(VO) Ryan's Space

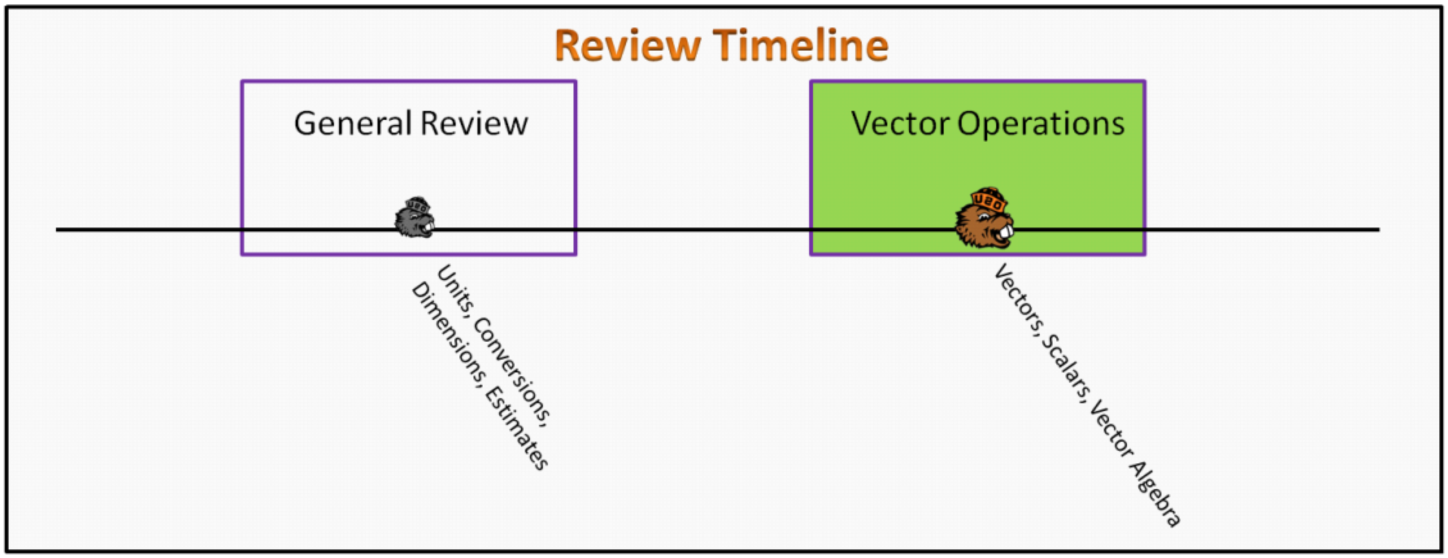
Monday, January 22, 2018 5:44 PM

Vector Operations

Foundation Stage (VO.2)

lecture 1

Vectors, scalars, vector algebra



Textbook Chapters

- **BoxSand** :: KC videos ([Vector representations and vector algebra](#))
- **Giancoli** (Physics Principles with Applications 7th) :: 3-1 ; 3-2 ; 3-3 ; 3-4
- **Knight** (College Physics : A strategic approach 3rd) :: 1.5 ; 3.1 ; 3.2 ; 3.3
- **Knight** (Physics for Scientists and Engineers 4th) :: 1.3 ; 3.1 ; 3.2 ; 3.3 ; 3.4

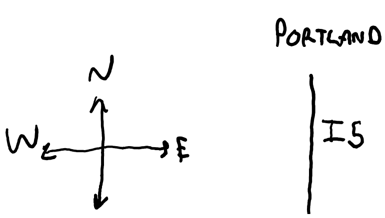
Warm up

VO.2-1

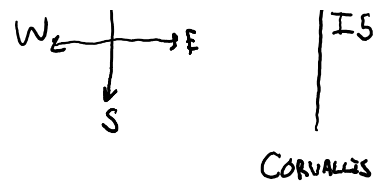
Description: Perform a unit conversion for a vector.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: While driving to Portland from Corvallis on I5, you notice your speed is 60 mph. What is your velocity in m/s?



$\vec{V} = 60$ mph , NORTH



Selected Learning Objectives

1. Identify if a physical quantity is a vector or a scalar.
2. Describe a vector using the descriptive, physical, and mathematical representation.
3. Convert between polar(magnitude and direction) and Cartesian coordinates (x & y).
4. Represent vectors using many standard notational representations, e.g. <x,y>; $\hat{x} + \hat{y}$; and $\hat{\imath} + \hat{\jmath}$.
5. Translate between the descriptive and physical representations of a vector.
6. Translate between the physical and mathematical representations of a vector.
7. Multiply a vector by a scalar in the mathematical and physical representations.
8. Add vectors using the physical representation with the head-to-tail method or parallelogram method when tail-to-tail.
9. Add vectors using the mathematical representation.
10. Subtract vectors with the tools of vector addition.
11. Subtract vectors using the physical representation with the tail-to-tail method.
12. Subtract vectors using the mathematical representation.
13. Determine the summation (net) of a set of more than two vectors.
14. Manipulate vector equations using the rules of algebra.
15. (UPMF) Select between using vector algebra and geometrical methods when finding distances.
16. Recognize that multiplication and division of two vectors are not possible operations - advanced operations of dot and cross products will be presented when the physical situation warrants it.

Key Terms

- Coordinate system
- Vector
- Scalar
- Vector addition
- Vector subtraction
- Magnitude

Key Equations

Mathematical representation of vector addition

$$\vec{A} + \vec{B} = \vec{C}$$

$\vec{C}$ is often referred to as the resultant vector

$$\boxed{\begin{aligned} \langle A_x, A_y\rangle + \langle B_x, B_y\rangle &= \langle C_x, C_y\rangle \\ \langle A_x + B_x, A_y + B_y\rangle &= \langle C_x, C_y\rangle \end{aligned}}$$

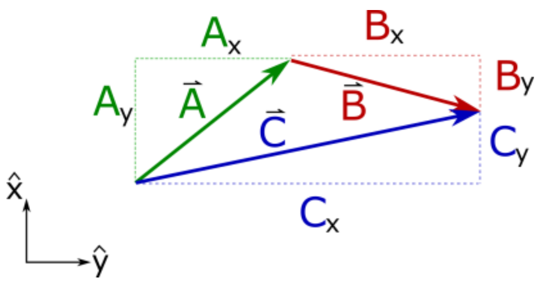
$$A_x + B_x = C_x$$

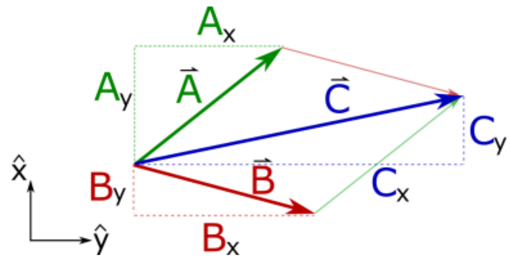
$$A_y + B_y = C_y$$

Physical representation of vector addition

Head-to-tail method

Tail-to-tail method





Mathematical representation of vector subtraction

$\vec{A} - \vec{B} = \vec{C}$

$\vec{C}$ is often referred to as the resultant vector

$$\langle A_x , A_y \rangle - \langle B_x , B_y \rangle = \langle C_x , C_y \rangle$$

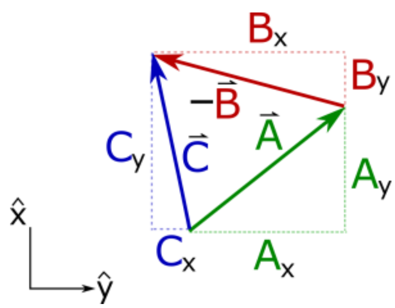
$$\langle A_x - B_x , A_y - B_y \rangle = \langle C_x , C_y \rangle$$

$$A_x - B_x = C_x$$

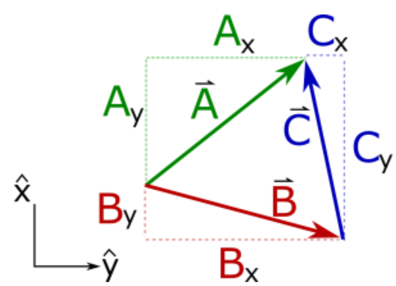
$$A_y - B_y = C_y$$

Physical representation of vector subtraction

Head-to-tail method



Tail-to-tail method (parallelogram method)



Mathematical representation of multiplying a vector by a scalar

$n\vec{A} = \vec{B}$

$$n\langle A_x , A_y \rangle = \langle B_x , B_y \rangle$$

$$\langle nA_x , nA_y \rangle = \langle B_x , B_y \rangle$$

$$nA_x = B_x$$

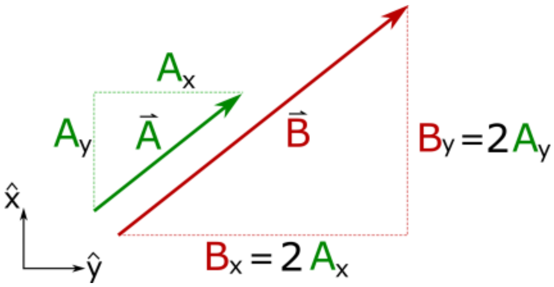
$$nA_y = B_y$$

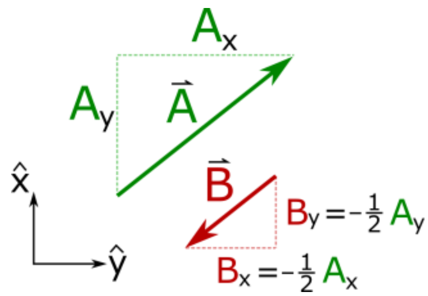
* n is any real number
* n can be positive or negative

Physical representation of multiplying a vector by a scalar

Example 1: n = 2

Example 2: n = -1/2





Key Concepts

- Scalars are quantities that are described with a single number, and that number can be positive or negative.
- Vectors are quantities that are described with a set of positive or negative numbers which can be expressed in component form (Cartesian) or magnitude and direction form (polar).
- A coordinate system must be chosen when a vector is written in the mathematical representation.
- Right triangle trigonometry is commonly used to find components of vectors.
- Vectors typically simplify an analysis compared to using a geometrical approach.

Act I: Vectors and Scalars

Questions

VO.2-2:

Description: Identify which quantities are vectors and which are scalars. (3 minutes)

Learning Objectives: [1]

Problem Statement: Which of the following are vector quantities?

S (1) Speed.
V (2) Velocity.
S (3) Temperature.
S (4) Volume.
S (5) Distance.
S (6) Time.
V (7) Force.
V (8) Position.

VO.2-3:

Description: Calculate a velocity and report the result in correct vector notation. (4 minutes + 2 minutes + 3 minutes)

Learning Objectives: [1,2,3,4]

Problem Statement: Benny the beaver waddles at a constant pace from Weniger Hall to Reser Stadium (1 km). The trip takes him 600 seconds.

(a) What is Benny's velocity?

(1) 6 kph

(2) 1.67 m/s
(3) 3.73 mph
(4) 3.23 knots
(5) None of the above.

$$\frac{1000\text{ m}}{600\text{ s}} = 1.67\text{ m/s}$$

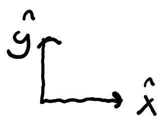
↑ SPEED ... ALSO NEED DIRECTION FOR VELOCITY

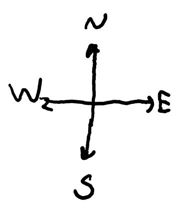
(b) Which one of these correctly differentiates vectors and scalars?

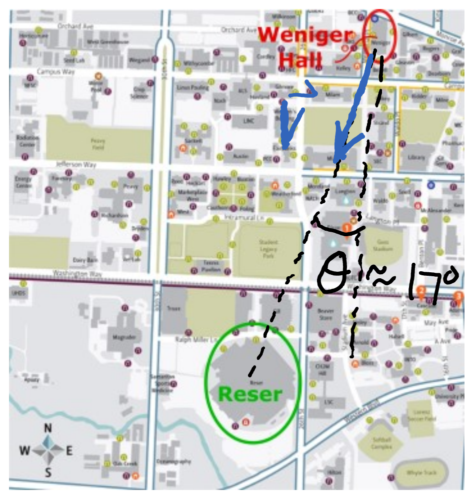
? (1) A vector is bigger.
NOT PHYSICS (2) A scalar involves fish.
T (3) A vector is multiple pieces of information.
NOT PHYSICS (4) Vectors involve diseases.
F (5) Scalars are always positive.

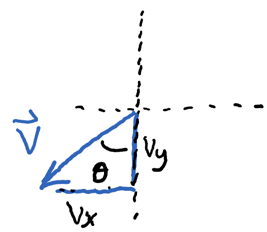
(c) Considering part (a) of this problem, which of the following expressions correctly states Benny's velocity using a standard coordinate system?

(1) 6 kph, 17° East of South
(2) 6 kph, 17° West of South
(3) 6 kph, 73° South of West
(4) <-1.75, -5.74> kph
(5) <1.75, 5.74> kph
(6) -1.75 kph $\hat{x}$ - 5.74 kph $\hat{y}$
(7) 1.75 kph $\hat{x}$ + 5.74 kph $\hat{y}$









$$\sin\theta = \frac{V_x}{|\vec{V}|} \qquad \cos\theta = \frac{V_y}{|\vec{V}|}$$

$$V_x = -|\vec{V}|\sin\theta \qquad V_y = -|\vec{V}|\cos\theta$$

"AD HOC"

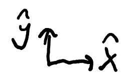
VO.2-4:

Description: Given a component form of a vector, convert to polar form. (6 minutes)

Learning Objectives: [2,3,4,5,6]

Problem Statement: The velocity of a cheetah is found to be <15, -10> m/s. What is the speed and direction of the cheetah's motion? Assume a standard coordinate system.

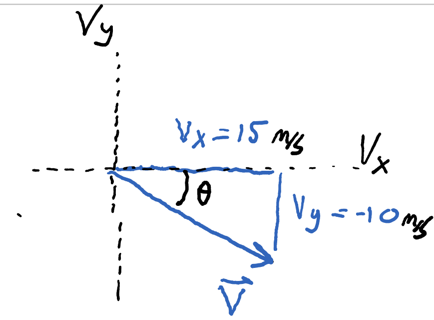
(1) 18 m/s, 33.7° from + $\hat{x}$ towards $-\hat{y}$
(2) 9 m/s, 44.4° from + $\hat{x}$ towards $-\hat{y}$
(3) 21 m/s, 12.0° from - $\hat{x}$ towards $+\hat{y}$
(4) 13.5 m/s, 33.3° from - $\hat{x}$ towards $-\hat{y}$
(5) 17.1 m/s, 58.7° from + $\hat{x}$ towards $+\hat{y}$



SPEED

$$|\vec{V}| = \sqrt{15^2 + (-10)^2}\ m/s$$

$$|\vec{V}| \approx 18\ m/s$$



$$\tan\theta = \frac{|V_y|}{|V_x|}$$

$$\theta = \tan^{-1}\left(\frac{10}{15}\right) \approx 33.7°$$

VO.2-5:

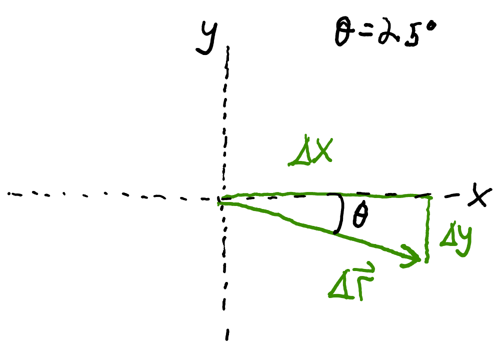
Description: Given a vector in polar form, convert to component form. (6 minutes)

Learning Objectives: [2,3,4,5,6]

$\Delta\vec{r}$

Problem Statement: Your change in position while riding a bike is 15 miles at an angle of 25° from + $\hat{x}$ to $-\hat{y}$. Find the change in position in component form.

$\theta = 25°$



$$\sin\theta = \frac{\Delta y}{|\Delta\vec{r}|} \qquad \cos\theta = \frac{\Delta x}{|\Delta\vec{r}|}$$

$$\Delta y = -|\Delta\vec{r}|\sin\theta \approx -6.34\ mi \qquad \Delta x = |\Delta\vec{r}|\cos\theta \approx 13.6\ mi$$

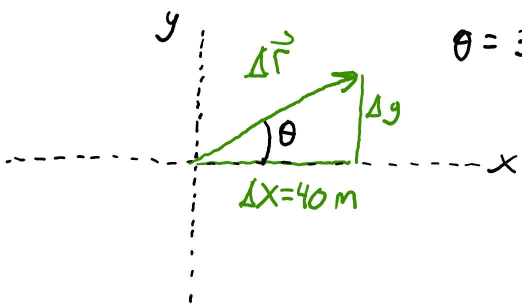
$$\boxed{\Delta\vec{r} = \langle 13.6, -6.34\rangle\ mi}$$

VO.2-6:

Description: Find the magnitude of a vector. (6 minutes)

Learning Objectives: [2,6]

Problem Statement: The change in position vector for an African Swallow you are studying is determined to be in a direction 36.87° from + $\hat{x}$ to + $\hat{y}$. The x-component is 40 m. What is the magnitude of the displacement in meters.



$\theta = 36.87°$

$$\cos\theta = \frac{\Delta x}{|\Delta\vec{r}|}$$

$$|\Delta\vec{r}| = \frac{\Delta x}{\cos\theta}$$

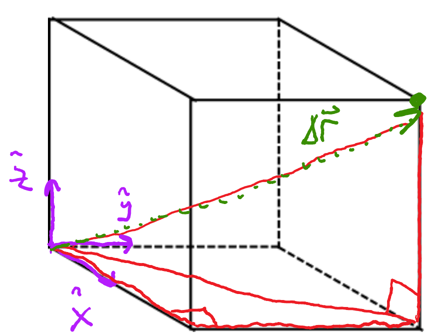
$$|\Delta\vec{r}| \approx 50.0\ m$$

VO.2-7:

Description: Apply vector algebra to simplify a challenging geometrical problem in 3D. (4 minutes)

Learning Objectives: [5,6,15]

Problem Statement: Start at the corner of a cube, of side length 1 m, and find the distance to the furthest point that still resides on the cube. Your answer should include units.



GEOMETRY

TOO MUCH EFFORT

VECTORS

$$\Delta\vec{r} = \langle 1,1,1\rangle\ m$$

$$|\Delta\vec{r}| = \sqrt{1^2+1^2+1^2}\ m$$

$$\boxed{|\Delta\vec{r}| = \sqrt{3}\ m = 1.73\ m}$$

VO.2-8:

Description: Apply the rules of vector addition and subtraction to construct a vector equation. (4 minutes)

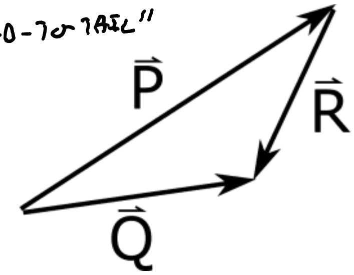
Learning Objectives: [2,6,8,10,11,14]

Problem Statement: Which of the following vector equations correctly describes the relationship among the vectors shown in the figure?

F (1) $\vec{P} + \vec{Q} = \vec{R}$
F (2) $\vec{P} = \vec{Q} + \vec{R}$
T (3) $\vec{P} + \vec{R} = \vec{Q}$
T (4) $\vec{P} = \vec{Q} - \vec{R}$
F (5) $\vec{P} + \vec{Q} + \vec{R} = \vec{0}$
F (6) None of the equations 1-5 are correct.

FROM FIGURE: "HEAD-TO-TAIL"

$\vec{P} + \vec{R} = \vec{Q}$

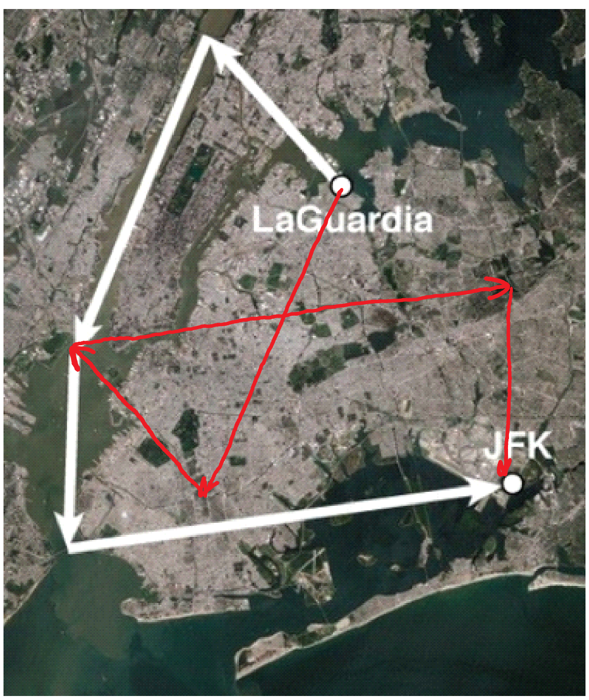


VO.2-9:

Description: Use physical representation of vectors to do vector algebra. (4 minutes)

Learning Objectives: [2,6,8,14]

Problem Statement: You take off from LaGuardia airport in a little airplane. First you fly **NW** for 5 miles. At the George Washington bridge, you turn **SSW** for 10 miles. Above the Statue of Liberty, you turn **S** towards the Verrazano-Narrows bridge, 5 miles ahead. Over the bridge you turn **ENE** for a landing at JFK airport, 10 miles ahead.



SAME LOCATION

COMMUTATIVE LAW OF ADDITION

$\Delta\vec{r}_1 + \vec{r}_2 = \Delta\vec{r}_2 + \Delta\vec{r}_1$

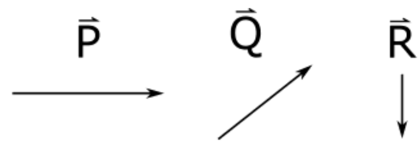
Suppose that instead of flying **NW-SSW-S-ENE**, you had started at LaGuardia but followed the same vectors in a different sequence: **SSW-NW-ENE-S**. Mark on the map where this path would have led you.

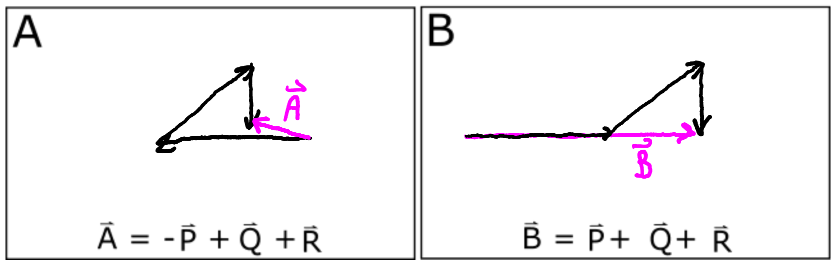
VO.2-10:

Description: Use physical representation of vectors to do vector algebra. (4 minutes)

Learning Objectives: [2,6,8,10,11,14]

Problem Statement: Three vectors, labeled $\vec{\mathbf{P}}$, $\vec{\mathbf{Q}}$, and $\vec{\mathbf{R}}$, are shown. The length of each vector is given in arbitrary units.





In each space provided above, construct a drawing of the indicated combinations of the vectors and rank the magnitude of these resultant vectors.

$\boxed{|\vec{B}| > |\vec{A}|}$

Conceptual questions for discussion

1. Is it possible to add or subtract two vectors of unequal magnitude and get a zero for the magnitude of the resultant vector? Is it possible to add or subtract three vectors of equal magnitude and get zero for the magnitude of the resultant vector?
2. Which of the following two statements is more appropriate?
   i. Two position vectors are added using the head-to-tail method because positions are vector quantities.
   ii. Position is a vector quantity because two positions are added using the head-to-tail method.
3. A situation may be described using different sets of coordinate axes having different orientations (e.g. A velocity vector can be described using a standard coordinate system, or the same velocity vector can be described using a coordinate system where the $\hat{v}_x$ and $\hat{v}_y$ axes are rotated by some angle relative to the standard coordinate system). Which of the following does not depend on the orientation of the axes?
   i. The value of a scalar.
   ii. A vector.
   iii. Components of a vector.
   iv. The magnitude of a vector.
4. Is it possible to add a scalar to a vector? If so, demonstrate this process using a physical representation. If not, explain.

Hints

VO.2-1: No hints.

VO.2-2: No hints.

VO.2-3: No hints.

VO.2-4: Draw a physical representation of the velocity and label the known sides and angles.

VO.2-5: Draw a physical representation of the change in position and label the known sides and angles.

VO.2-6: Draw a physical representation of the displacement and label the known sides and angles.

VO.2-7: The magnitude of a vector in three dimensions is the square root of the summation of the square of all three components of the vector.

VO.2-8: Which vectors are head to tail?...think vector addition.

VO.2-9: Don't try to trace the new flight path in your head, sketch the new path on a piece of paper while being careful to keep the same length and direction of each vector.

VO.2-10: No hints.