Work - Energy / Conservation of Energy

- $\Delta E = W \Rightarrow \sum_i \Delta E_i = W$

- Isolated system $\Rightarrow$ No energy is transfered into or out of the system

- $\Delta E = 0 \Rightarrow \Delta K + \Delta U_g + \Delta U_s + \Delta E_{th} + \Delta E_{chem} + \cdots = 0$

$K = \frac{1}{2} m V^2 \qquad U_g = mgy \qquad U_s = \frac{1}{2} k x^2$

- The maximum energy a bone can absorb without breaking is surprisingly small. For a healthy human of mass 60 kg, experimental data shows that the leg bones can absorb about 200 J.

$\Rightarrow$ From what maximum height could a person jump and land rigidly upright on both feet without breaking their legs?

$\Rightarrow$ Assume all the energy is absorbed in the leg bones in a rigid landing.

Known	Unknown
$W_{max} = 200\ J$	y

Use work energy-equation

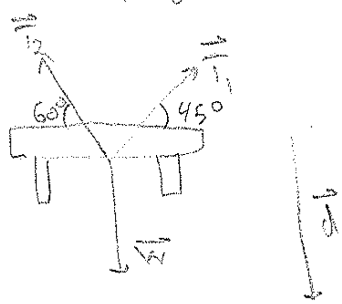
$$\Delta E = W \qquad \Delta E = \Delta U_g$$

$$\Rightarrow mg\,y_{max} = W_{max}$$

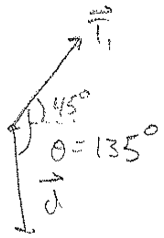
$$y_{max} = \frac{W_{max}}{mg} = 0.34\ m = 34\ cm$$

$K_i + W = K_f$

$W = \vec{F} \cdot \vec{d} = F d \cos\theta$



$W_1 = T_1 d \cos\theta$



$W_1 = T_1 d \cos(135)$

$W_2 = T_2 d \cos(150)$

$T_1 = 1295\ N$

$T_2 = 1830\ N$

$W = 2500\ N$

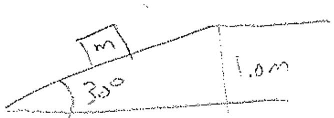
$d = 5m$

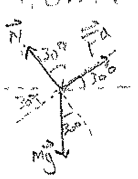
$W_w = W d$

A 20 g plastic ball is moving to the left at 30 m/s. How much work must be done on the ball to cause it to move to the right at 30 m/s.

$W = K_f - K_i = \frac{1}{2} m V_f^2 - \frac{1}{2} m V_i^2 = \frac{1}{2} m [V_f^2 - V_i^2] = 0$

How much work do you do on a 20 kg box if you push it up a ramp, @ 30°, to a height of 1.0 m?





$\sum F_x = m a_x$

$F^a - mg \sin\theta = 0$

$F^a = mg \sin\theta$

$\sum F_y = m a_y$

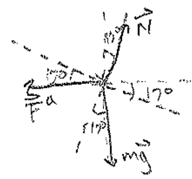
$N - mg \cos\theta = 0$

$\Delta x = \frac{1}{\sin\theta}$

$W = F d \cos\theta = F^a \Delta x = mg \sin\theta \left(\frac{1}{\sin\theta}\right) = mg$ Joules $= 20$ Joules

A 2.3 kg box, starting from rest, is pushed up a ramp by a 10 N force parallel to the floor. The ramp is 2.0 m long and tilted @ 17°. The speed of the box at the top of the ramp is 0.80 m/s

a) How much work W does the force do on the box



$W_a = F d \cos\theta$

$= (10N)(2.0m)\cos 17°$

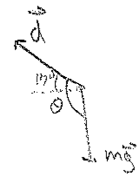
$= 19.13\ J$

b) How much work does gravity do on the box ?

$W_g = mg\, d \cos\theta$

$= mg\, d \cos 107°$

$= -13.18\ J$



c) What is the change in kinetic energy of the system?

$\Delta K = K_f - K_i \qquad V_i = 0 \Rightarrow K_i = 0$

$= K_f = \frac{1}{2}mv^2 = 0.736\ J$

d) What is the change in thermal energy of the system?

Work-energy equation

$\Delta E = W \Rightarrow \Delta K + \Delta E_{th} = W$

$\Delta E_{th} = W - \Delta K = (19.13 - 13.18) - (0.736\ J)$

$= 4.594\ J$

e) What is the coefficient of kinetic friction between the box and the incline?

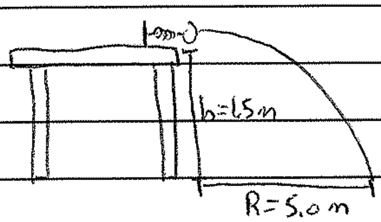
$N - mg\cos 17 - F^a \sin 17 = 0$

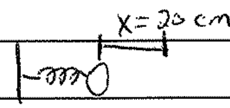
$N = mg\cos 17 + F^a \sin 17$

$\Delta E_{th} = W = f_k \Delta x = \mu_k N \Delta x$

$\mu_k = \frac{\Delta E_{th}}{(mg\cos 17 + F^a \sin 17)\Delta x} = 0.094$

In a physics lab experiment, a spring clamped to the edge of a table shoots a 20 g ball horizontally. When the spring is compressed 20 cm, the ball travels horizontally 5.0 m and lands on the floor 1.5 m below the point at which it left the spring. What is the spring constant





No energy transfer to the ball or from the ball

2 stages ⇒ Stage 1 Spring compression
Stage 2 projectile motion

Stage 1

$E_i = U_{si} + K_i = \frac{1}{2} k x_i^2$ $\quad V_i = 0 \quad X_i = 20$ cm

$m = 20$ g

$E_f = U_{sf} + K_f = \frac{1}{2} m V_{f1}^2 \quad X_f = 0$

Stage 2

$E_i = U_{gi} + K_i = mgh + \frac{1}{2} m V_{i2}^2 \quad V_{i2} = V_{f1}$

$E_f = K_f = \frac{1}{2} m V_f^2$

Apply conservation of energy (obtain 2 equations)

(1) $\frac{1}{2} k X_i^2 = \frac{1}{2} m V_{f1}^2$ $\quad$ (2) $mgh + \frac{1}{2} m V_{i2}^2 = \frac{1}{2} m V_{f2}^2$

Stage 2: $V_{i2}^2 = V_{ix}^2 + \cancel{V_{iy}^2}^{0} = V_{ix}^2$

$$V_{f2}^2 = V_{fx}^2 + V_{fy}^2 = V_{ix}^2 + V_{fy}^2$$

$\Rightarrow mgh = \frac{1}{2} m V_{fy}^2 \Rightarrow V_{fy}^2 = 2gh$

kinematics on Δy

$V_{fy} = \cancel{V_{iy}}^{0} - g\Delta t = -g\Delta t \Rightarrow V_{fy}^2 = g^2 \Delta t^2$

Sub into energy result

$\Rightarrow \Delta t^2 = \frac{2h}{g}$

kinematics on Δx

$\Delta x = V_{ix}\Delta t + \cancel{\frac{1}{2} a \Delta t^2}^{0} \Rightarrow R = V_{ix}\Delta t$ or $R^2 = V_{ix}^2 \Delta t^2$

$\Rightarrow V_{ix}^2 = \frac{R^2}{\Delta t^2} = V_{f1}^2$

Stage 1:

$k x_i^2 = m V_{f1}^2 = m \frac{R^2}{\Delta t^2} \Rightarrow k = m \frac{R^2}{x_i^2 \Delta t^2} = m \frac{g R^2}{2h x_i^2}$

$k = (20g) \frac{(9.8 \, m/s^2)(5.0m)}{2(1.5m)(20 cm)} = \frac{5}{3}(0.2 \, kg) \frac{(9.8 \, m/s^2)}{(0.2 m)} = \frac{5}{3}(9.8)$

$= 16.33 \frac{N}{m}$