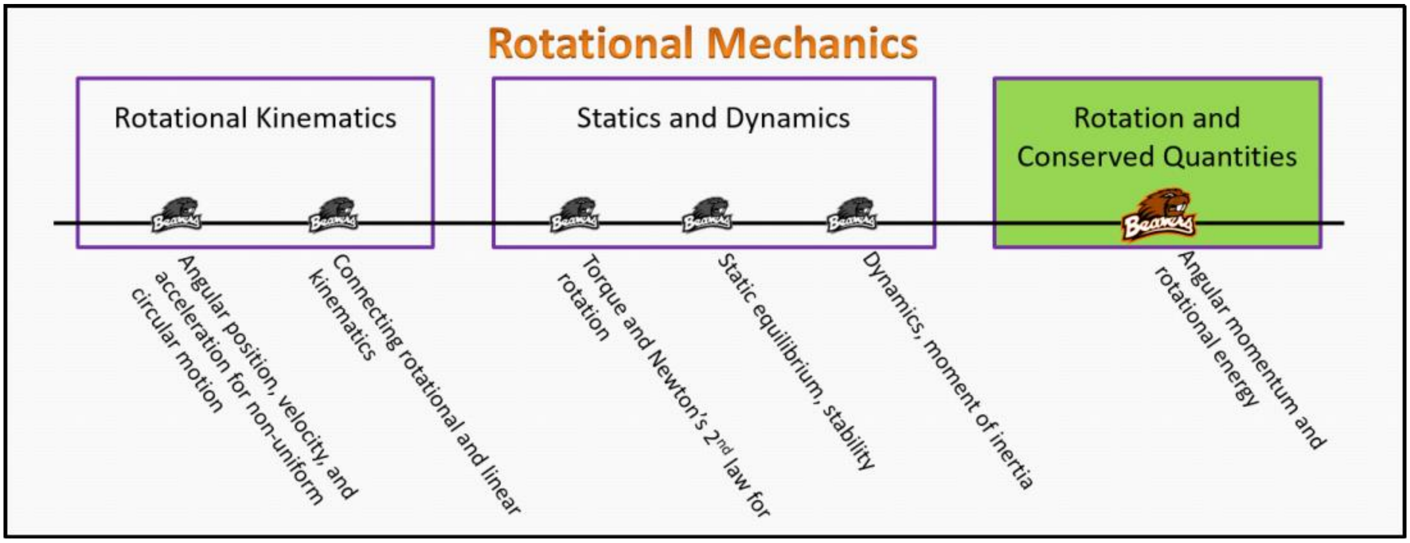
(RC.2.L1) Ryan's Space

Monday, January 22, 2018 5:44 PM

Statics and Dynamics
Foundation Stage (RC.2.L1)

lecture 1
Angular momentum and rotational energy



Textbook Chapters (* Calculus version)

- **BoxSand** :: KC videos (statics & dynamics)
- **Knight** (College Physics : A strategic approach 3rd) :: 7.5 ; 7.6
- ***Knight** (Physics for Scientists and Engineers 4th) :: 12.6 ; 12.7
- **Giancoli** (Physics Principles with Applications 7th) :: 8-5 ; 8-6

Warm up

RC.2.L1-1:

Description: Apply Newton's 2nd law for rotation given torques and moment of inertia to solve for angular acceleration.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: Consider a system with a moment of inertia of 4 kg·m^2 with one counter-clockwise torque about reference axis **o** of magnitude 12 N·m, and two clockwise torques about reference axis **o** of magnitudes 8 N·m and 2 N·m. What is the angular acceleration of this system?

Selected Learning Objectives

1. Coming soon to a lecture template near you.

Key Terms

- Rotational kinetic energy
- Angular momentum
- Conservation of angular momentum
- Zero angular impulse approximation

Key Equations

Key Concepts

- Recall that the net force acting on a system caused the center of mass of the system to accelerate. For a given net force, the magnitude of the acceleration was scaled by the mass of the system. Similarly, Newton's 2nd law for rotation tells us that a net torque acting on a system about a reference axis will cause an angular acceleration. For a given net torque, the magnitude of the angular acceleration is scaled by a new quantity known as the moment of inertia. Thus the moment of inertia is analogous to mass in the sense that both scale the magnitude of acceleration/angular-acceleration given a net force/net torque.
- The moment of inertia depends on the reference axis and on the distribution of the mass about that reference axis. For example, the moment of inertia about a reference axis orientated vertically going through your head to the floor is larger if you stretch your arms out (more mass further away from axis) compared to if you kept your arms at your side.
- For objects that are not point-like particles about a reference axis, you can break the object up into multiple point particles to approximate the moment of inertia of the object.
- A force analysis and a torque analysis can be used together to analysis a scenario, just remember that the net force relates to the acceleration of the center of mass and the net torque relates to the angular acceleration about a reference axis.

Questions

Act I: Angular momentum

RC.2.L1-2:

Description: Identify equilibrium status of given systems. (2 minutes + 2 minutes)

Learning Objectives: [1, 12, 13]

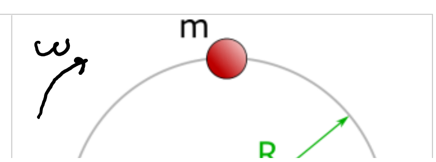
Problem Statement: Four point particles each of mass m are fixed to a negligible mass wire bent into a circle of radius R as shown below. If the masses are spinning clockwise around the center at a constant 60 RPM, what is the angular momentum of the 4-mass-system?

$\frac{60\ \text{REV}}{\text{MIN}} \times \frac{1\ \text{MIN}}{60\ \text{SEC}} = 1 \frac{\text{REV}}{\text{S}} \longleftrightarrow \omega = 2\pi f$

(1) 0 **m·R**^{2}
(2) -4 **m·R**^{2}
(3) -25.1 **m·R**^{2}
(4) 60 **m·R**^{2}
(5) -240 **m·R**^{2}

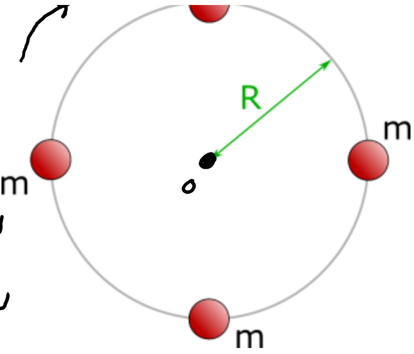
SYSTEM: 4 MASSES

$L_{sys,o} = \sum L_o$



(2) -4 $m{\cdot}R^2$
(3) -25.1 $m{\cdot}R^2$
(4) 60 $m{\cdot}R^2$
(5) -240 $m{\cdot}R^2$
(6) 1510 $m{\cdot}R^2$

$$L_{sys,o} = \sum L_o$$
$$= L_{1,o} + L_{2,o} + L_{3,o} + L_{4,o}$$
$$= -I_{1,o}\omega - I_{2,o}\omega - I_{3,o}\omega - I_{4,o}\omega$$
$$= -mR^2 - mR^2\omega - mR^2\omega - mR^2\omega$$
$$= -4mR^2\omega$$
$$= -4mR^2(2\pi f)$$
$$\approx -25.1\, mR^2$$



RC.2.L1-3:

Description: Conceptual application of torque analysis and force analysis to determine motion of object. (5 minutes + 3 minutes + 2 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Recall the impulse-momentum theorem: $\Sigma\vec{\bar{F}}_{ext}\,\Delta t = \Delta\vec{p}_{sys}$. Which of the following expressions could be angular impulse - angular momentum theorem?

(1) $\Sigma\vec{\bar{F}}_{ext}\,\Delta t = \Delta\vec{p}_{sys}$

(2) $\Sigma\vec{\bar{\tau}}_{ext,o}\,\Delta t = \Delta\vec{p}_{sys}$

(3) $\Sigma\vec{\bar{\tau}}_{ext,o}\,\Delta t = \Delta\vec{\omega}_{sys,o}$

(4) $\Sigma\vec{\bar{\tau}}_{ext,o}\,\Delta t = \Delta\vec{L}_{sys,o}$

RC.2.L1-4:

Description: Conceptual application of torque analysis and force analysis to determine motion of object. (5 minutes + 3 minutes + 2 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Dizzy the dog is ice skating spinning in circles when she stands up on her hind legs. Assume frictionless.

(a) With the figure dog-skater as the system, is there any net external torque?

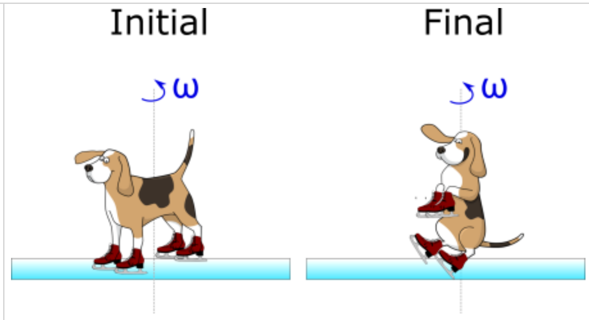
$\cancel{\Sigma \tau_{EXT,cm}}^{0} \Delta t = \Delta L_{sys,cm}$

(1) Yes
(2) No

$\Delta L_{sys,cm} = 0$

(b) When the dog stands up, her angular velocity ______________ because her ______________ .

(1) increases ; mass decreases
(2) decreases ; moment of inertia increases
(3) increases ; moment of inertia decreases
(4) increases ; moment of inertia increases



$L_i = L_F$

$I_i \omega_i = I_F \omega_F$

RC.2.L1-5:

Description: Conceptual application of torque analysis and force analysis to determine motion of object. (5 minutes + 3 minutes + 2 minutes)

Learning Objectives: [1, 12, 13]

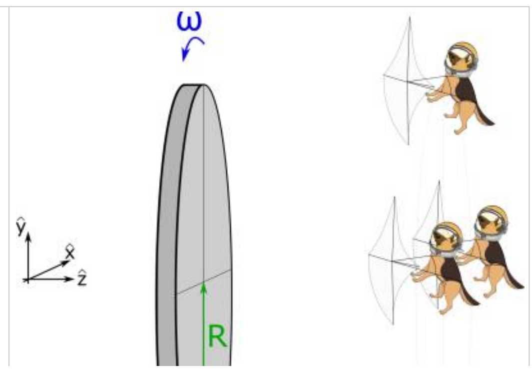
Problem Statement: A uniform 600 kg disk with a radius of 5 meters is in space rotating counter-clockwise at 1 RPM about the **z**-axis. Four 30-kg space-parachuting dogs are floating perpendicular to the face of the disk and have velocities in the $-\hat{z}$ direction as shown below. The four dogs land simultaneously on the rotating space disk. There is sufficient friction between the dogs and the disk that when they land they rotate with the disk and do not slide relative to the disk's surface.

(a) For a system of just the space disk, is there a net external torque about the z-axis as the dogs land?

YES, FROM DOGS

(b) For a system of disk+dogs, is there any net external torque about the z-axis as the dogs land?

NO, ISOLATED



A uniform 600 kg disk with a radius of 5 meters is in space rotating counter-clockwise at 1 RPM about the **z**-axis. Four 30-kg space-parachuting dogs are floating perpendicular to the face of the disk and have velocities in the $-\hat{\mathbf{z}}$ direction as shown below. The four dogs land simultaneously on the rotating space disk. There is sufficient friction between the dogs and the disk that when they land they rotate with the disk and do not slide relative to the disk's surface.

$$1\,\frac{\text{REV}}{\text{MIN}} \times \frac{1\,\text{min}}{60\,\text{sec}} = \frac{1}{60}\,\text{Hz} \equiv f$$

(c) What is the angular moment about the z-axis of the disk+dogs system before the dogs land?	(d) How does the angular moment of the disk+dogs system before and after the dogs land compare?	(e) What is the moment of inertia of the system about the z-axis after the dogs land?
$L_{i\,sys,z} = \cancel{L_{i\,Dogs}}^{0} + L_{i\,DISK}$ $= I_{DISK,z}\,\omega$ $= \frac{1}{2}M_{DISK}R^2\,2\pi f$	(1) $\mathbf{L}_{i,z} > \mathbf{L}_{f,z}$ (2) $\mathbf{L}_{i,z} < \mathbf{L}_{f,z}$ (3) $\mathbf{L}_{i,z} = \mathbf{L}_{f,z}$	$I_f = I_{Dogs} + I_{DISK}$ $= 4M_{Dog}R^2 + \frac{1}{2}M_{DISK}R^2$

$$\boxed{\Sigma L_{i,z} \approx 785.4\ \frac{kg\,m^2}{s}}$$

$$\cancel{\Sigma \tau_{Ext,z}}^{0}\,\Delta t = \Delta L_{sys,z}$$

$$0 = \Delta L_{sys,z}$$

(f) What is the final rotational rate (in RPM) after the dogs land?

$$L_i = L_f$$

$$785.4\ \frac{kg\,m^2}{s} = 4L_{Dog\,f} + L_{DISK\,f}$$

$$= 4 I_{Dog}\omega_f + I_{Disk}\omega_f$$

$$= 4 M_{Dog}R^2\omega_f + \frac{1}{2}M_{DISK}R^2\omega_f$$

$$= 3000\,\omega_f + 7500\,\omega_f$$

$$785.4\ \frac{kg\,m^2}{s} = 10500\,\omega_f$$

$$\omega_f = 0.0748\ \frac{\text{RAD}}{s}$$

$$\omega = 2\pi f$$

$$\frac{0.0748}{2\pi} = f$$

$$f = 0.011905\ \text{Hz}$$

$$\boxed{f \approx 0.714\ \text{RPM}}$$

(g) The center of mass of the disk is initially stationary. What happens to the center of mass of the rotating disk after the dogs land?

F (1) The center of mass velocity doesn't change because linear momentum is conserved..
F (2) The center of mass velocity doesn't change because kinetic energy is conserved.
T (2) The center of mass begins to move in the $-\hat{\mathbf{z}}$ direction because there is a net external force acting on it when the dogs land.
F (3) The center of mass begins to move in the $-\hat{\mathbf{z}}$ direction because there is a net external torque acting on it when the dogs land.

$$\cancel{\sum \bar{F}_{EXT_z}}^{\,0}\, \Delta t = \Delta P_{sys,z}$$

<u>Sys Dog + DISK</u>

$$\Delta P_{sys_z} = 0$$

$$P_{i_z} = P_{f_z}$$

$$P_{i\,DOGS,z} + \cancel{P_{i\,DISK\,z}}^{\,0} = P_{f\,(DISK+DOGS)\,z}$$

$$-4M_{Dog} V_{i\,Dog\,z} = (4M_{Dog} + M_{DISK}) V_{f\,z}$$

$$\boxed{V_{f\,z} = -\frac{4M_{Dog}}{(4M_{Dog} + M_{DISK})} V_{i\,Dog\,z}}$$

NEED TO KNOW

RC.2.L1-6:

Description: Conceptual application of torque analysis and force analysis to determine motion of object. (5 minutes + 3 minutes + 2 minutes)

Learning Objectives: [1, 12, 13]

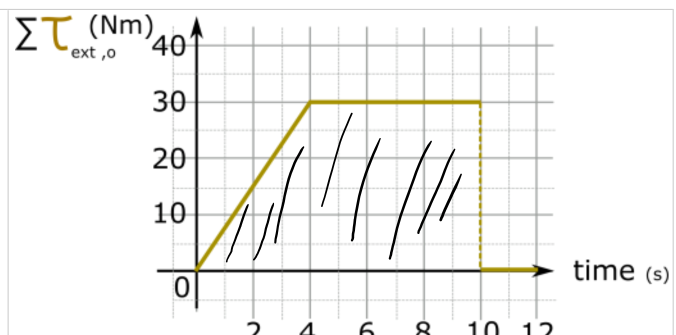
Problem Statement: A metal smith using a bench grinder applies a net torque as a function of time from their newly forged knife on a grinding wheel shown by the graph below. The grinder is not plugged in so it spinning freely.

(a) If the grinder's wheel started with an initial angular momentum of -300 kg·m^2/s , what is its final angular moment after 10 seconds?

AREA = ΔL

$240\ \frac{kgm^2}{s} = L_f - L_i$

$$\boxed{L_f = -60\ \frac{kgm^2}{s}}$$



(b) The moment of inertia is a constant 0.550 k·gm^2 , what is the final angular velocity of the grinder wheel?

$L_f = I_f \omega_f$

$-60 = (0.25)\, \omega_f$

$\boxed{\omega_f = 109 \ \frac{\text{RAD}}{\text{S}}}$

Act II: Rotational energy

RC.2.L1-7:

Description: Calculate moment of inertia for system of point particles. (4 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: We wish to explore the graphical representation of rotational work.

(a) The rotational work due to a torque is the area under a

(1) force vs position graph.
(2) position vs force graph.
(3) torque vs angular position graph.
(4) torque vs time graph.

(b) Consider your answer to part (a). What can this area also represent?

(1) Rotational momentum.
(2) Change in rotational momentum.
(3) Rotational kinetic energy.
(4) Change in rotational kinetic energy.
(5) Change in energy.

RC.2.L1-8:

Description: Rank moment of inertias. (3 minutes + 3 minute)

Learning Objectives: [1, 12, 13]

Problem Statement: Match the following energy flow diagrams with the given scenario.

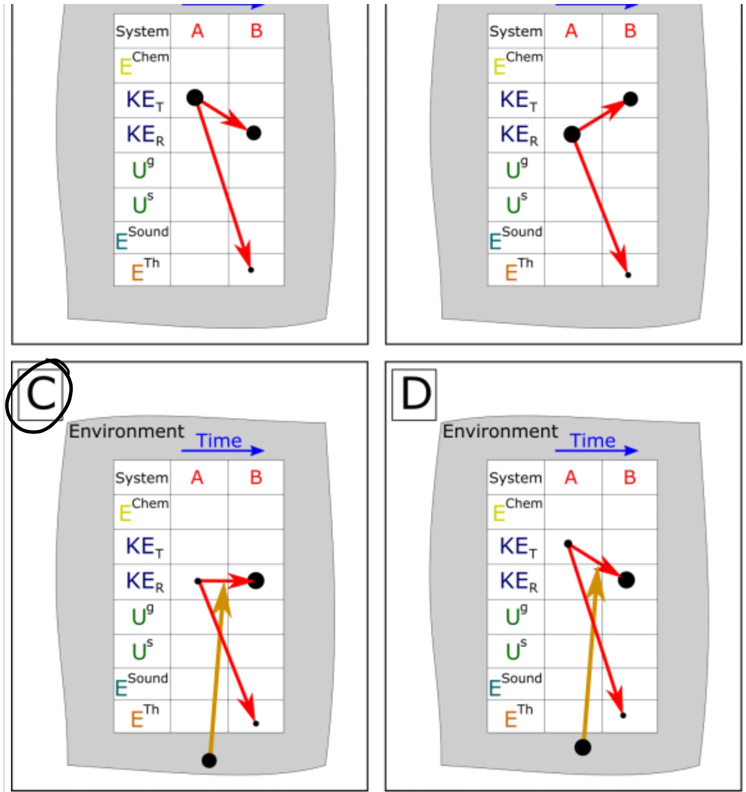
(a) While fishing, you hook into a killer Oregon steelhead and it begins taking line, swimming directly away from you. Snapshots were taken when the fish was at the following

locations:

A: The moment after the fish bit and slowly begins swimming away.

B: Some time later when the fishy is still hooked and swimming away quickly.

System: fishing reel



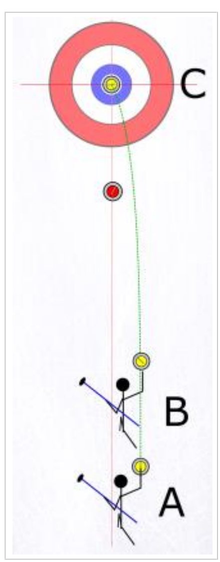
(b) Consider the game of curling. Three snapshots are taken when the stone is at locations **A**, **B**, and **C**. The dashed green line shows the trajectory of the stone's center of mass. Snapshots are taken when the stone is at the following locations:

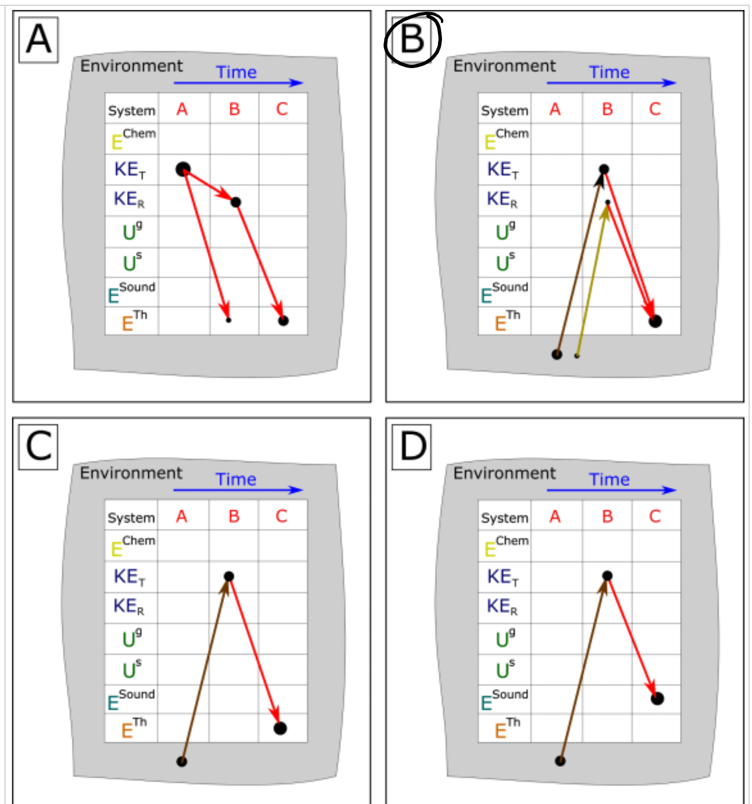
A: The person and the stone are at rest.

B: The stone has just left the person's hand rotating counter-clockwise.

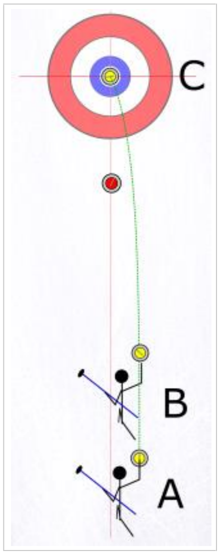
C: The stone has stopped on the button.

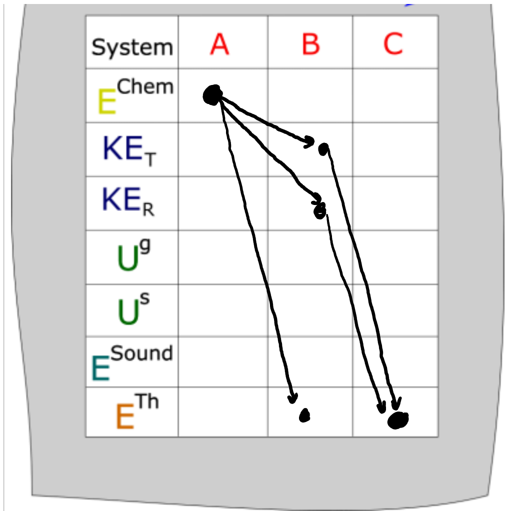
System: stone + surface





(c) Consider the same scenario as part (b), but this time the system is stone+surface+person. Fill out the energy flow diagram below.

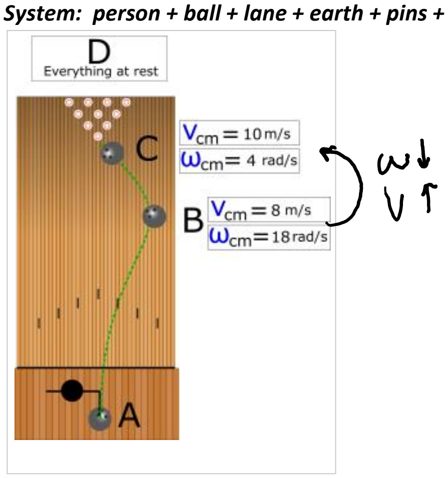


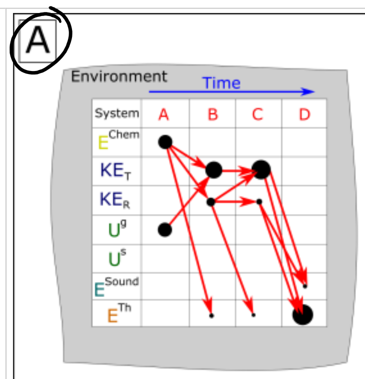


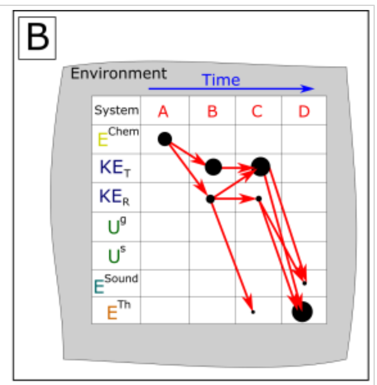
(d) A round of bowling begins with a bowling ball in a person's hand raised backwards above the level ground. Snapshots are taken when the ball is at the following locations:

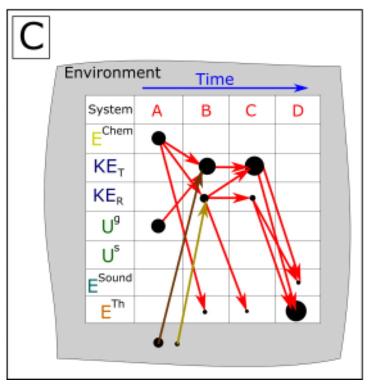
- **A:** The ball and person are at rest with the ball at some height above the ground cocked backwards.
- **B:** The ball is two thirds down the lane with the velocity of the center of mass and angular velocity about the center of mass given.
- **C:** The ball is about to hit the pins with the velocity of the center of mass and angular velocity about the center of mass given.
- **D:** The ball and pins are at rest.

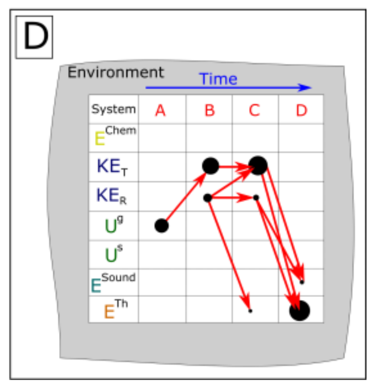
System: person + ball + lane + earth + pins + atmosphere











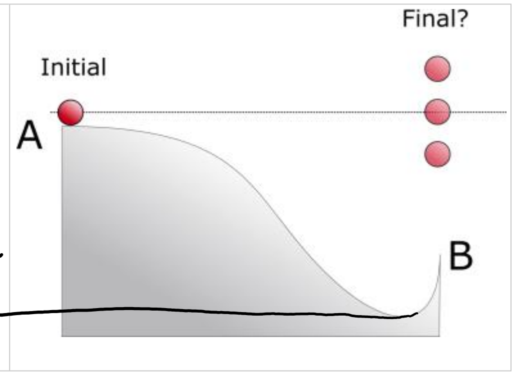
RC.2.L1-9:

Description: Rank moment of inertias. (4 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A solid sphere rolls without slipping along a track shaped as shown below. It starts from rest at location **A** and is moving vertically when it leaves the track at location **B**. At its highest point in the air, the sphere will be ___________ location **A**.

(1) above
(2) below
(3) at the same height as



QUICK

$U^g \to KE_T$, $U^g \to KE_R$

@ MAX $KE_T = 0$

$KE_R \neq 0$

so $U_g \to U_g$, $U_g \to KE_R$

$y=0$ $\hat{y}\uparrow$

CONS

$$\cancel{KE_{Ti}}^{0} + \cancel{KE_{Ri}}^{0} + U^g_i + \cancel{W_{EXT}}^{0} = \cancel{KE_{Tf}}^{0} + KE_{Rf} + U^g_f$$

$$U^g_i = \underbrace{KE_{Rf} + U^g_f}$$

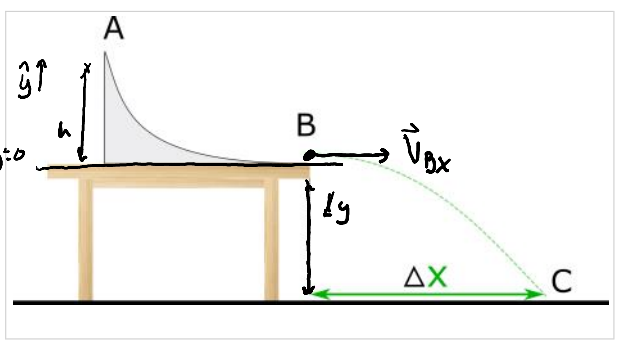
$$\text{so } U^g_i > U^g_f$$

RC.2.L1-10:

Description: Rotational dynamics problem solving for moment of inertia. (2 minutes + 2 minutes + 6 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A solid sphere, solid disk, and hollow ring of equivalent mass and radius are rolled without slipping down a ramp on a table. They both start from rest at **A**, then fly horizontally off the edge of the table at **B**. Rank the horizontal distance, Δ**x**, each travels during their flight in the air to the moment they land at **C**.



$\hat{y}\uparrow$ h $y=0$ $\vec{V}_{Bx}$ Δy

$\underline{B \to C}$

$$\Delta x = V_{ix}\Delta t + \tfrac{1}{2}\cancel{a_x}^{0}\Delta t^2$$

$$\Delta X = V_{ix}\Delta t$$

so

$$\Delta X \propto V_{Bx}$$

$\underline{B \to C}$

$$\Delta y = \cancel{V_{iy}}^{0}\Delta t + \tfrac{1}{2}a_y\Delta t^2$$

$$\Delta y = -\tfrac{1}{2}g\Delta t^2$$

SAME Δy SAME g

So Δt = CONST.

$$\cancel{KE_{TA}}^{0} + \cancel{KE_{RA}}^{0} + U^g_A + \cancel{W_{EXT}}^{0} = KE_{TB} + KE_{RB} + \cancel{U^g_B}^{0}$$

$$\cancel{KE_{TA}} + \cancel{KE_{RA}} + U^g_A + \cancel{W_{ext}} = KE_{TB} + KE_{RB} + \cancel{U^g_B}$$

$$U^g_A = KE_{TB} + KE_{RB}$$

$$mgh = \frac{1}{2} m V_B^2 + \frac{1}{2} I_{cm} \omega^2$$

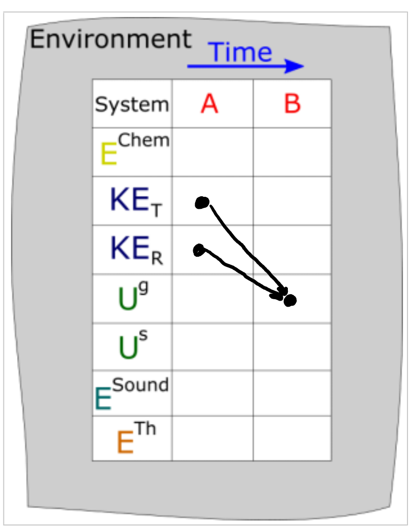
RC.2.L1-11:

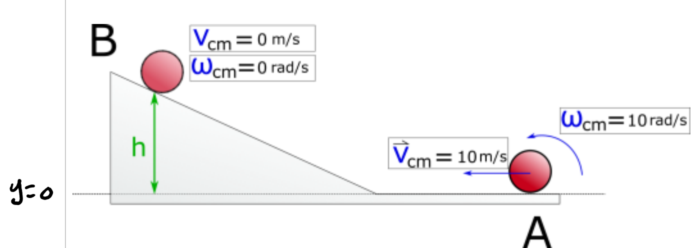
Description: Rotational dynamics and translational static problem solving for linear and rotational quantities. (4 minutes + 5 minutes + ...)

Learning Objectives: [1, 12, 13]

Problem Statement: A thin hoop with a radius of 2 meters is moving so that its center of mass is initially moving at 20 m/s while also rolling without slipping at 10 rad/s along a horizontal surface. It rolls up an incline, coming to rest as shown below.

(a) Fill out an energy flow diagram for the hoop+earth system. Ignore friction.





(b) Below is the work-energy equation with all of the forms of energy we have discussed up to this point. Which of the following energy terms are zero?

0 0 0 0 0

(b) Below is the work-energy equation with all of the forms of energy we have discussed up to this point. Which of the following energy terms are zero?

$$\Delta E^{Chem} + \Delta KE_T + \cancel{\Delta KE_R}^{0} + \Delta U^g + \Delta U^s + \Delta E^{Sound} \cancel{\Delta E^{Th}}^{0} = \cancel{W_{ext}}^{0}$$

$$KE_{Ti} + KE_{Ri} + \cancel{U^g_i}^{0} = \cancel{KE_{TF}}^{0} + \cancel{KE_{RF}}^{0} + U^g_f$$

(c) A thin 20 gram hoop with a radius of 2 meters is moving so that its center of mass is initially moving at 20 m/s while also rolling without slipping at 10 rad/s along a horizontal surface. **Use your simplified work-energy equation from part (b) to find the vertical height up the incline the hoop reaches when it stops.**

$$\frac{1}{2}MV_{cm}^2 + \underline{\frac{1}{2}I_{cm}\omega^2} = Mgy_f$$

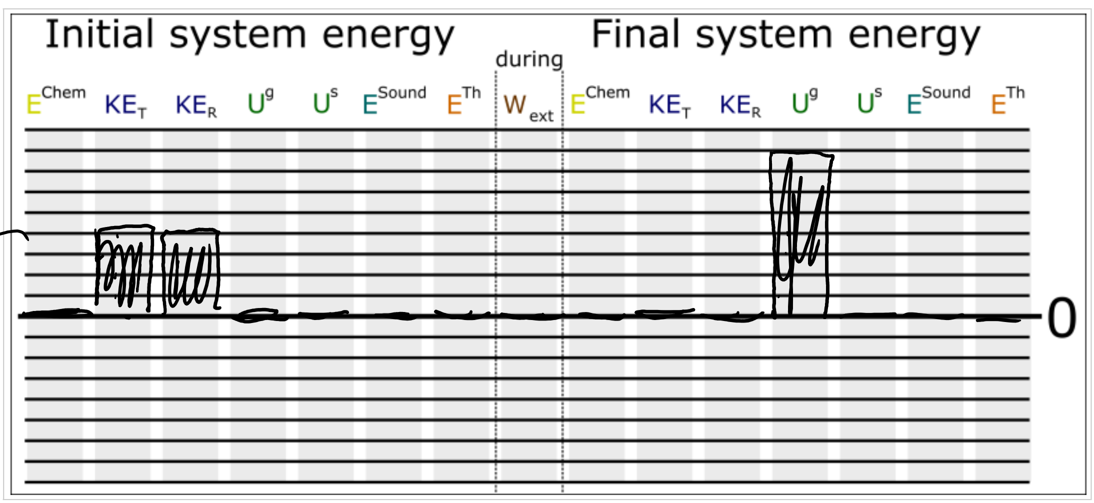
$$\frac{1}{2}\cancel{M}V_{cm}^2 + \frac{1}{2}(\cancel{M}R^2)\omega^2 = \cancel{M}gy_f$$

$$\frac{1}{2}V_{cm}^2 + \frac{1}{2}R^2\omega^2 = gy_f$$

$$\boxed{y_f = 40.8\ m}$$

(d) Another useful physical representation to show energy transformations and transfers is an energy bar chart. Fill in the energy bar chart below for this scenario.

4J



$\frac{1}{2}mv_i^2 = 4J$

$\frac{1}{2}mR^2\omega^2 = 4J$

RC.2.L1-12:

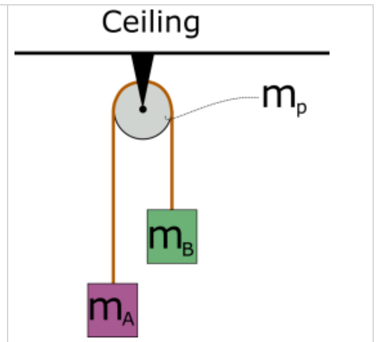
Description: Rotational dynamics and translational dynamics problem solving for linear and rotational quantities. (3 minutes + 3 minutes + 4 minutes + 8 minutes + ...)

Learning Objectives: [1, 12, 13]

Problem Statement: Two unequal masses are connected across a solid disk pulley. A few moments after releasing them from rest, the speed of one of the masses is recorded.

(a) If the pulley is replaced with one of a smaller radius but equivalent mass, and the experiment is repeated, the same hanging mass be going _______________ after the same amount of time elapses?

(1) faster
(2) slower
(3) the same speed



(b) Below shows semi-simplified work-energy equations. Which one is the correct simplification for this scenario?

(c) What is the final speed of one of the hanging masses after it goes through a magnitude of displacement of 0.50 meters? The radius of the solid disk pulley is 0.20 meters and has a mass of 0.100 kg.

RC.2.L1-13:

Description: Rotational dynamics and translational dynamics problem solving for linear and rotational quantities. (3 minutes + 3 minutes + 4 minutes + 8 minutes + ...)

Learning Objectives: [1, 12, 13]

Problem Statement: A figure skater stands on one spot on ice (assumed frictionless) and spins around with her arms extended. When she pulls in her arms, she reduces her moment of inertia and her angular speed increases. Compared to her initial rotational kinetic energy, her rotational kinetic energy after she has pulled in her arms must be:

(1) the same because no external work is done on her.
(2) larger because she's rotating faster.
(3) smaller because her moment of inertia is smaller.

$$\cancelto{0}{\Sigma \bar{\tau}_{ext,cm}} \Delta t = \Delta L_{cm}$$

$$0 = \Delta L_{cm}$$

$$L_{cm} = \text{const.}$$

$$L_{cm} = I_{cm} \omega$$

$$KE_R = \frac{1}{2} I \omega^2$$

Both change

$I \downarrow \quad \omega \uparrow$

$$KE_R = \frac{1}{2} L \omega$$

const

$KE_R \propto \omega$ so w/ $\omega \uparrow$

$KE_R \propto \omega$ so w/ $\omega \uparrow$

$KE_R \uparrow$

WHAT? WHERE DID THE EXTRA ENERGY COME FROM?

$\boxed{E^{CHEM} \rightarrow KE_R}$

Conceptual questions for discussion

1. Do you agree with the following statement: Every object has only one moment of inertia. Support your answer with examples.
2. What happens to the moment of inertia about Earth's rotational axis, if anything, when tall building are built near the equator?
3. Is there a location on Earth that you could build a tall building without affecting the moment of inertia of Earth about its rotational axis?
4. Use your knowledge of Newton's laws of motion for rotation to explain why sports car use wheels of lighter mass than normal wheels.
5. Tight rope walkers often use long poles as seen in the cartoon below. Use your knowledge of Newton's laws of motion for rotation to explain why it is advantage to use such poles when walking along a tight rope.

Hints

RC.2.L1-1: No hints.

RC.2.L1-2: No hints.

RC.2.L1-3: Draw FBDs for each spool to determine which one lands first and where A lands relative to the X.

RC.2.L1-4: The "**r**" in the moment of inertia equation is the perpendicular distance from the reference axis.

RC.2.L1-5: No hints.

RC.2.L1-6: Break the T-handle up into multiple pieces of equal mass and look at their "**r**" distance away from the reference axis.

RC.2.L1-7: No hints.

RC.2.L1-8: No hints.

RC.2.L1-9: No hints.