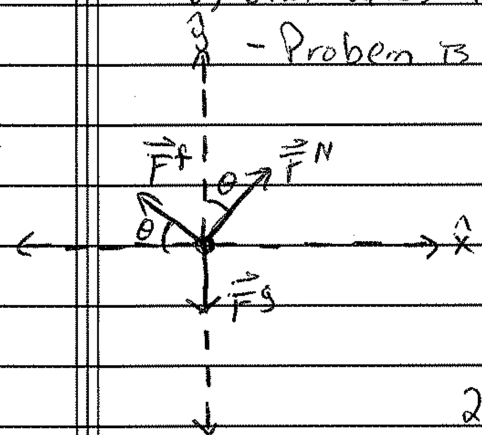


2) FBD



i) tilt axes to simplify forces

- Probem is much more difficult without tilted axes



3) Break forces into components and sum forces.

$$\sum F_x = m a_x \qquad \sum F_y = m a_y$$

2)(i) $-F^f_x + F^N_x = m a_x$ (x)

$$-F^f \cos\theta + F^N \sin\theta = m a_x$$

$$-F^g_y + F^f_y + F^N_y = m a_y \quad (y)$$

$$-mg + F^f \sin\theta + F^N \cos\theta = m a_y$$

2) (x) ⇒ $-F^f_x + F^g_x = m a_x$

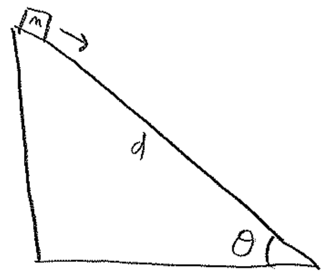
$$-F^f + mg \sin\theta = m a_x$$

(y) ⇒ $F^N_y - F^g_y = m a_y$

$$F^N - mg\cos\theta = m a_y$$

[Which way is simpler? (2) or (2)(i)?]

(3) Box of mass m slides down incline plane. How long until it reaches the bottom?

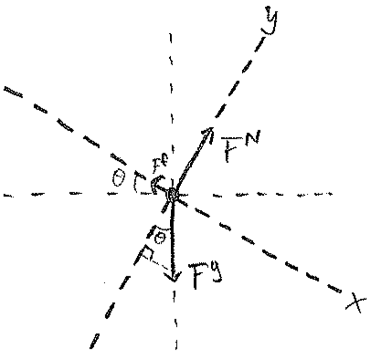


Newtons 2^{nd} law

$$\sum F_x = m a_x \qquad \sum F_y = m a_y$$

$$\Rightarrow F_x^g - F_x^f = m a_x \qquad F_y^N - F_y^g = m \cancel{a_y}^{\,0}$$

$$\Rightarrow mg\sin\theta - F^f = ma \qquad N - mg\cos\theta = 0$$



$$F^f \propto N \Rightarrow F^f = \mu_k N = \mu_k mg\cos\theta$$

$$\Rightarrow mg\sin\theta - \mu_k mg\cos\theta = ma$$

$$\Rightarrow a = g\sin\theta - \mu_k g\cos\theta$$

$$= g(\sin\theta - \mu_k\cos\theta)$$

Now use Kinematics

Known	Unknown
$\Delta x = d$	Δt
$a = g(\dots)$	V_f
$V_i = 0$	

$$(1)\ V_f^2 = V_i^2 + 2a\Delta x \qquad (2)\ V_f = V_i + a\Delta t$$

$$(1) \Rightarrow V_f^2 = 2ad$$

$$V_f = \sqrt{2g(\sin\theta - \mu_k\cos\theta)d}$$

$$(2)\quad V_f = a\Delta t$$

$$\Rightarrow \Delta t = \frac{V_f}{a} = \frac{\sqrt{2g(\sin\theta - \mu_k\cos\theta)d}}{g(\sin\theta - \mu_k\cos\theta)}$$

$$= \sqrt{\frac{2d}{g(\sin\theta - \mu_k\cos\theta)}}$$

If the ramp was 5.2 meters long, $\theta = 30°$, and it took $t = 2.31s$

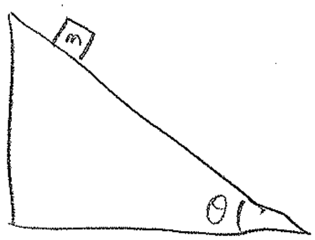
$$\Delta t^2 = \frac{2d}{g(\sin\theta - \mu_k\cos\theta)}$$

$$\sin\theta - \mu_k\cos\theta = \frac{2d}{g\Delta t^2}$$

$$\mu_k = \tan\theta - \frac{2d}{g\Delta t^2\cos\theta} = \tan(30°) - \frac{2(5.2m)}{(9.8 m/s^2)(2.31s)^2\cos(30°)} = 0.35$$

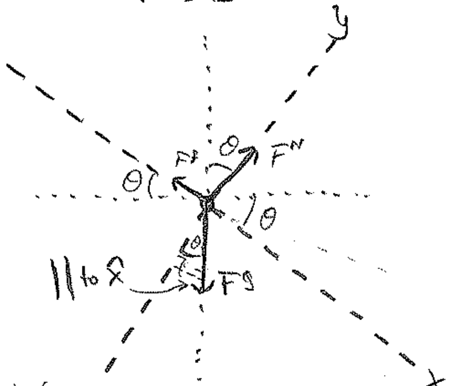
μ_s for wood-wood interface is between 0.25 - 0.5

Incline Planes



(1) stationary mass m on incline which makes angle θ with horizontal.

FBD



Newton's Second law

$$\Sigma F_x = m a_x \qquad \Sigma F_y = m a_y$$

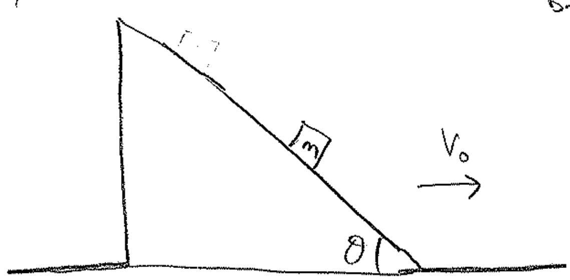
$$F_x^g - F_x^f = m a_x \qquad F^N - F_y^g = m a_y$$

$$mg\sin\theta - F^f = 0 \qquad N - mg\cos\theta = 0$$

$$F^f = mg\sin\theta \qquad N = mg\cos\theta$$

$\Rightarrow \mu_s = \tan\theta$ ← maximum θ before block will slide. Angle of friction.

☆ (Remember hand nuemonic). Pointer finger in $\hat{x}$ direction, thumb in $\hat{y}$ direction - rotate by angle theta. Both thumb and finger rotate by θ.

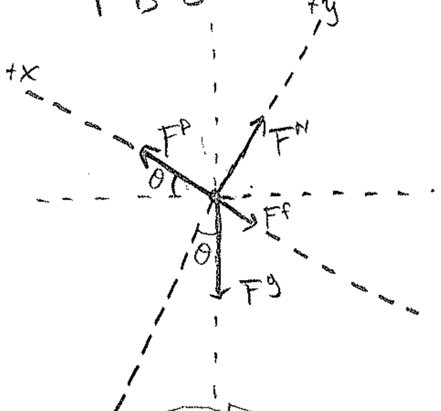


(2) - Allow stationary mass/wedge to move with constant velocity.

- Does FBD change?
  - No

(4) Next add a fan, blowing up the ramp such that the mass moves upward. What velocity does the mass leave the ramp with, and how far does it travel away from the ramp?

FBD for mass M

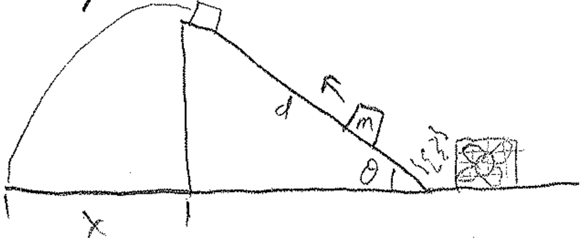


$$\Sigma F_x = m a_x \qquad \Sigma F_y = m a_y$$

$$-F_x^f - F_x^g + F_x^P = m a_x \qquad F_y^N - F_y^g = m a_y^{\,0}$$

$$-F^f - mg\sin\theta + F_x^P = m a_x \qquad F^N = mg\cos\theta$$

$$F^f = \mu F^N = \mu_k mg\cos\theta$$



$$\Rightarrow F^P - mg(\mu_k\cos\theta + \sin\theta) = ma$$

$$a = \frac{1}{m}F^P - g(\mu_k\cos\theta + \sin\theta)$$

Kinematics equations ...

$$V_f^2 = V_i^2 + 2a\Delta x$$

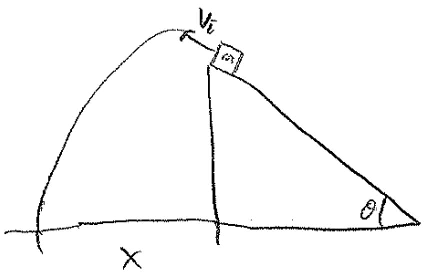
$V_i = 0$

$\Delta x = d$

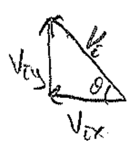
$$V_f^2 = 2\left(\frac{1}{m}F^P - g(\mu_k\cos\theta + \sin\theta)\right)d$$

$$V_{f1} = \sqrt{2\left[\frac{1}{m}F^P - g(\mu_k\cos\theta + \sin\theta)\right]d}$$

Now projectile motion, assume negligible air resistance.



Velocity triangle



$V_i = V_{f1}$

$V_{ix} = V_i\cos\theta$

$V_{iy} = V_i\sin\theta$

Knowns	Unknowns
$V_i = V_{f1}$	Δx
$a_y = -g$	Δt
$V_{ix} = V_i\cos\theta$	V_f
$V_{iy} = V_i\sin\theta$	
$y_i = d\sin\theta$	
$y_f = 0$	
$a_x = 0$	

y - component

$$\Delta y = V_{iy}t + \frac{1}{2}a_y t^2$$

$$-d\sin\theta = V_i\sin\theta\, t - \frac{1}{2}g t^2$$

$$0 = d\sin\theta + V_i\sin\theta\, t - \frac{1}{2}g t^2$$

$$t = \frac{-V_i\sin\theta \pm \sqrt{(V_i\sin\theta)^2 - 4(-\frac{1}{2}g)(d\sin\theta)}}{2(-\frac{1}{2}g)}$$

$$= \frac{-V_i\sin\theta \pm \sqrt{(V_i\sin\theta)^2 + 2gd\sin\theta}}{-g}$$

$$\Rightarrow t_y = \frac{V_i\sin\theta + \sqrt{(V_i\sin\theta)^2 + 2gd\sin\theta}}{g}$$

$$= \frac{V_{f1}\sin\theta + \sqrt{(V_{f1}\sin\theta)^2 + 2gd\sin\theta}}{g}$$

x - component

$$\Delta x = V_{ix}t + \cancel{\frac{1}{2}a_x t^2} \to 0$$

$$\Rightarrow x = V_i\cos\theta\, t_y$$

$$= V_{f1}\cos\theta\, t_y$$